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National Senior Certificate

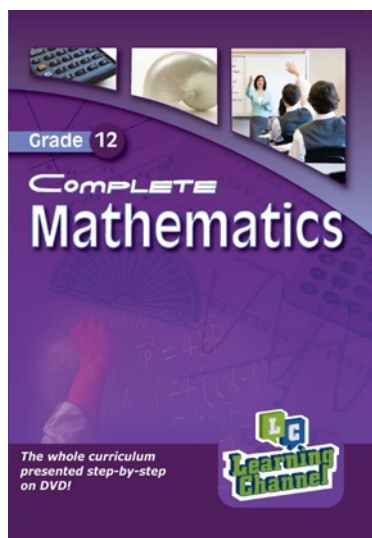
Grade 12

Mathematics

Paper 1

MEMORANDUM

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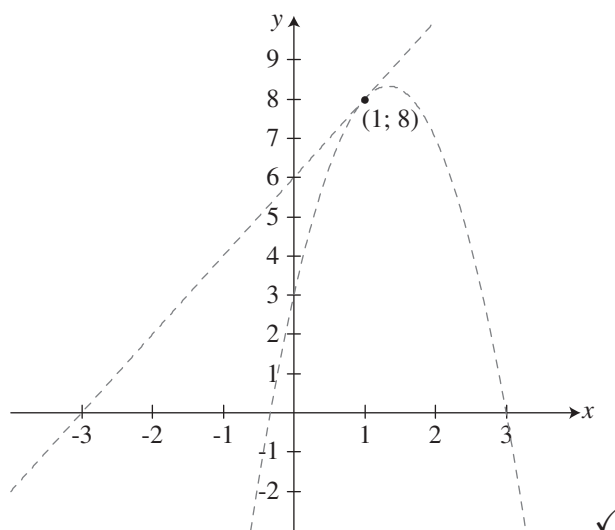
$$\begin{aligned}
 1.1 \quad 1.1.1 \quad & 2x^2 + 7x = 30 \\
 & 2x^2 + 7x - 30 = 0 \checkmark \\
 & (2x - 5)(x + 6) = 0 \checkmark \\
 & \therefore x = 2.5 \text{ or } x = -6 \checkmark\checkmark \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 1.1.2 \quad & 2x(x - 2) - 5 = 0 \\
 & 2x^2 - 4x - 5 = 0 \checkmark \\
 & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 & x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times (2)(-5)}}{2 \times 2} \checkmark \\
 & \therefore x = \frac{4 + \sqrt{56}}{4} \text{ or } x = \frac{4 - \sqrt{56}}{4} \checkmark\checkmark \\
 & \text{Or } x = 2.87 \text{ or } x = -0.87 \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 1.1.3 \quad & 4x^2 + 5x - 2 < 0 \\
 & (4x - 1)(x + 2) < 0 \checkmark\checkmark \\
 & x < \frac{1}{4} \text{ or } x > -2 \checkmark\checkmark \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad 1.2.1 \quad & 2x + 6 - y = 0 \\
 & 2x + 6 = y \\
 & y + 3x^2 = 8x + 3 \\
 & y = -3x^2 + 8x + 3 \checkmark \\
 & \therefore 2x + 6 = -3x^2 + 8x + 3 \checkmark \\
 & 3x^2 - 6x + 3 = 0 \\
 & x^2 - 2x + 1 = 0 \\
 & (x - 1)^2 = 0 \checkmark \\
 & \therefore x = 1 \checkmark \\
 & \therefore y = 2x + 6 = 8 \checkmark \quad (5)
 \end{aligned}$$

1.2.2



(1; 8) is the point of contact between the straight line $y = 2x + 6$ and the parabola $y = -3x^2 + 8x + 3$. There is one point of contact.

