learning channel

STUDY Mate

Department of Education

GRADES 8–11 ISSUE 13 SEPTEMBER 2007

Learning activities for Grades 8–11

Education Minister Naledi Pandor

Content

Dear Learner

We hope that your Heritage Day long weekend was both relaxing and productive. Remember that a sharp mind relies on a healthy body and enough sleep (but eight hours a night should do you fine). Another way to keep your mind sharp is to use this issue of Study Mate to practise your school subjects. Our English lesson asks you to recognise that there is a difference between fact and opinion. Being able to distinguish between the two is a skill that you will use for the rest of your life, especially when you watch television and read newspapers - because don't be misled to believe that the news is filled with hard facts! We also have a substantial Physical Sciences lesson for you - this is a good opportunity to revise what you learnt about electrostatics. If you think that the Linear Programming maths lesson looks familiar, you'd be spot on. Due to some technical problems that crept in when we placed this lesson a few weeks ago, we wanted to put it in again so that we could all get it exactly right. That is something you can aim for in your exams - to get all those difficult questions exactly right! Have fun trying.

ENGLISH GRADE 11 LESSON NOTES

FACT AND OPINION

OVERVIEW

You need to understand

- The difference between statements of **fact** and statements of **opinion**.

This will help you in your reading and comprehension and will also help you in your persuasive and expository writing and speaking.

A **FACT** is a statement that can be proved or disproved. There must be concrete evidence to show that the statement is true or to prove it false.

An **OPINION** is a statement that expresses a belief or an idea. An opinion expresses someone's view, attitude or judgment.

To answer the question, a fact must be supported by evidence and an opinion is simply one person's ideas, not supported by evidence.

How do you know if you can accept the facts someone gives you?

You may know from your general knowledge or common sense, but find out if you are unsure.

Lesson

There are certain kinds of writing which we expect to contain a lot of facts, such as your school Physical Science or Life Sciences textbooks, recipe books, and instruction manuals. When we turn to newspapers and magazines, we know we are going to find a lot of facts in the news reports. Here are some headlines which include only facts:

- The SA National Blood Service needs blood urgently
- Runaway schoolboy returned to his family
- Hoax e-mails warning
- Film opening on circuit
- Doctors removed from the practitioners' register
- Male appears in court
- Dust from comet brought to Earth
- Coal mining accident
- New woman leader in Liberia

Some articles comment on news or offer an opinion or try to persuade the reader about how to respond to news.

Examples:

1.

"Telephones seem innocent enough but they're not. Affair-consumed lovers whisper into them, revenge-fuelled mob bosses bark into them, and gamblers and drug dealers rely on them for business."

(Sunday Times, February 19, 2006)

Here, the writer presents the idea that telephones are not innocent because of the kind of people who use them and the purposes for which they are used. Notice how emotive the language is.

2.

We must protect our women and children

The focus should not only be on the government and the police when it comes to dealing with abusers of women and children.

It is our duty as South Africans to make sure that peace prevails in our country.

We must help stop this growing epidemic of child abuse.
We blame the police and say that they are not doing their jobs properly, but we do not help them with information.

3.

Abuse is a national issue and we should ask ourselves how we can make South Africa a better and safer place to live in.

Mandla Morare, Pretoria (*Sowetan*, Friday October 28, 2005)

This writer wants everyone to be involved in the fight against violence against women and children. The writer is presenting a point of view and trying to persuade others to agree.

Newspaper articles do contain a lot of facts, but it is not possible that they can present all of the facts related to an incident that is being reported. Journalists have to make decisions about which facts to include and which to leave out. These choices may distort the article so that a biased picture of the event is painted and the writer may subtly convey his or her opinion through the way the chosen facts are presented.

For example, if a group of learners is called a "mob", the writer is hostile to the learners. If the writer simply says that this "mob" of learners refused to go to class, the reader will have little sympathy for the learners. Perhaps the learners had a legitimate reason for refusing to go to class. By leaving out the facts the journalist removes the reasons for the learners' actions and so makes the readers less sympathetic.

Activities

Activity 1

Read the article; "Your favourite paper keeps getting better".

Your favourite paper keeps getting better

The Sunday Times has listened to you, the reader, and as of this week, you will get more of all your favourite things – including exciting new columnists and features.

Look out for our brand-new 24-page Soccer Life & Sport pull-out section. Blockbuster soccer and rugby action dominate this week's edition alongside our new Sport on Television page, where we make sure you don't miss a game and give expert opinion on what to look out for during the week.

Don't miss a new column by outgoing Bafana coach and soccer guru Pitso Mosimane.

Sport's move out of the main section means more space for breaking news and more in-depth coverage of the week's events. Our new News & Opinion section includes all our favourite commentators. Look out for the Fred Khumalo Page and a weekly column by Sunday Times editor Mondli Makhanya.

Gwen Gill gets more space, and is joined by social guru Craig Jacobs. Another new edition is our celebrity column from New York, written by Nadine Rubin. And our new Health and Technology pages will keep you up to date with advances that make your life easier.

You can enjoy more space than ever to air your views on our new Readers' Views page.

And you can tell us what you think about something that's important to you at our new universal e-mail address: tellus@sundaytimes.co.za.

Turn to page two, where we will tell you what's hot in the paper and on our website. And read our commentary on the week in the column Eish! by Andrew Donaldson.

This week, the newspaper includes the occasional section News & Review, which will give you even more quality writing.

Enjoy your new-look Sunday Times. And remember to keep telling us what you like or hate about the nation's favourite newspaper at tellus@sundaytimes.co.za.

ENGLISH GRADE 11 LESSON NOTES

1. What is the main idea in the headline?
2. What facts are used to support this idea?

Activity 2

Now look at the article; "Principals face the axe for matric failures."

Principals face the axe for matric failures

Education departments will crack the whip after the pass rate declines for the second year running

PREGA GOVENDER

PROVINCIAL education authorities are to crack down on underperforming principals as the national Education Department blames them for contributing to the second consecutive drop in the matric pass rate.

Planned punitive measures include denying pay increases based on seniority to heads of schools that continually miss pass-rate targets, and demoting others.

As the national matric pass rate slid from 70.7% in 2004 to 66.6% in 2006, analysts are describing the results as "quite appalling" and slamming the education system as "seriously dysfunctional".

The decline in the pass rate in some provinces suggests that access may have been promoted without due attention to the provision of qualitative support to ensure positive senior certificate outcomes.

Taking a swipe at underperforming provinces, Education Minister Naledi Pandor said the decline in some provinces' pass rates might have been caused by schools accepting more matric pupils without the necessary support.

"There is a clear lesson for our departments — we cannot expect schools and teachers to do more with less."

Pandor's deputy, Enver Surty, said provinces were tasked with executing department policy.

"They receive the budgets, they receive the resources, and schools are within their jurisdiction," he said.

About the results, Surty said: "If you have about a third of your learners not passing, it should be a matter of concern. I can't pretend to say I am over-excited about it. There's a measure of disappointment."

Provincial plans to stem the pass-rate decline include:

- In the Western Cape, seniority-linked salary increases will not be paid to bad principals, or to their circuit managers and district directors, who will also lose out on performance bonuses. The province will also transfer problem principals and their senior staff to circuit offices to do administrative work;
- In Mpumalanga, principals of bad schools will be moved to top schools to be mentored;
- In Gauteng, bad school heads will be moved to other jobs and their managers will be called to account;
- The North West department plans to move or demote problem principals and transfer bad teachers. The principal and deputy principal of Kgononyane High School outside Vryburg were suspended after only one of their 651 pupils passed the exams in Grades 8 to 11;
- The Free State will send top department staff to schools regularly to monitor problems; and
- The Northern Cape will introduce pass-rate targets for every matric subject.

Western Cape Education MEC Cameron Dugmore said high schools had to set targets for their overall pass rate, university entrance endorsements and scores for maths and science by the end of February.

"Should a school continuously fail to reach targets, the procedures laid down in the Employment of Educators Act will be applied, which could lead to demotion, redeployment and even dismissal [of principals]."

