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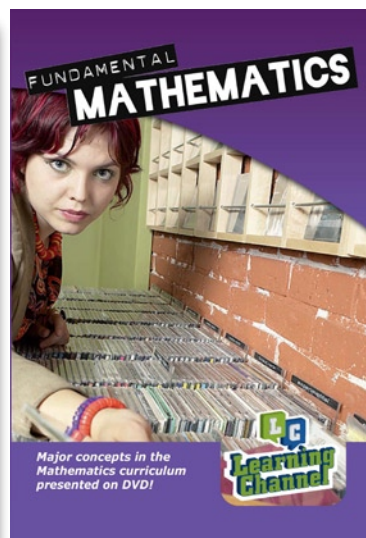
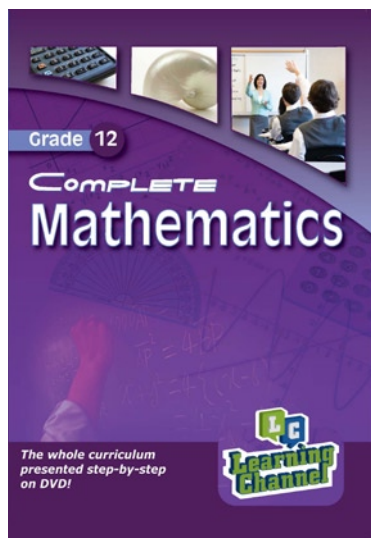
Nasionale Seniorsertifikaat

Graad 12

Wiskunde

Vraestel 2

MEMORANDUM



$$1.1 \quad M = \left(\frac{-1+3}{2}; \frac{3+(-1)}{2} \right) \checkmark$$

$$M = (1; 1) \checkmark \quad (2)$$

$$1.2 \quad \text{Middelpunt FH} = \left(\frac{4+(-2)}{2}; \frac{4+(-2)}{2} \right) \checkmark$$

$$= (1; 1) \checkmark$$

$\therefore$ middelpunt FH = middelpunt EG

$\therefore$ lyne halveer mekaar. (2)

$$1.3 \quad m_{EG} = \frac{3-(-1)}{-1-3} = \frac{4}{-4} = -1 \checkmark$$

$$m_{FH} = \frac{4-(-2)}{4-(-2)} = \frac{6}{6} = 1 \checkmark$$

$\therefore m_{EG} \times m_{FH} = -1$

$\Rightarrow EG \perp FH \checkmark$

$\therefore$ gebruik 1.2, hoeklyn van EFGH halveer teen $90^\circ \checkmark$

$\therefore$ EFGH is 'n ruit.

OF

$$\text{lengte}_{EH} = \sqrt{(-1-(-2))^2 + (3-(-2))^2}$$

$$= \sqrt{1+25}$$

$$= \sqrt{26} \checkmark$$

$$\text{lengte}_{EF} = \sqrt{(-1-4)^2 + (3-4)^2}$$

$$= \sqrt{25+1}$$

$$= \sqrt{26} \checkmark$$

$\therefore$ gebruik 1.2, EFGH is 'n parallelogram (hoeklyne halveer mekaar) $\checkmark$

Maar EH = EF $\checkmark$

$\therefore$ EFGH is 'n ruit. (4)

$$1.4 \quad m_{EG} = -1 \text{ uit bogenoemde } \checkmark$$

$\therefore y = -x + c$ Vervang $(-1; 3)$: $\checkmark$

$$3 = -(-1) + c$$

$$2 = c$$

$\therefore y = -x + 2 \checkmark$ (3)

$$1.5 \quad y = -x + \quad \text{Laat } x = \frac{5}{2}.$$

$$y = -\frac{5}{2} + 2 \checkmark$$

$$y = -\frac{1}{2} \checkmark$$

$\therefore \left(\frac{5}{2}; -\frac{3}{4} \right)$ lê nie op die lyn nie. $\checkmark$ (3)