$\Rightarrow$ line is a tangent to the curve. ✓✓

(3)

2.1

$$P = \frac{x[1 - (1 + i)^{-n}]}{i} \checkmark$$

$$800\,000 = \frac{7\,500 \left[1 - \left(1 + \frac{\frac{9}{100}}{12} \right)^{-n} \right]}{\frac{\frac{9}{100}}{12}} \checkmark \checkmark$$

$$800\,000 = \frac{7\,500 \left[1 - \left(1 + \frac{3}{400} \right)^{-n} \right]}{\frac{3}{400}}$$

$$800\,000 \times \frac{3}{400} \div 7\,500 = 1 - \left(\frac{403}{400} \right)^{-n}$$

$$\frac{20}{25} = 1 - \left(\frac{403}{400} \right)^{-n} \checkmark$$

$$\left(\frac{403}{400} \right)^{-n} = \frac{5}{25}$$

$$= \frac{1}{5}$$

$$-n = \left[\frac{\log \frac{1}{5}}{\log \frac{403}{400}} \right] \checkmark$$

$$-n = -215,4 \text{ months} \checkmark$$

$\therefore$ It will take 17,95 years to pay back the loan. ✓

(Simplifications at various stages are not necessary.)

(7)

2.2 2.2.1 $A = P(1 - i)^n \checkmark$

$$A = 1\,500\,000 \left(1 - \frac{18}{100} \right)^5 \checkmark$$

$$A = 556\,109,76 \checkmark$$

$\therefore$ It is worth R556 109,76.

(3)

$$2.2.2 \quad A = P(1 - i)^n \checkmark$$

$$A = 1\,500\,000 \left(1 + \frac{4,5}{100}\right)^5 \checkmark$$

$$A = 1\,869\,272,91 \checkmark$$

$\therefore$ Sinking fund required is:

$$R1\,869\,272,91 - R556\,109,76 = R1\,313\,163,15 \checkmark \quad (4)$$

2.2.3

$$F = \frac{x[(1+i)^2 - 1]}{i} \checkmark$$

$$1\,313\,163,15 = \frac{x \left[\left(1 + \frac{6}{100}\right)^5 - 1 \right]}{\frac{6}{100}} \checkmark \checkmark$$

$$\frac{1\,313\,163,15 \times 6 \div 100 \div 12}{\left[\left(1 + \frac{6}{100}\right)^5 - 1 \right]} = x \checkmark$$

$$260\,019,40 = x \checkmark$$

$\therefore$ They will need to pay R260 019,40 per month. (5)

3.1 3.1.1 Quadratic sequence $\checkmark$ (1)

$$\begin{array}{cccccc}
 T_0 & T_1 & T_2 & T_3 & T_4 & T_5 \\
 9 & 9 & 11 & 15 & 21 & 29 \\
 \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \\
 0 & 2 & 4 & 6 & 8 & \\
 \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \\
 2 & 2 & 2 & 2 & &
 \end{array}$$

Second difference = 2

$$\therefore 2a = 2$$

$$a = 1 \checkmark$$

By inspection $T_0 = 9$

$$\therefore c = 9 \checkmark$$

$$\therefore T_n = n^2 + bn + 9$$

$$T_1 = 1^2 + b + 9 = 9$$

$$\therefore b = -1 \checkmark$$

$$\therefore T_n = n^2 - n + 9 \checkmark$$

(or equivalent method) (4)

3.2 Sammy:

$$a = 2 \quad T_4 = a + 3d \checkmark$$

$$= 2 + 3d = 17$$

$$3d = 15$$

$$d = 5 \checkmark$$

$$\therefore T_{20} = a + 19d$$

$$= 2 + 19 \times 5$$

$$= 97$$

$$\therefore x = 97 \checkmark$$

Solly:

$$T_4 = a + 3d = 59$$

$$T_{20} = a + 19d = 107 \checkmark$$

$$16d = 48$$

$$d = 3 \checkmark$$

$$\therefore T_4 = a + 3d = 59$$

$$a + 9 = 59$$

$$a = 50 \checkmark$$

$$\therefore y = 50$$

(6)

4.1 4.1.1 2010

$$109\% \text{ of } 85\,000 = R92\,650 \checkmark$$

$$109\% \text{ of } 92\,650 = R100\,988,50 \checkmark$$

(2)

4.1.2 GP with $a = 85\,000$

$$r = 109\% = 1,09 \checkmark$$

$$T_n = ar^{n-1} \checkmark$$

$$T_{15} = 85\,000(1,09)^{15} \checkmark$$

$$= R309\,611,01 \checkmark$$

(4)

$$4.2 \quad \sum_{k=0}^3 x = \frac{2}{5^k}$$

$$= 2 + 25 + 225 + 2\,125 \checkmark \checkmark$$

$$= \frac{312}{125} (2,496) \checkmark$$

(3)