Don Pasquallie, deputy general secretary of the South African Democratic Teachers' Union, said his union would support the provinces' redeployment plans only if they were not done punitively but as a "corrective measure".

Mpumalanga's Education MEC, Siphosezwe Masango, said: "I don't think it's correct that a bad principal must be kept at a school to keep his or her job when learners' futures are being doomed."

Gauteng Education MEC Angie Motshekga said she knew of three principals who had to be removed from their posts. "I visited one of the principals more than five times. He's got to go back to the classroom," she said.

Limpopo's Education MEC, Aaron Motsoaledi, blamed his province's poor performance — the lowest in the country at 55.6% — on the large number of pupils who wrote subjects on higher grade. KwaZulu-Natal had the highest number of schools that scored a zero pass rate.

The national education department's Senior Certificate Technical Report, released this week, listed 33 government and private schools that could do no worse — two more than last year. Of the worst schools, 14 are in KwaZulu-Natal, six in the Western Cape and five in the Eastern Cape.

Besides schools at which all matrics failed, 173 institutions obtained pass rates of less than 20% this year — up from last year's 148.

Salim Vally, the chief researcher of the University of the Witwatersrand's Education Policy Unit, said the department had poor results coming. Early childhood development had been neglected for too long, which was "no excuse", and that research revealed teachers to be disillusioned, overburdened by administration and not receiving any "professional development".

Professor Mary Metcalfe, head of the Wits School of Education, said the solution lay in the lower grades. "We need to look rather at quality improvement in the primary and junior secondary school. Long-term improvement is based on what happens in the primary school." — Additional reporting by Thabo Mkhize

PROVINCIAL MATRIC EXAM RESULTS

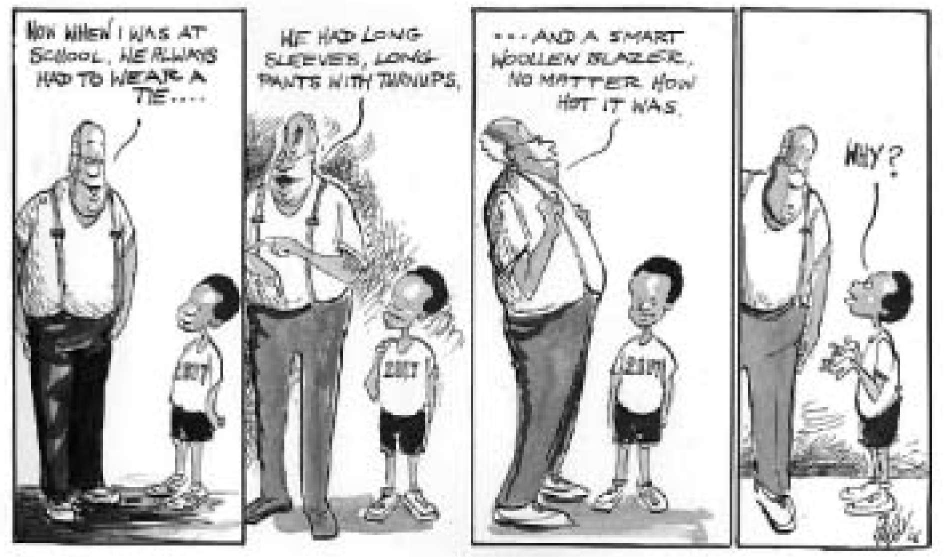
Province	Year	Pass rate	Endorsement rate
Western Cape	2004	85.0%	27.1%
	2005	84.4%	26.6%
	2006	83.7%	26.9%
Gauteng	2004	76.8%	22.1%
	2005	74.9%	21.1%
	2006	78.3%	23.2%
Northern Cape	2004	83.4%	18.7%
	2005	78.9%	15.9%
	2006	76.8%	15.5%
Free State	2004	78.7%	22.2%
	2005	77.8%	21.9%
	2006	72.2%	19.7%
North West	2004	64.9%	12.4%
	2005	63.0%	12.1%
	2006	67.0%	14.8%
KwaZulu-Natal	2004	74.0%	18.9%
	2005	70.5%	17.4%
	2006	65.6%	15.2%
Mpumalanga	2004	61.8%	12.0%
	2005	58.6%	12.7%
	2006	65.3%	14.0%
Eastern Cape	2004	53.5%	8.6%
	2005	56.7%	8.8%
	2006	59.3%	10.1%
Limpopo	2004	70.6%	20.9%
	2005	64.7%	17.7%
	2006	55.6%	13.3%
National	2004	70.7%	18.2%
	2005	68.3%	17.0%
	2006	66.6%	16.2%

1. Look at the table of statistics. What is its function?
2. Now read the article itself. What facts is the journalist reporting? Who is blamed for the problem that is reported? What action will be taken? Is the headline accurate?

3

Activity 3

Now look at the article; "Schools still need a lot of help, but we're getting there".



Schools still need a lot of help, but we're getting there

The matric pass rate was down, but that bald fact conceals some achievements — including higher enrolments, which strained the system in some provinces, writes **Duncan Hindle**

But there is also good news in the results. The minister noted with some pride that we are confident enough as a nation to set high standards, and to strive to achieve them

AT THE official announcement of the 2006 matric results on Thursday, the Minister of Education, Naledi Pandor, spelt out the agenda for the next period. Expressing concern about the disappointing achievements of too many of our children, the minister indicated that "all 10 departments of education — national and provincial", would be expected to give renewed attention to various systemic matters, and to the shared pursuit of agreed national priorities.

These sentiments are driven by a decline in the overall matric pass rate, in the number of candidates passing maths and science on the higher grade, and fewer endorsements. This is despite various interventions, as well as additional resources, that have supported better achievements in many of these areas. Extensive work will therefore be needed to identify and correct these problems.

But there is also good news in the results. The minister noted with some pride that we are confident enough as a nation to set high standards, and to strive to achieve them. Umalusi, an independent quality assurer, confirmed that the matric examinations have become more demanding.

Despite this, we have welcomed more than 85000 new students into Grade 12 over the past five years — a significant growth in access and retention rates. More than 50 000 of them have passed matric — a valuable addition to our skills. In 2006 alone, 4000 more passed matric than in 2005. And this in a year in which some 90% of learners wrote national papers, acknowledged by all to have been much more demanding.

There is much more analysis still to be done, but what stands out is that the two provinces whose matric classes grew most since 2002 — Limpopo (34300) and KwaZulu-Natal (28300) — have had the greatest difficulty in meeting the needs of learners, and quality has apparently been compromised when inputs have not matched this growth. By contrast, Western Cape and North West were able to sustain or grow their pass rates.

Following this, 2007 will be an exciting year for education in South Africa. Schools, Further Education and Training (FET) colleges and universities will all benefit from significant new policy directions, as well as major additional funding. The Department of Education will strive to ensure that these policies and resources serve the twin goals of government: accelerated growth and shared growth.

The Department of Education will contribute to growth through targeted interventions in key areas where we have a shortage of knowledge and skills. It is often said that for sustained social and economic development "the most valuable skill is a good education". Our development process therefore starts in Grade R [the Reception year before Grade 1], continues through general and further education levels, and is completed by some form of tertiary education.

It is acknowledged that the contribution of our schools to economic growth is below standard, and steps are being taken at different levels to erode this highly resilient legacy of apartheid. At grades 3 and 6 we run regular Systemic Evaluations — massive testing of literacy, numeracy and life skills, involving a sample of more than 30000 children. In 2007 we will repeat this at Grade 3 level, with the benefit of a baseline to see if we are indeed getting better following various interventions.

In Grade 9, all children now complete the Common Tasks for Assessment, which are nationally set and moderated, and which schools can use to track the development of learners and the quality of teaching.

The introduction of the new National Curriculum Statement in Grade 10 last year clearly caught some by surprise, despite repeated warnings that this was a much more demanding curriculum, with higher demands than previous Grade 10 syllabuses. Teachers and learners were urged to put in more effort and, where they did, they have done well.

Others have felt the consequence of not doing so, and will now need to do some catching up. In 2007, the new curriculum will be offered in grades 8, 9 and 11, and with maths now a compulsory subject throughout school, we can expect to see some top-class matrics as from 2008.