$$1.6 \quad m_{FH} = \frac{4-(-1)}{4-3} = \frac{5}{1} = 5 \checkmark$$

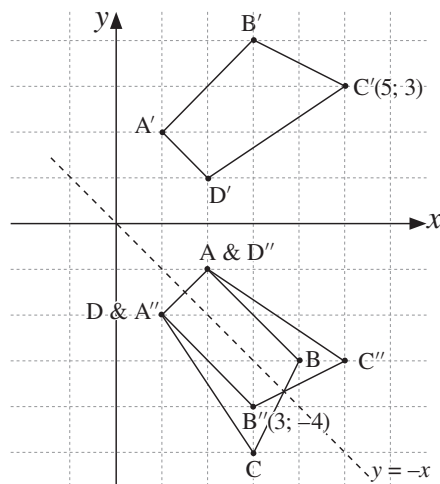
$\therefore \tan \alpha = 5$

$$A = \tan^{-1}(5) \checkmark$$

$$= 78,69^\circ \checkmark \quad (3)$$

- 1.7
$$\begin{aligned} \text{lengte}_{\text{EG}} &= \sqrt{(3 - (-1))^2 + (-1 - 3)^2} \\ &= \sqrt{32} \checkmark \\ \text{lengte}_{\text{HM}} &= \sqrt{(1 - (-2))^2 + (1 - (-2))^2} \\ &= \sqrt{18} \checkmark \\ \therefore \text{oppervlakte } \triangle \text{EGH} &= \frac{1}{2} \sqrt{18} \times \sqrt{32} \checkmark \\ &= 5,05 \text{ eenhede}^2 \checkmark \end{aligned} \quad (4)$$
- 1.8 $G \rightarrow H$ 5 terug 1 af
 $\therefore E \rightarrow P$ is dieselfde $\checkmark$
 $\therefore P(-6; 2) \checkmark$ (of soortgelyk) (2)
- 2.1 $\triangle \text{EDC}$ is reghoekig by C (raaklyn, radius)
 $\text{ED}^2 = \text{EC}^2 + \text{DC}^2 \checkmark$
 $13^2 = 12^2 + \text{DC}^2$
 $25 = \text{DC}^2$
 $5 = \text{DC} \checkmark \quad (2)$
- 2.2 $\text{DC}^2 = (a - 1)^2 + (2 - (-1))^2 \checkmark$
 $25 = a^2 - 2a + 1 + 9 \checkmark$
 $0 = a^2 - 2a - 15 \checkmark$
 $a = -3; a = 5 \checkmark$
 Deur inspeksie, vir die skets $a = 5. \checkmark \quad (5)$
- 2.3 $m_{\text{DC}} = \frac{2 - (-1)}{5 - 1} = \frac{3}{4} \checkmark$
 $m_{\text{tangent}} = \frac{-4}{3} \checkmark$
 $y = \frac{-4}{3}x + c$ Vervang $(1; -1)$:
 $-1 = \frac{-4}{3}x + c \checkmark$
 $\frac{1}{3} = c \checkmark$
 $y = \frac{-4}{3}x + \frac{1}{3} \quad (4)$
- 2.4 y-as is raaklyn aan sirkel by A.
 $\therefore \text{AD}$ is horisontaal $\therefore A(0; 2) \checkmark \checkmark$ (inspeksie) (2)
- 2.5 $(x - a)^2 + (y - b)^2 = c^2 \checkmark$
 Vervang $(1; -1)$ en $(0; 2)$:
 $(0 - 1)^2 + (2 - (-1))^2 = c^2 \checkmark \checkmark$
 $10 = c^2 \checkmark$
 $(x - 1)^2 + (y + 1)^2 = 10 \checkmark \quad (5)$
-

3.1
en
3.2



(10)

3.3 $P(x; y)$

$$\rightarrow P'(-x; -y)$$

$$\rightarrow \text{finale beeld } (-45x; -45y) \checkmark\checkmark$$

(2)

3.4 Lineêre faktor $\frac{4}{5}$

$$\Rightarrow \text{oppervlaktefaktor } \frac{16}{25} \checkmark$$

$$\therefore \text{oppervlakte} = \frac{16}{25}p \text{ eenhede}^2 \checkmark$$

(2)

4.1 $(x \cos \theta - y \sin \theta; x \sin \theta + y \cos \theta) \checkmark$

$$= (6 \cos 60^\circ - 3 \sin 60^\circ; 6 \sin 60^\circ + 3 \cos 60^\circ) \checkmark\checkmark$$

$$= \left(\frac{6 - 3\sqrt{3}}{2}, \frac{3 + 6\sqrt{3}}{2} \right) \checkmark\checkmark$$

OF

$$= \left(3 - \frac{3\sqrt{3}}{2}, 3\sqrt{3} + \frac{3}{2} \right)$$

(5)

4.2 $(6; 3) \rightarrow (3; 6) \rightarrow (3; -6) \checkmark\checkmark$

(2)

5.1 5.1.1 $\tan \theta = \frac{y}{x} = \frac{a}{b} \checkmark$

(1)

$$5.1.2 \quad \cos(-\theta) = \cos \theta = \frac{x}{r} \checkmark\checkmark$$

$$r = \sqrt{a^2 + b^2} \text{ (Pythagoras)} \checkmark$$

$$\therefore \cos(-\theta) = \frac{a}{\sqrt{a^2 + b^2}} \checkmark$$

(4)

5.2 5.2.1 $\cos 53^\circ$

$$= \sin(90^\circ - 53^\circ)$$

$$= \sin 37^\circ \checkmark$$

$$= k \checkmark$$

(2)