4.3 4.3.1 GP with $a = 20$

$$r = \frac{3}{4}$$

$$S_n = \frac{a[1-r^n]}{1-r} \checkmark$$

$$S_n = \frac{20 \left[1 - \left(\frac{3}{4} \right)^{10} \right]}{1 - \frac{3}{4}} \checkmark$$

$$= 75,49 \checkmark$$

$$\therefore \text{Depth is } 75,49 \text{ m.}$$

(3)

$$\begin{aligned}
 4.3.2 \quad S_{\infty} &= \frac{a}{1-r} \checkmark \\
 &= \frac{20}{1-\frac{3}{4}} \checkmark \\
 &= 80
 \end{aligned}$$

$$s_{\infty} = 80 \text{ m} \checkmark$$

$\therefore$ Company will never reach the water. $\checkmark$

(4)

$$5.1 \quad 5.1.1 \quad f(x) = \frac{-3}{x-1} + 2$$

asymptotes at $x = 1$; $y = 2$

$$\text{y-intercept: } x = 0 \quad \therefore f(0) = \frac{-3}{0-1} + 2 = 5$$

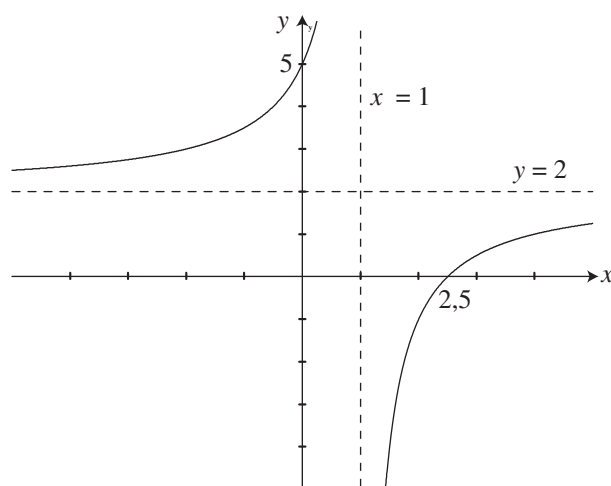
$$\text{x-intercept: } y = 0 \quad \therefore 0 = \frac{-3}{x-1} + 2$$

$$\frac{3}{x-1} = 2$$

$$3 = 2x - 2$$

$$5 = 2x$$

$$\frac{5}{2} = x$$



✓✓✓✓✓

(4)

5.1.2 Axes of symmetry:

$$y = x + c \text{ and } y = -x + c$$

Pass through $(1; 2)$ $\checkmark$

$$\therefore 2 = 1 + c \quad 2 = -1 + c$$

$$1 = c \quad 3 = c$$

$$\therefore y = x + 1 \text{ and } y = -x + 3 \checkmark \checkmark$$

(3)

- 5.2 5.2.1 $y = a(x - x_1)(x - x_2) \checkmark$
 $y = a(x + 1)(x - 3) \checkmark$
 Substitute (0; -1):
 $-1 = a(0 + 1)(0 - 3) \checkmark$
 $-1 = -3a$
 $\frac{1}{3} = a \checkmark$
 $\therefore p(x) = \frac{1}{3}(x + 1)(x - 3)$
 $= \frac{1}{3}(x^2 - 2x - 3)$
 $= \frac{1}{3}x^2 - \frac{2}{3}x - 1 \checkmark$ (5)
- 5.2.2 $q(x) = 12b^x - 4$
 Substitute (3; 0):
 $0 = \frac{1}{2}b^3 - 4 \checkmark$
 $4 = \frac{1}{2}b^3$
 $8 = b^3 \checkmark$
 $2 = b$ (2)
- 5.2.3 Domain: $x \in \mathbb{R} \checkmark$
 Range: $y \in \mathbb{R}; y > -4 \checkmark$ (2)
- 5.2.4 $q(x) = y = \frac{1}{2} \cdot 2^x - 4$
 $q^{-1}(x): x = \frac{1}{2} \cdot 2^y - 4 \checkmark$
 $2x = 2^y - 8$
 $2x + 8 = 2^y \checkmark$
 $\therefore y = \log_2(2x + 8) \checkmark$
 $q^{-1}(x) = \log_2(2x + 8)$ (3)
- 5.2.5 Domain of $q^{-1}(x) = \text{Range of } q(x)$
 $= x \in \mathbb{R}; x > -4 \checkmark$ (1)
- 5.3 5.3.1 $a = -4 \checkmark$ (1)
- 5.3.2 $x = 180^\circ; x = -180^\circ \checkmark$
 $x = 0^\circ \checkmark$ (2)
- 5.3.3 $y = \tan(x + 90^\circ + 45^\circ) + 2 \checkmark$
 $y = \tan(x + 135^\circ) + 2 \checkmark$
 or $y = \tan(x - 45^\circ) + 2$
 by inspection/reduction (2)
-