With the passage of the FET Colleges Act in 2006, the new year will see the effects of this legislation, including provisions for self-governance. FET colleges will be expected to offer courses in direct response to industry needs, as well as high-quality, three-year programmes to meet the demand for intermediate and high-level skills.

In support of accelerated growth, the country also needs high-level skills, research and innovation. In 2006, additional funds were allocated to selected universities to increase output in specialised areas like engineering, and in 2007 significant resources will again be

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ENGLISH GRADE 11 LESSON NOTES

given to institutions that have demonstrated the capacity to increase enrolments and success rates, especially among African students.

The Department of Education must help to share the benefits of this accelerated growth by giving more attention to the quality of education that we offer poor, mostly rural, communities. The continuing absence of water and sanitation in more than 1000 schools is unacceptable, and decisive steps will be taken on all infrastructure questions.

More than 5000 poor schools will enjoy in 2007 their new status as "no-fee schools". This will undoubtedly benefit many households that have struggled to find school fees.Some 40% of children will attend no-fee schools, thanks to increased provincial allocations to these schools. These resources must now be well used by the schools to ensure that "no-fee schools" become schools of choice, offering the best education to the poorest, who need it most.

Parents will also be assisted by the Guidelines on School Uniforms, published in 2006, which suggest that schools adopt simple uniforms, suitable for our subtropical African climate. They also prescribe that a child who genuinely cannot afford a uniform should be assisted by the school.

The further provision of resources to poor schools will be given attention through the QIDS [Quality Improvement and Development Strategy] upliftment programme, which has already seen the delivery of reading books to more than 4000 schools, with another 6000 targeted for 2007.

Bursaries are on offer to future teachers, who will be contracted to the Department of Education on completion of their studies. Students will be selected according to their teaching subjects, with preference going to students from rural areas. But students will also be considered for their commitment, and their suitability for the profession — we want the brightest and the best of our youth.

A major expansion of the education system is also needed, by vastly increasing access to lifelong learning.

Education continues to be a central pillar of our development, and the department will continue to eliminate bad practice, and replicate the good. Of the 114 schools that performed below 20% in 2005, 99 improved this year. Some moved just above 20% and some to around 50%, but seven of them moved to above 90% in 2006 — a remarkable achievement.

This is the evidence that our children are indeed capable, that more of them can do well, and that schools can make a difference provided they receive the necessary support and basic resources for teaching and learning.

Our appeal is that as a nation, we should endeavour to provide all our schools with the affirmation, the support and the resources they need, so we can in turn demand of them better performance. That is our path to the future.

Duncan Hindle is Director-General of the Department of Education

1. This article appears on page 25 whereas the previous article appeared on page 4. Is this significant?
2. What is the function of the cartoon?
3. Who wrote the article? Is this significant? Write a response to the article and compare with a friend.

Answers and Assessment

Activity 1

1. The headline declares two things:

   That Sunday Times is the reader's favourite newspaper.

   That Sunday Times is getting better.

2. In the article itself, there are no facts whatsoever to prove that this newspaper is any particular reader's favourite or to prove that this newspaper is the most popular newspaper printed on a Sunday. You might argue that there are facts to show the circulation figures but the article itself does not provide these facts.

   As for the claim that the newspaper is getting better: you would have to read the various articles and decide for yourself. All you know from the article is that the newspaper has made various changes which the editor hopes will prove exciting to the readers.

Activity 2

1. The statistics provide one kind of evidence: these statistics give the results of three years of matric examinations in terms of the pass rate and the endorsement rate (those who can attend tertiary education). This kind of factual evidence helps to make the article more authoritative and reliable. The reader feels she can judge for herself by looking at the evidence.

2. The journalist focuses on schools whose matric pass rates are appalling.

   The government is blaming principals of schools where too many learners fail matric.

   The action that will be taken:

   ➢ Principals will be removed and given administrative jobs.

   ➢ Principals will be sent to good schools and mentored.

   ➢ Principals will have their salaries decreased.

   However, it is also reported that:

   ➢ Government officials in charge of those schools will also be financially penalised or demoted or given a different job.

   ➢ Poor teachers will also be dealt with.

   Therefore, the headline is mostly accurate but does not cover everyone whom the department will target as being responsible for poor education.

Activity 3

1. We expect the earlier articles in the newspaper to be news reports with mostly facts. We expect the later articles to be under the heading: News and Opinion, so we expect these articles to be giving us the journalist's point of view, together with the factual evidence to support that point of view.

2. The cartoon at the top of the article alerts us to the fact that this section of the newspaper will include opinion. You should discuss what point the cartoonist is making and whether or not you agree with it.

   The cartoon introduces the idea of discipline and what discipline in schools should be. Is this focus continued in the article itself?

3. Look at who wrote the article: Duncan Hindle. Look at the end of the article where this person is identified: he is the Director-General of the Department of Education.

   If this person is a D-G, are you expecting the article as a whole to praise or criticise the Education Department? We would expect him to be fair and balanced in his opinions but we would not expect open criticism of his own department in so public a forum!

   Read the article and decide what opinion is presented and what facts are used to support that opinion. Then decide whether or not you agree with Duncan Hindle. Plan your response: make sure you have a few main points to make (your opinion). Then write down all the facts or evidence you will use to support that opinion.

Helpful tips

One way of conveying bias is through the choice of facts. All the information that is reported can be true, but the author can deliberately choose to report only the information that suits his or her purposes.

Opinion or bias is often quite difficult to see. The more subtle kinds of bias often go undetected. Look out for articles where facts have been selected to create a certain impression about an incident that occurred.

For example, a Sunday Times article stated that a white school principal was racist because she refused a black child accommodation in her boarding school. This needs lots of thought. What might be reasons OTHER THAN racism? What questions would you like to ask the principal, the child or the child's parents BEFORE deciding that this principal is racist? She might very well be racist, but can we make that decision based solely on her refusal to allow the black child into her boarding school?

You need to start looking carefully at what is provided in texts but also at what is left out!

Communication: Listen up and make yourself heard

Good communication means others easily understand your messages and you listen to others with understanding. Poor communication can lead to misunderstandings, frustration, resentment and even broken relationships.

How do we communicate?

Messages aren't just sent using words. We also communicate in a number of other ways:

➢ Body language – the use of eye contact, facial expressions, stance and gestures can tell people if we're excited, scared, bored, amused or nervous.

➢ Voice – the tone, pitch, volume and expression of voice helps to express humour, emotions and sarcasm.

➢ Dress – our clothes and appearance send messages about our status, profession and personality.

Think about your body language, voice and dress in different situations. What messages were you sending out? Were these the messages you meant to send?

Tips to communicate better

- Use "I" statements

Saying, "I am not sure I completely understand" instead of, "You aren't making yourself clear", communicates your concern without blaming someone.

- Listen to hear and understand

Let people know you are listening by making eye contact, asking questions and encouraging them to continue speaking. When the person has finished speaking, check that you understood by recapping the main points in your own words.

- Handle conflict with grace

When dealing with a bully, aggression or unfair criticism, respond in a calm and polite manner. Shouting back will only make the situation worse and it lowers you to the other person's level.

- Acknowledge other people's emotions

Every story has two sides. Listen to the other person's views and realise that everyone has the right to their opinion. Once the other person has finished speaking, present your side without criticising or ignoring the other person's view.

- Don't make assumptions

If you aren't sure, ASK! If something is bothering you, don't assume it will get better – ask for help or express your concern before a small problem becomes bigger.

Linear Programming

1. Historical background

Problems seeking to maximize (or minimize) a numerical function of a number of variables, subject to certain constraints, form a general class known as optimization problems. These problems were first encountered in the physical sciences and led to the development of such techniques as differential calculus and the calculus of variations. More recently new optimization problems have emerged in the field of economics for which the above techniques were found to be of limited use. These problems are known as programming problems and are concerned with determining optimal allocations of limited resources to meet given objectives. More specifically, they are concerned with situations where a number of resources (e.g. personnel, materials, land, etc.) are combined to yield one or more products. There are, however, restrictions on the total amount of each resource, the total amount of products required, the quality of the products, etc. The problem is to determine that allocation of resources which will either minimize the costs involved, or alternatively maximize the profit.