5.2.2 $\sin(-74^\circ)$

$$= -\sin 74^\circ \checkmark$$

$$= -2\sin 37^\circ \cos 37^\circ \checkmark\checkmark$$

$$= -2k \sqrt{1 - k^2} \checkmark$$

(4)

5.3 5.3.1 LK

$$= \frac{\sin \alpha \sin 2\alpha}{\cos \alpha} + \cos 2\alpha \checkmark$$

$$= \frac{\sin \alpha 2 \sin \alpha \cos \alpha}{\cos \alpha} + 1 - 2 \sin^2 \alpha \checkmark \checkmark$$

$$= 1 \checkmark$$

$$= \text{RK}$$

(4)

5.3.2 LK

$$= \frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin (x - 90^\circ) \cos (90^\circ - 2x)}{\sin (x - 360^\circ)}$$

$$= \frac{-\sin 54^\circ}{\cos 36^\circ} - \frac{(-\cos x) \sin 2x}{\sin x} \checkmark \checkmark \checkmark \checkmark$$

$$= \frac{-\sin 54^\circ}{\sin 54^\circ} - \frac{\cos x \cdot 2 \sin x \cos x}{\sin x} \checkmark \checkmark$$

$$= -1 + 2 \cos^2 x \checkmark$$

$$= \cos 2x \checkmark$$

$$= \text{RK}$$

(8)

$$5.4 \quad 3 \cos^2 x + 5 \sin x = 3$$

$$3(1 - \sin^2 x) + 5 \sin x = 3 \checkmark$$

$$3 - 3 \sin^2 x + 5 \sin x = 3$$

$$0 = 3 \sin^2 x - 5 \sin x$$

$$0 = \sin x (3 \sin x - 5) \checkmark$$

$$\sin x = 0 \text{ or } \sin x = \frac{5}{3} \checkmark \checkmark$$

$$\therefore x = 0^\circ + n180^\circ \text{ of } x \text{ is ongedefinieerd } \checkmark \checkmark$$

$$(n \in \mathbb{Z})$$

(6)

$$6.1 \quad \widehat{ACB} = 110^\circ - 50^\circ$$

$$= 60^\circ \checkmark$$

$$\therefore AB^2 = 150^2 + 260^2 - (2 \cdot 150 \cdot 260) \cos 60^\circ \checkmark$$

$$AB^2 = 51\,100 \checkmark$$

$$\therefore AB = 226,05 \checkmark$$

$$AB = 226 \text{ km } \checkmark$$

(5)

$$6.2 \quad 6.2.1 \quad \widehat{CDB} = 180^\circ - (\theta + 30^\circ) \checkmark$$

(1)

$$6.2.2 \quad \text{In } \triangle ABC:$$

$$\tan \theta = \frac{p}{CB}$$

$$CB \tan \theta = p \dots\dots\dots(i) \checkmark$$

$$\text{In } \triangle CBD: \frac{CB}{\sin [180^\circ - (\theta + 30^\circ)]} = \frac{8}{\sin \theta} \checkmark$$

$$\frac{CB}{(\sin \theta + 30^\circ)} = \frac{8}{\sin \theta} \checkmark$$

$$CB = \frac{8(\sin \theta + 30^\circ)}{\sin \theta} \dots\dots\dots(ii) \checkmark$$

Kombineer (i) en (ii):

$$p = \frac{8 \sin (\theta + 30^\circ)}{\sin \theta} \cdot \tan \theta$$

$$p = \frac{8 \sin (\theta + 30^\circ)}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} \checkmark$$

$$p = \frac{8(\sin \theta + 30^\circ)}{\cos \theta} \checkmark$$

(6)

7.1 $\sin 2x = \cos (x - 45^\circ)$

$$\sin 2x = \sin [90^\circ - (x - 45^\circ)]$$

$$\sin 2x = \sin (135^\circ - x) \checkmark$$

$$\therefore 2x = 135^\circ - x + n \cdot 360^\circ \ (n \in \mathbb{Z}) \checkmark$$

$$3x = 135^\circ + n \cdot 360^\circ$$

$$x = 45^\circ + n \cdot 120^\circ \checkmark$$

OF

$$2x = 180^\circ - (135^\circ - x) + n \cdot 360^\circ \ (n \in \mathbb{Z}) \checkmark$$