$$\begin{aligned}
 6.1 \quad f(x) &= \frac{1}{2}x^2 \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{2}(x+h)^2 - \frac{1}{2}x^2}{h} \checkmark \\
 \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - \frac{1}{2}x^2}{h} \checkmark \\
 \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{h(x + \frac{1}{2}h)}{h} \checkmark \\
 f'(x) &= \lim_{h \rightarrow 0} x + \frac{1}{2}h \checkmark \\
 f(x) &= x \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 6.2 \quad 6.2.1 \quad f(x) &= x^4 + \sqrt{x} - \frac{9}{x} \\
 f(x) &= x^4 + x^{\frac{1}{2}} - x^{-1} \checkmark \\
 \therefore f'(x) &= 4x^3 + \frac{1}{2}x^{-\frac{1}{2}} - 9x^{-2} \checkmark \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 6.2.2 \quad y &= t(t+1) \\
 y &= 3x(3x+1) \checkmark \\
 y &= 9x^2 + 3x \checkmark \\
 \therefore \frac{dy}{dx} &= 18x + 3 \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 6.3 \quad 6.3.1 \quad A(x) &= -12x^3 + 12x^2 \\
 A'(x) &= -32x^2 + 24x \checkmark \\
 \therefore 0 &= -32x^2 + 24x \checkmark \\
 0 &= x^2 - 16x \checkmark \\
 0 &= x(x-16) \\
 x &= 0 \text{ or } x = 16 \\
 \therefore \text{Maximum area will be covered after 16 months.} \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 6.3.2 \quad A'(x) &= -\frac{3}{2}x^2 + 24x \\
 A'(1) &= -\frac{3}{2}(1)^2 + 24(1) \checkmark \\
 &= 22\frac{1}{2} \checkmark \\
 \therefore \text{The rate of growth was } 22\frac{1}{2} \text{ m}^2/\text{month one month after the study had} \\
 &\text{begun.}
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 7.1 \quad f(x) &= x^3 - 4x^2 - 3x + 18 \\
 f(3) &= 27 - (4 \times 9) - (3 \times 3) + 18 = 0 \checkmark
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 7.2 \quad 0 &= x^3 - 4x^2 - 3x + 18 \\
 0 &= (x-3)(x^2 - x - 6) \checkmark \\
 0 &= (x-3)(x-3)(x+2) \checkmark \\
 \therefore x\text{-intercepts at } x = 3 \text{ and } x = 2 \checkmark \checkmark
 \end{aligned} \tag{4}$$

7.3 $f'(x) = 3x^2 - 8x - 3 = 0$ for turning points ✓

$\therefore (3x + 1)(x - 3) = 0$

$x = -\frac{1}{3}$ and $x = 3$ ✓✓

$\therefore f(x) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18$

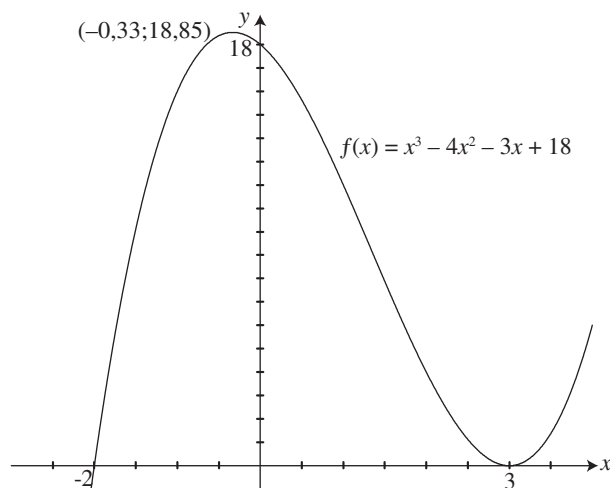
$= \frac{500}{27} (= 18,52)$ ✓

$f(3) = 0$

$\therefore$ Turning points at $\left(-\frac{1}{3}; \frac{500}{27}\right)$ and $(3; 0)$. ✓