Linear programming problems are programming problems in which all relationships between variables are linear. The general linear programming problem was first formulated and solved by George B. Dantzig in 1947. His method of solution is known as the simplex algorithm.

2. Example of a Linear Programming problem

Consider a factory which has three types of machines, Machines A, B and C, and which turns out four products, Products 1, 2, 3 and 4. Each of the products has to undergo some sort of processing on each machine (in the same sequence of activity).

The table below shows the processing time per product per machine, the unit profit per product and the total time available per machine per week. For example, Machine type A requires 1,5 hours to produce 1 unit of Product 1. We wish to determine the weekly output for each product in order to maximize profits.

Machine type	Products				Total time available
	1	2	3	4	
A	1,5	1	2,4	1	2 000
B	1	5	1	3,5	8 000
C	1,5	3	3,5	1	5 000
Unit profit	5,24	7,30	8,34	4,18	

Suppose x_j represents the number of units produced of product j. The value of x_j cannot be increased arbitrarily, because machine time is limited.

Thus, the machines of type A are in use for:

$(1{,}5x_1 + 1x_2 + 2{,}4x_3 + 1x_4)$ hours per week,

so that we must have:

$1{,}5x_1 + x_2 + 2{,}4x_3 + x_4 \le 2\,000$ ... (3.1)

(It would be wrong to use only an = sign in the above inequality, because there might not be any combination of production rates which would use each machine type to capacity, and we do not wish to predict which machines are to be used to capacity.)

Similarly, we must have for type B and C machines, respectively:

$x_1 + 5x_2 + x_3 + 3{,}5x_4 \le 8\,000$ ... (3.2)

$1{,}5x_1 + 3x_2 + 3{,}5x_3 + x_4 \le 5\,000$ ... (3.3)

Since, furthermore, we cannot produce negative quantities of any product, we must have:

$x_j \ge 0,\ j = 1, 2, 3, 4$ ... (3.4)

The total profit per week to be maximized is z, where:

$z = 5{,}24x_1 + 7{,}30x_2 + 8{,}34x_3 + 4{,}18x_4$ ... (3.5)

The above is clearly a programming problem, and furthermore it is linear, since all relationships (3.1) to (3.5) are linear.

Inequalities (3.1) to (3.3) are known as the **constraints.**

Inequality (3.4) are known as the **non-negativity restriction**.

Equation (3.5) is known as the **objective function.**

It is worth noting that even for the above very simple case the problem is non-trivial (and hence will not be solved here) and the optimal product mix is far from obvious.

3. Graphical methods

Simple Linear Programming problems (LPPs) involving only two variables can easily be solved by a graphical method. The geometrical (graphical) interpretation of LPPs with two variables is quite important and provides a great deal of insight into the structure of problems with more variables. The following examples are given to develop a greater understanding of the graphs of inequalities, and the relevant optimization techniques.

1

3.1 Examples

3.1.1 Maximize $z = 5x + 3y$ subject to the following constraints:

$3x + 5y \le 15$

$5x + 2y \le 10$

$y \ge 0$

$x \ge 0$

These constraints become, respectively:

$y \le -\frac{3}{5}x + 3$

$y \le -\frac{5}{2}x + 5$

$y \ge 0$

$x \ge 0$

The feasible region (i.e. the set of points in the plane satisfying the constraints and the non-negativity restrictions) is the shaded region in the figure below. In your classroom you would probably have a number of differently shaded regions, representing the different constraints. The inequality $y \le -\frac{3}{5}x + 3$ is represented by shading all points on or below the line $y \le -\frac{3}{5}x + 3$; similarly the inequality $y \le -\frac{5}{2}x + 5$ is represented by shading all points on or below the line $y \le -\frac{5}{2}x + 5$. The inequality $y \ge 0$ is represented by shading all points on or above the x-axis; the inequality $x \ge 0$ is represented by shading all points on or to the right of the y-axis. For now it would be quite difficult to see which region is shaded in four different ways (for the four inequalities) and we thus shade only the feasible region in **Figure 3.1**. It would be a good idea to practise shading the different regions, as described above, so that you can see how the feasible region was determined.

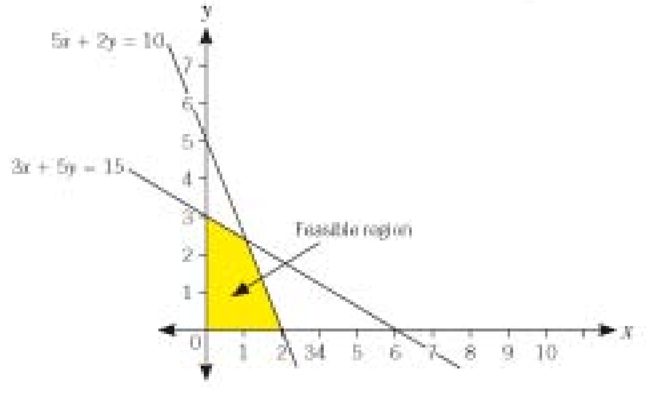

Figure 3.1

Any point in the feasible region is known as a **feasible solution**.

The feasible region is bounded by:

(i) the two co-ordinate axes (the non-negativity restrictions ensure that all feasible solutions must lie in the first quadrant)

(ii) the line $3x + 5y = 15$ (the first constraint $3x + 5y \le 15$ ensures that the feasible region is on or below the line $y = -\frac{3}{5}x + 3$); the line $5x + 2y = 10$ (the second constraint $5x + 2y \le 10$ ensures that the feasible region is on or below the line $y = -\frac{5}{2}x + 5$).

Now, for each value of z, the line $z = 5x + 3y$ is a straight line. It can be rewritten as $y = -\frac{5}{3}x + \frac{z}{3}$. Allowing z to vary generates a set of parallel straight lines (z increases when these lines are shifted to the upper right as indicated by the broken lines in **Figure 3.2**).

It is now immediately obvious that the optimal solution to the problem is given by the upper right corner of the feasible region. This is the point where the two lines $3x + 5y \le 15$ and $5x + 2y \le 10$ intersect. Using a calculator we find that this point has the coordinates $y = 1{,}053$ and $x = 2{,}368$. The associated value of z is 12,37.

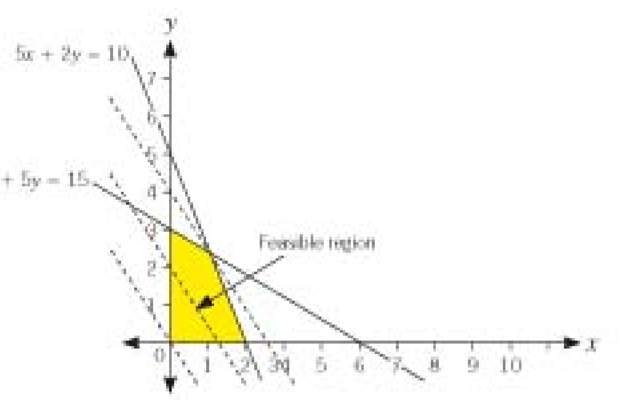

Figure 3.2

3.1.2 The same problem as 3.1.1, but with the objective function changed to $z = 5x + 2y$, gives the following (only the feasible region has been shaded):

2

Linear Programming

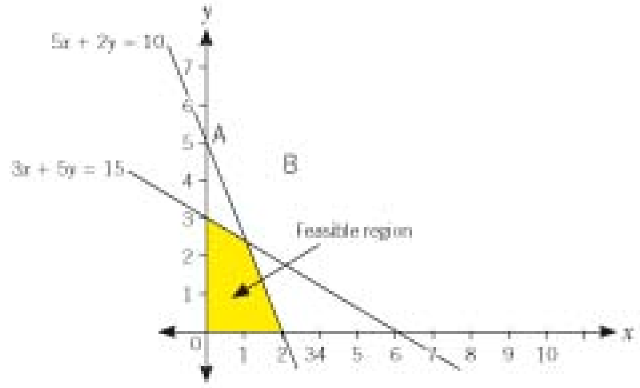

Figure 3.3

The lines representing $z = 5x + 2y$ are now parallel to one of the edges of the feasible region, since both the objective function and one of the constraints have the same slope. This means that there is no longer a unique solution, but that the entire line segment from the point of intersection of the two constraint lines to B gives optimal solutions. Note however that as in 3.1.1 at least one corner of the feasible region gives an optimal solution.