$$2x = 45^\circ + x + n \cdot 360^\circ$$

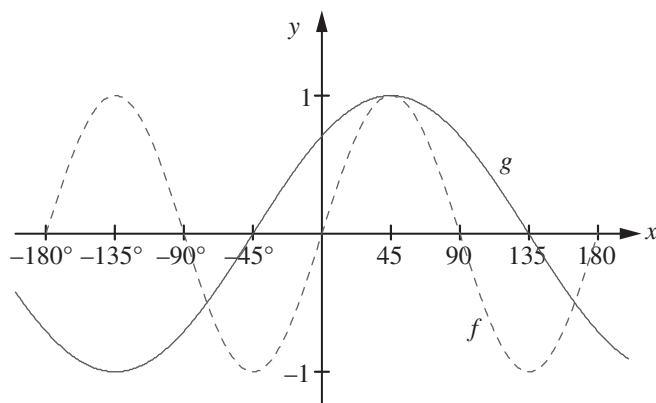
$$x = 45^\circ + n \cdot 360^\circ \checkmark$$

$$\therefore \text{vir } x \in [-180^\circ; 180^\circ]$$

$$x = 45^\circ; 165^\circ; -75^\circ \checkmark\checkmark\checkmark$$

(4 × 2) (8)

7.2



$$\checkmark\checkmark\checkmark \text{ vir } g \quad \checkmark\checkmark\checkmark \text{ vir } f$$

(6)

7.3 7.3.1 $g(x) \leq f(x) \text{ vir } [-180^\circ; 90^\circ]$

$$\Rightarrow -180^\circ \leq x \leq -75^\circ \checkmark\checkmark \checkmark$$

(3)

7.3.2 $\frac{f(x)}{g(x)}$ ongedefinieer $\Rightarrow g(x) = 0 \checkmark$

$$\therefore x = -45^\circ \text{ slegs vir } [-180^\circ; 90^\circ] \checkmark$$

(2)

8.1 8.1.1 $\bar{x} = 65,27 \checkmark\checkmark \checkmark$

Gebruik statist. modus op sakrekenaar of met die hand

(3)

8.1.2 $SD = 8,71 \checkmark\checkmark$

(2)

8.1.3 Boonste grens

$$= 65,27 + 8,71$$

$$= 73,98 \checkmark$$

Onderste grens

$$= 65,27 - 8,71$$

$$= 56,56 \checkmark$$

$$\therefore \text{verwerp } 50; 45; 80$$

$$\text{d.i. 3 sakke moet afgekeur word } \checkmark$$

(3)

8.2 8.2.1 Geordende lys

11 000

12 600

14 200 $Q_1 = 14\,200$

14 500

15 300

Mediaan = 15 350

15 400

16 500

16 800 $Q_3 = 16\,800$

18 600

19 600

 $\therefore$ Minimum = 11 000 ✓

Onderste kwartiel = 14 200 ✓

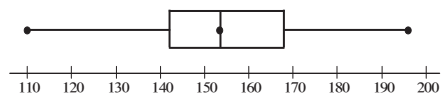
Mediaan = 15 350 ✓

Boonste kwartiel = 16 800 ✓

Maksimum = 19 600 ✓

(5)

8.2.2 Skaal in 100e



✓✓✓

(3)

8.2.3 Maksimum per dag = $\frac{19\,600}{28} = 700$ pasiënte per dag ✓

Minimum per dag = $\frac{11\,000}{28} = 392$ pasiënte per dag ✓

(2)

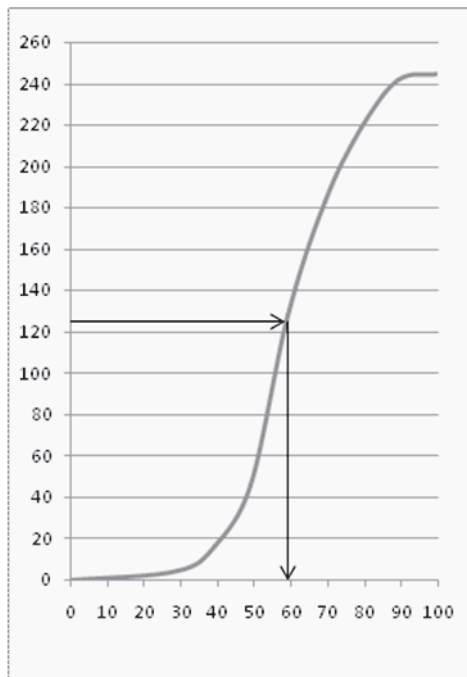
9.1

Punte	F	CF
$20 \leq x \leq 29$	4	4
$30 \leq x \leq 39$	12	16
$40 \leq x \leq 49$	30	46
$50 \leq x \leq 59$	82	128
$60 \leq x \leq 69$	55	183
$70 \leq x \leq 79$	35	218
$80 \leq x \leq 89$	24	242
$90 \leq x \leq 100$	3	245

✓✓

(2)

9.2



✓✓✓✓

(4)

9.3 Mediaan at $\frac{245 + 1}{2} = 123$

Mediaan ongeveer 59 ✓✓

(2)

9.4 Data is gegroepeer, dus is oorspronklike rou data verlore.

∴ gemiddeld of mediaan sal gepas wees. ✓✓

(2)