(5)

7.4



✓✓ for each of the points

(4)

7.5 $f''(x) = 6x - 8 = 0$ ✓

for point of inflection.

$\therefore x = \frac{8}{6} = \frac{4}{3}$ ✓

(2)

8.1 $V = \text{Area of base} \times \text{Height}$

$2,5 = x^2 h$ ✓

$\frac{2,5}{x^2} = h$ ✓

(2)

8.2 $\text{Area} = 2x^2 + 4xh$ ✓

$= 2x^2 + 4x \times \frac{2,5}{x^2}$ ✓

$= 2x^2 + \frac{10}{x}$ ✓

(3)

8.3 $\text{Area} = 2x^2 + 10x^{-1}$ ✓

$\text{Area} = 4x - 10x^{-2}$ ✓

$4x - \frac{10}{x^2} = 0$ (for minimum) ✓

$4x^3 - 10 = 0$

$x^3 = \frac{10}{4}$

$x = \sqrt[3]{\frac{5}{2}}$ ✓

$x = 1,36$ ✓

(5)

9.1 x = No. of guitars of type A

y = No. of guitars of type B

$$x + y \leq 20 \checkmark$$

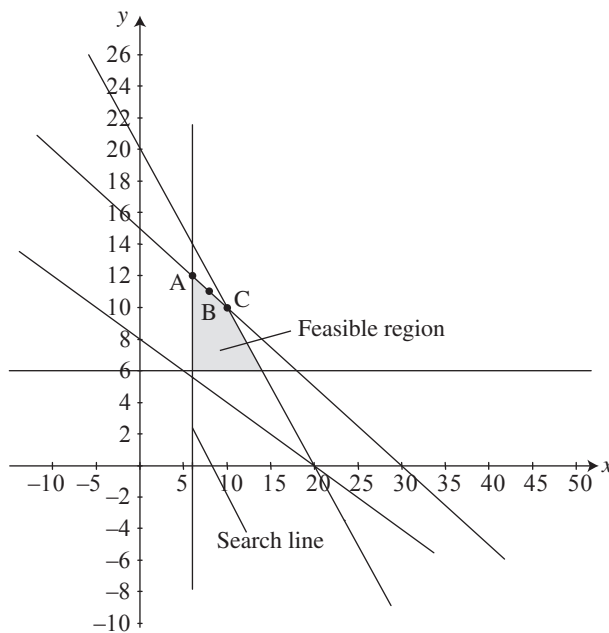
$$1\,500x + 3\,000y \leq 45\,000 \checkmark$$

$$x \geq 6 \checkmark$$

$$y \geq 6 \checkmark$$

(4)

9.2



✓✓✓✓✓

(5)

9.3 $P = 400x + 1\,000y \checkmark$

(1)

9.4 $-400x + P = 1\,000y$

$$-\frac{400}{1\,000}x + \frac{P}{1\,000} = y$$

$$-\frac{2}{5}x + \frac{P}{1\,000} = y \checkmark$$

$\therefore$ Use search line of slope $-\frac{2}{5}$... see sketch. ✓

$\therefore$ Maximum profit for $x = 6$ and $y = 12$ at point A. ✓

(3)

9.5 New profit equation:

$$P = 500x + 1\,000y$$

$$-\frac{500}{1\,000}x + \frac{P}{1\,000} = y$$

$$-\frac{1}{2}x + \frac{P}{1\,000} = y \checkmark$$

This line is parallel to one of the borders of the feasible region. Therefore maximum profit occurs at any whole-number point on this line. ✓

i.e. A(6; 12)

B(8; 11)

C(10; 10) ✓

(3)