The two examples above illustrate normal (well-behaved) LPPs. The next two will illustrate two possible exceptional cases.

3.1.3 Maximize $z = 2x + 2y$ subject to the following constraints:

$x - y \geq -1$

$-\frac{1}{2}x + y \leq 2$

$y \geq 0$

$x \geq 0$

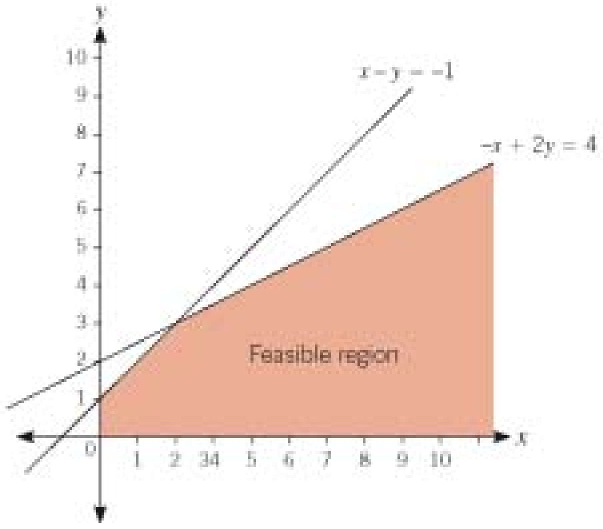

Figure 3.4

In this case the feasible region is unbounded i.e. there are many feasible solutions This is indicated by the shaded area.

3.1.4 Maximize $z = 3x - 2y$ subject to the following constraints:

$x + y \leq 1$

$2x + 2y \geq 4$

$y \geq 0$

$x \geq 0$

These constraints become:

$y \leq -x + 1$

$y \geq -x + 2$

$y \geq 0$

$x \geq 0$

The feasible region thus consists of all points

on or below the line $y = -x + 1$,

on or above the line $y = -x + 2$,

on or above the x-axis,

on or to the right of the y-axis.

In **Figure 3.5** only the first two inequalities are represented, and from these it is immediately clear that there are no points satisfying the constraints.

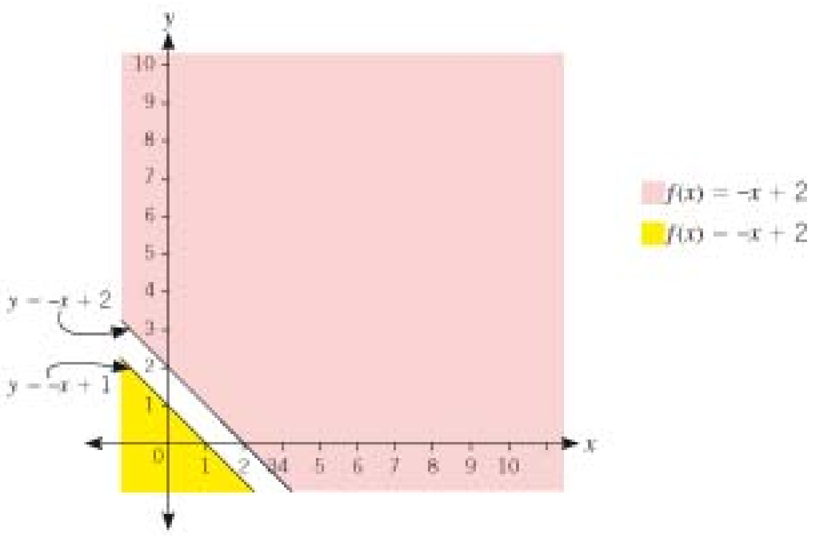

Figure 3.5

Since there are no points satisfying the constraints we say the problem is infeasible. (This is indicated by the fact that the shaded areas on the graph do not overlap).

NOTE: The occurrence of unbounded regions and infeasible solutions is in practice usually due to incorrect formulation of the problem.

From these few examples we can make the following generalizations:

(i) The feasible region (if there is one) is always a convex set (i.e. for any two points in the feasible region, the entire line segment joining these points lies in the feasible region).

(ii) Whenever the optimal value of z is finite, a corner of the feasible region will give the optimal solution.

It can be proved that these two observations are in general true.

The above examples also illustrate important facts concerning linear programming problems. These facts can be summarized as follows:

(i) A linear programming (LP) problem is the problem of maximizing or minimizing a linear function subject to a finite number of linear constraints.

(ii) The linear function that is to be maximized or minimized in a LP problem is called the objective function.

(iii) Numbers that satisfy all the constraints of a LP problem constitute a feasible solution of that problem.

(iv) A feasible solution that maximizes or minimizes the objective function is called an optimal solution. The corresponding value of the objective function is called the optimal value of the problem.

(v) Not every LP problem has a unique optimal solution. Some problems have many different solutions and others have no optimal solution at all.

(vi) Problems that have no feasible solutions are called infeasible.

(vii) Problems that have many feasible solutions are called unbounded.

(viii) A LP problem is said to be in standard form when all the constraints are linear inequalities and all variables are non-negative.

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Linear Programming

4. Prior knowledge required for Linear Programming

(a) Knowledge of linear equations:

(i) solution of linear equations

(ii) standard form of linear equations

(iii) graphs of linear equations

(iv) algebraic solutions of simultaneous linear equations

(b) Knowledge of linear inequalities:

(i) solution of linear inequalities

5. Revision Exercises

5.1 Graphs of linear equations and inequalities

(i) Revise linear equations and sketch graphs of these functions (start with the equation in non-standard form; write in standard form and then point out gradient, y-intercept and x-intercept; use intercept method to draw graphs indicating positive and negative slopes; find the point of intersection of line graphs using the algebraic method).

Example 1

Write the following equations in standard form and then draw the graph that represents each equation:

(a) $4 - 2y - x = 0$ (negative slope)

(b) $2x - 3y + 6 = 0$ (positive slope)

Solution

(a) $y = -\frac{1}{2}x + 2$

(b) $y = \frac{2}{3}x + 2$

The point of intersection of the two lines, i.e. the simultaneous solution of (a) and (b), is $x = 0$ and $y = 2$.

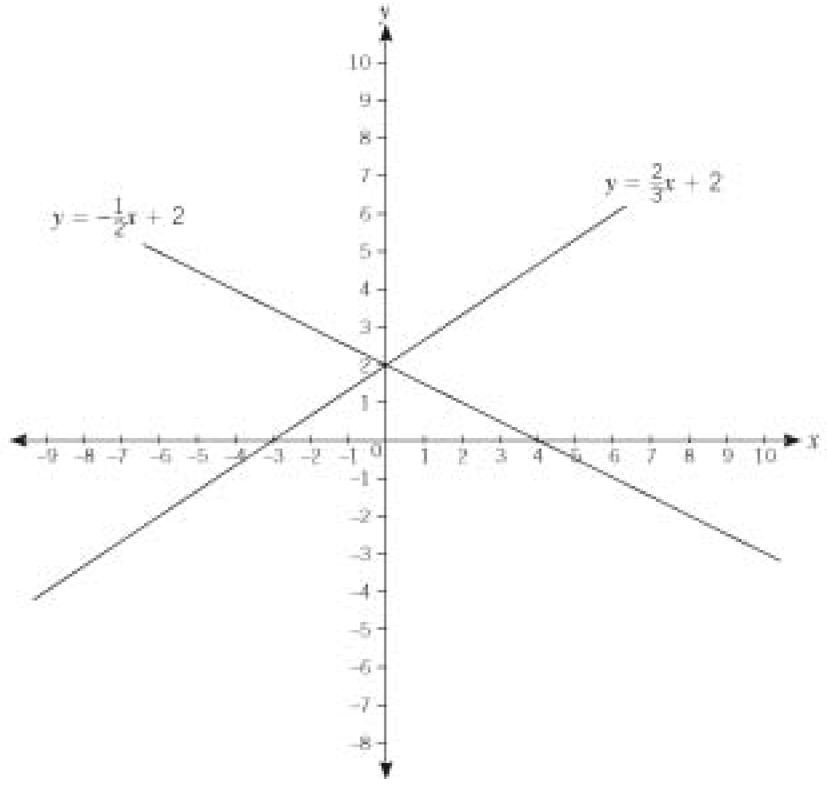

Figure 5.1

(ii) Revise linear inequalities and sketch graphs of inequalities:

Example 2

Draw the graphs of:

(a) $y - 1 = x$

(b) $y + 2x = 1$

(c) $y = -4$

Then shade in your diagram to show the regions representing:

(d) $y - 1 > x$

(e) $y = 2x \leq 1$

(f) $y \geq -4$

Solutions

(a), (b), (c)

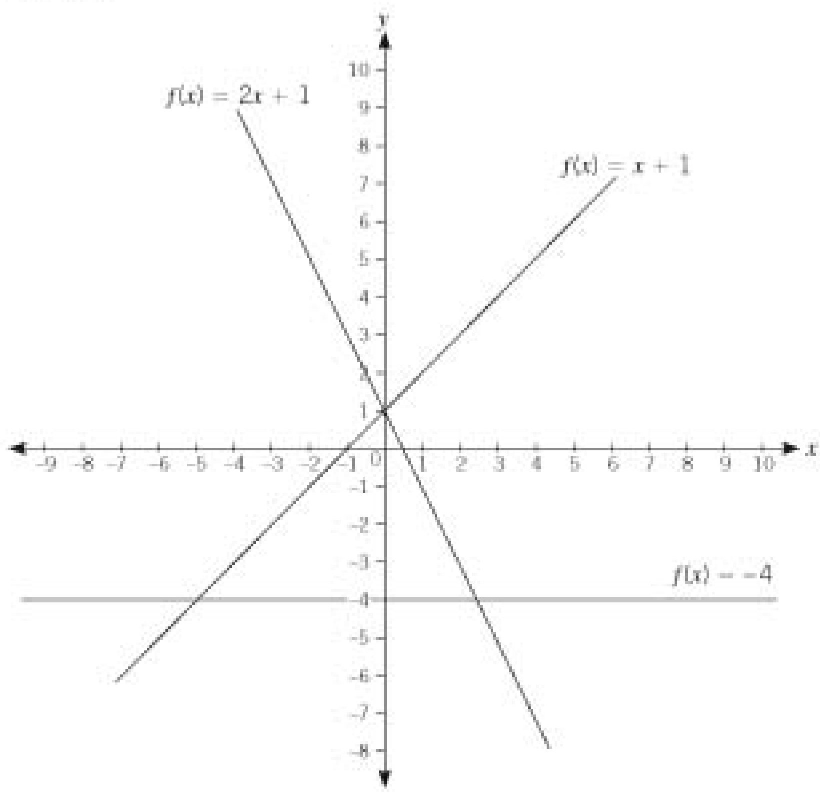

Figure 5.2

(d) The graph of $y > x + 1$:

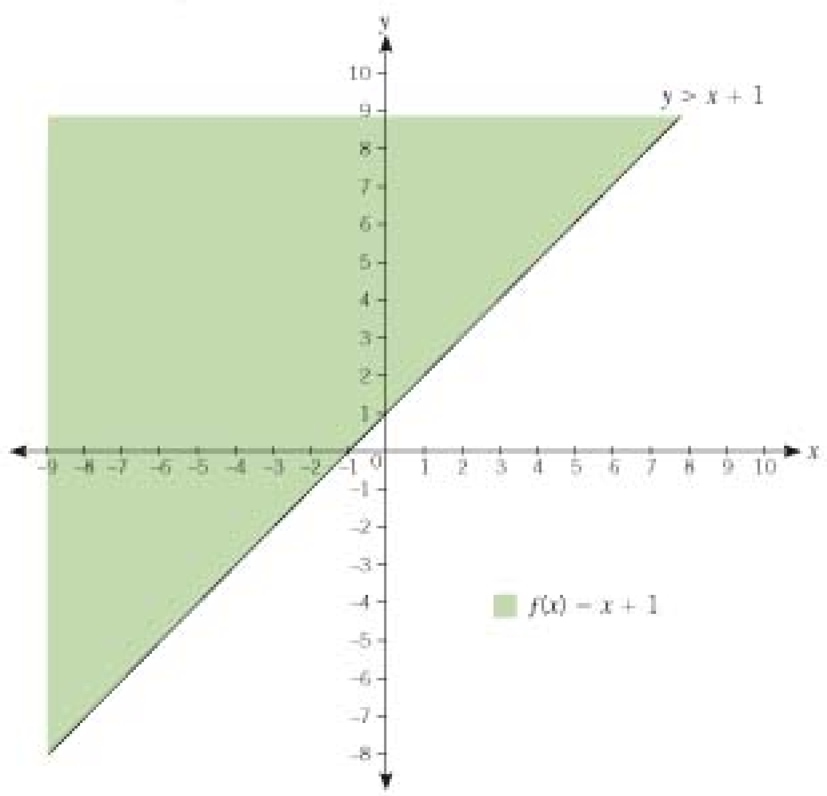

Figure 5.3

NB: The line $y = x + 1$ is shown as a dotted line since only points that satisfy $y > x + 1$ are required.

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Linear Programming

(e) The graph of $y + 2x \leq 1$:

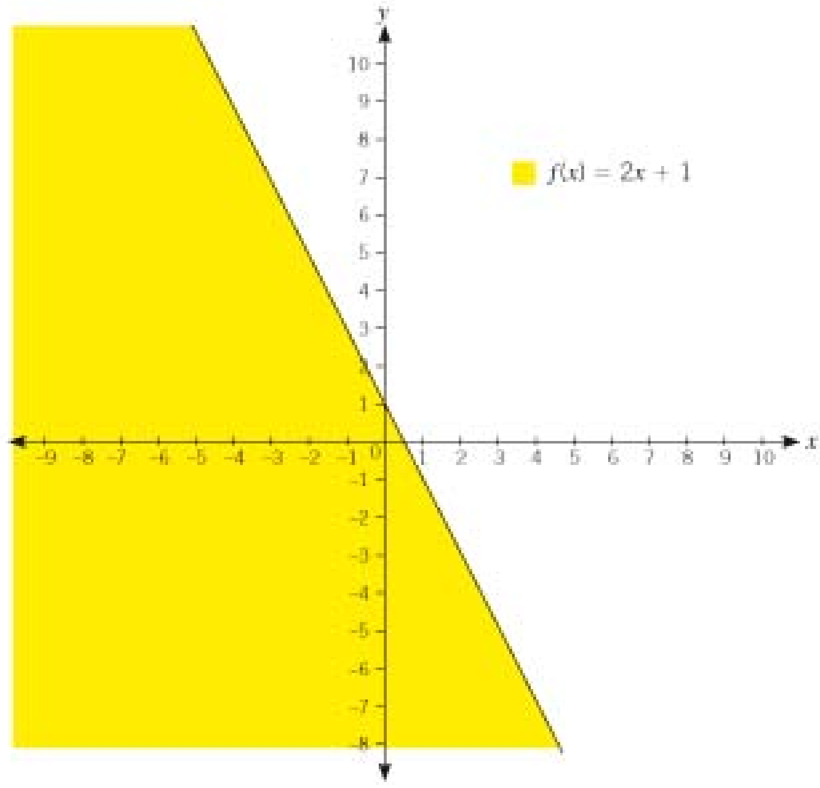

Figure 5.4

(f) The graph of $y \geq -4$:

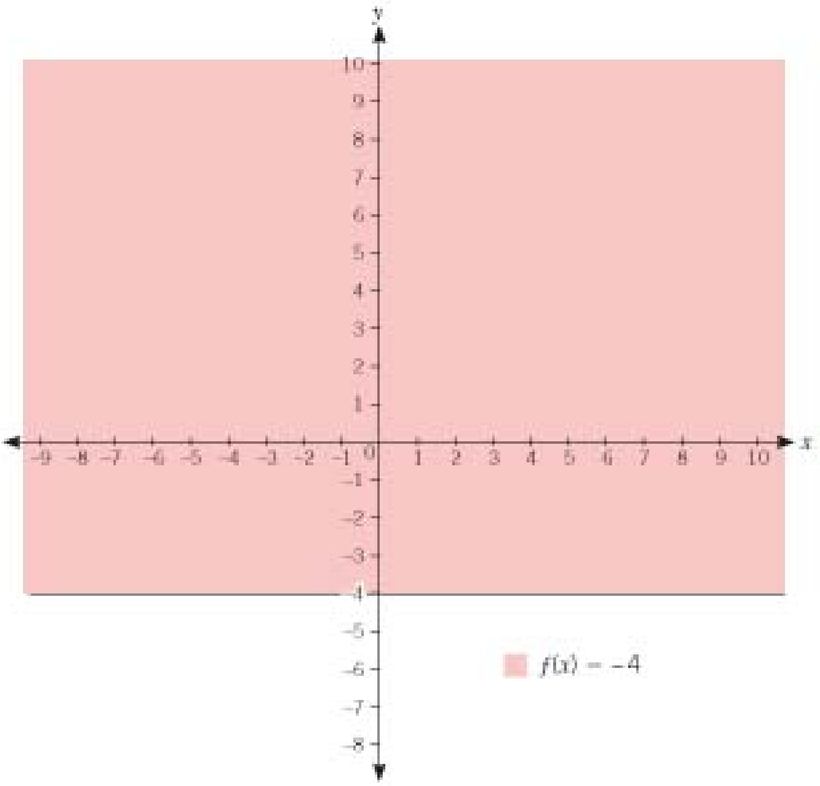

Figure 5.5

5.2 Graphs of simultaneous linear inequalities

(i) In the following example one of the inequalities involves a line parallel to one of the axes.

Example 3

Graph the solution set of the following system of linear inequalities:

$y \geq -1$

$x + 2y - 8 \leq 0$

Solution

$y \geq -1$

$y \leq -\frac{1}{2}x + 4$

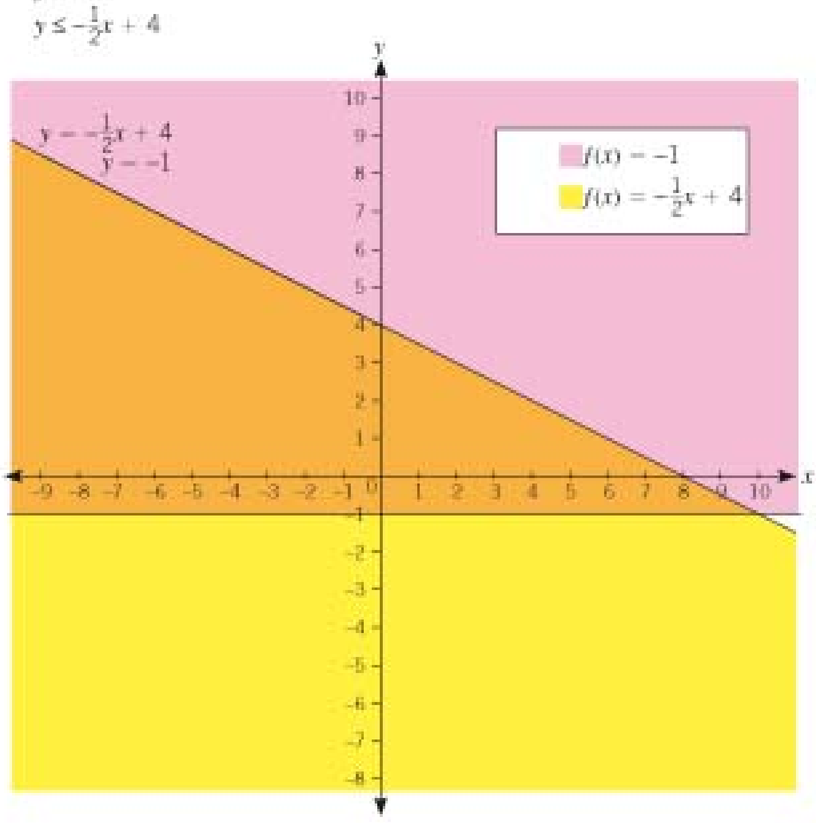

Figure 5.6

The solution set is where the shaded areas overlap.

In the following example the two lines are parallel.

Example 4

Graph the solution set of the following system of inequalities:

$x - 2y + 4 \geq 0$

$2y - x + 2 \geq 0$

Solution

The inequalities may be rewritten as follows:

$y \leq \frac{1}{2}x + 2$

$y \geq \frac{1}{2}x - 1$

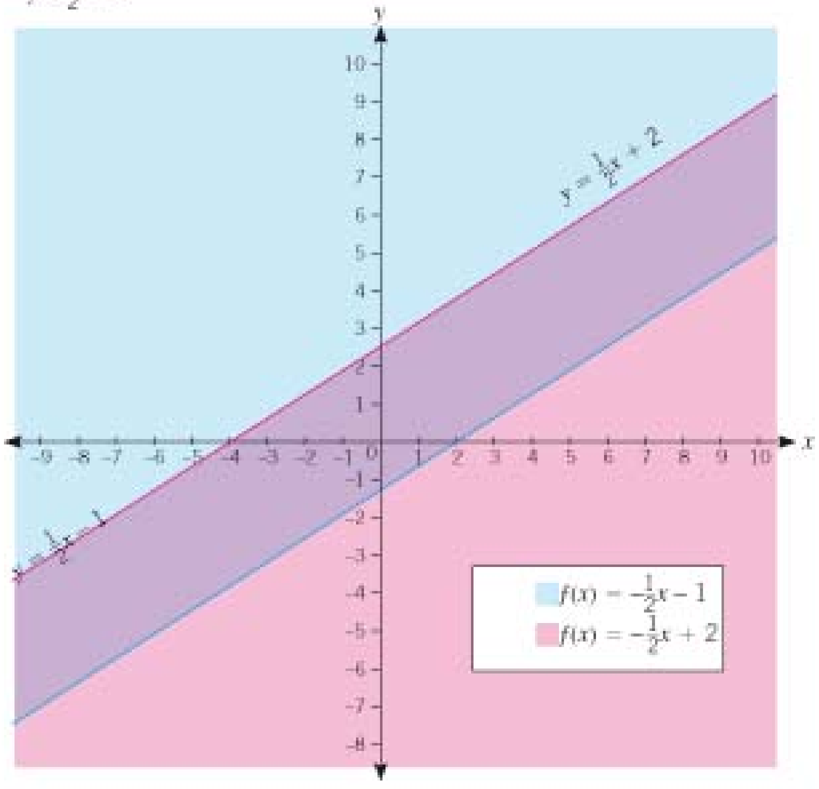

Figure 5.7

The solution set is where the shaded areas overlap.

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Linear Programming

In the following example we have three inequalities.

Example 5

Graph the solution set of the following inequalities:

$y \geq -6$
$x + 2y - 8 \leq 0$
$2x - y - 6 \leq 0$

Solution

The inequalities can be rewritten, respectively, as follows:

$y \geq -6$
$y \leq -\frac{1}{2}x + 4$
$y \geq 2x - 6$

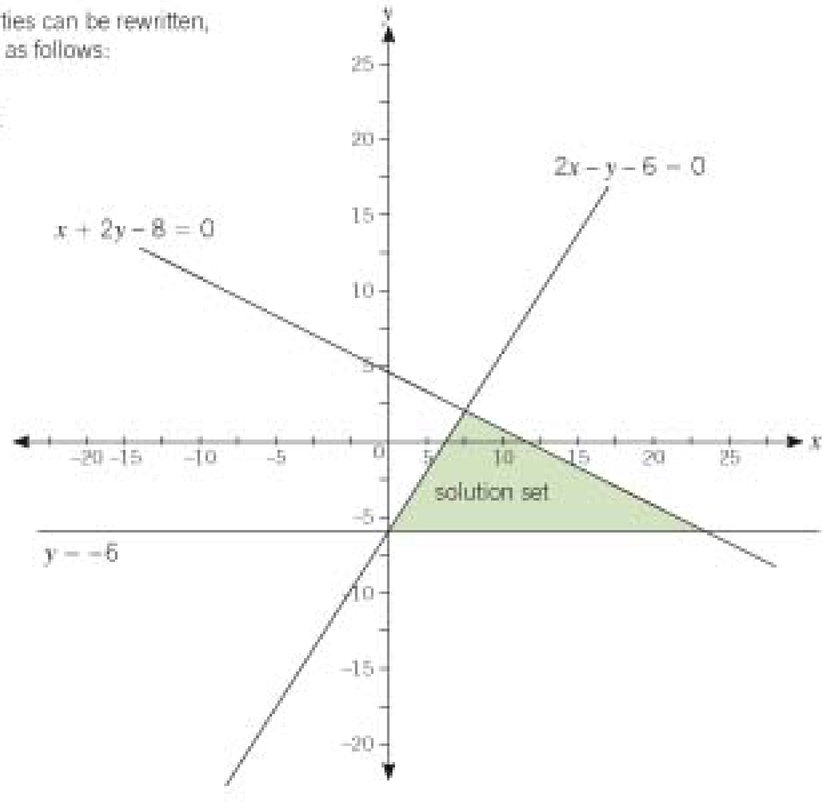

Figure 5.8

The solution set is given by the shaded area.

6. Linear Programming Exercises

6.1 Hints on solving LP problems using the graphical method

(i) Always choose the variables of the objective function first (i.e. first find what must be maximized or minimized and then assign variables to these). All the constraints must then be written in terms of these variables.

(ii) Like constraints must be grouped (i.e. money with money, masses with masses, etc.) to assign the variables.

(iii) The slope of the objective function is used to draw the search line. The search line is the line used to find the maximum or minimum of the objective function when it is written in standard form, that is, y is the subject of the formula. Note that for grade 11 the maximum or minimum is found by a numerical search along the boundary of the feasible region.

(iv) The minimum or maximum always occurs at a vertex of the feasible region or on a boundary line if there is no unique solution to the problem. So when you want to maximize you are looking for a vertex as far as possible from the origin. If you want to minimize you are looking for a vertex as close as possible to the origin.

(v) The search line must not go through the feasible region but must still touch it (this will happen at one of the vertices).

Example 6

A certain manufacturer of television sets operates on a weekly basis and produces two models, i.e. a model X (colour) and a model Y (black and white). At least one set of both models and at most 6 sets of model X and 5 sets of model Y, must be manufactured weekly. It takes 5 hours to produce model X and 4 hours for model Y, while a working week comprises of 40 hours. Twice as many workers are necessary to produce model X than model Y, while at least eight workers are constantly occupied in the manufacturing process.

(a) Represent the data algebraically and graphically. Use your graph to determine the feasible region for the number of each model which can be manufactured weekly, taking into account the restrictions mentioned.

(b) The profit on model X is R500 and on model Y it is R250. Determine:

(i) the number of sets of each model to be manufactured weekly for a maximum profit.

(ii) the maximum profit.

Solution

We must choose the variables of the objective function first. What we want to do is to manufacture a certain number of sets to maximize the profit. Hence the number of sets is what will determine the maximum. Therefore we let:

number of TV sets of model X produced = x

number of TV sets of model Y produced = y

profit = P

The objective function in this case is given by $P = 500x + 250y$.

We now need to find the constraints (pay particular attention to the words in bold font):

(i) At least one set of both models must be produced:
$x \geq 1$ and $y \geq 1$

NB: 'At least one' means 'one or more than one', and we thus use $\geq$.

(iii) At most 6 sets of model X must be produced:
$x \leq 6$

NB: 'At most 6' means '6 or less than 6', and we thus use $\leq$.

(iii) At most 5 sets of model Y must be produced:
$y \leq 5$

NB: 'At most 5' means '5 or less than 5', so we use $\leq$.

(iv) It takes 5 hours to produce model X and 4 hours for model Y, while a working week comprises of 40 hours. It follows that:
$5x + 4y \leq 40$

NB: a working week of 40 hours means that the number of available hours is 40 or less, so we use $\leq$; note also that if it takes 5 hours to produce 1 set of model X, it takes 10 hours (i.e. 2×5 hours) to produce 2 sets, 15 hours (i.e. 3×5 hours) to produce 3 sets, etc., so that it takes $5 \times x$ hours, i.e. $5x$ hours, to produce x sets of Model X. The term $4y$ is obtained in a similar way.

(v) Twice as many workers are necessary to produce model X than model Y, while at least 8 workers are constantly occupied in the manufacturing process. We thus have:
$2x + y \geq 8$

NB: 'Twice as many as' means the 'number we are comparing with, multiplied by 2', and we thus have $x \times 2$, i.e. $2x$. 'at least 8' means 'exactly 8 or more than 8' and we thus use $\geq$.

In order to draw the graph we use the following table:

	INTERCEPTS	
CONSTRAINTS	x	y
(i) $x = 1$	1	–
(ii) $y = 1$	–	1
(iii) $x = 6$	6	–
(iv) $y = 5$	–	5
(v) $5x + 4y = 40$	8	10
(vi) $2x + y = 8$	4	8

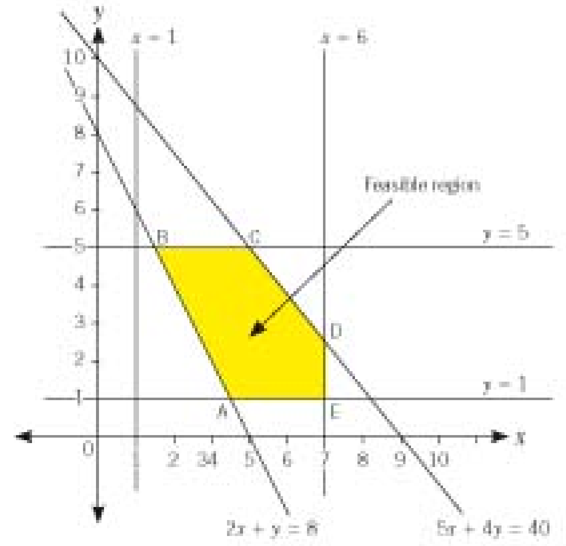

Figure 5.9

The feasible region is the shaded area.

(b) From the information given we know that the profit (P) can be stated as follows:
$P = 500x + 250y$

This is the objective function that needs to be maximized subject to the above constraints.

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Linear Programming

(i) **For grade 11 we do the following:**

The grade 11 method is based on a numerical search along the boundary of the feasible region. This requires us to find the vertices A, B, C, D, and E. To find vertices A, B, C, D and E, respectively, we must find the points of intersection of the lines

$y = 1$ and $y = -2x + 8$ (A)

$y = 5$ and $y = -2x + 8$ (B)

$y = 5$ and $y = -\frac{5}{4}x + 10$ (C)

$x = 6$ and $y = -\frac{5}{4}x + 10$ (D)

$x = 6$ and $y = 1$. (E)

The co-ordinates of the vertices as well as the corresponding values for the objective function are given in the following table:

Vertex	Coordinates		Profit P
	x	y	$P = 500x + 250y$
A	3,5* change to 4	1	2 250
B	1,5* change to 2	5	2 250
C	4	5	3 250
D	6	2,5* change to 2.	3 500
E	6	1	3 250

* Since fractions of television sets are not allowed these values have to change to whole numbers. The new value must however still be in the feasible region.

From the above table it is clear that maximum profit occurs at vertex D.

Hence 6 sets of model X and 2 sets of model Y should be manufactured for maximum profit.

For grade 12 we do the following:

We make y the subject of the formula $P = 500x + 250y$ and we obtain $y = -2x + \frac{P}{250}$.

We draw a search line using the $-\frac{2}{1}$ for slope. The search line is then

moved parallel to itself in order to find a vertex where the search line touches the feasible area without passing through it. From the graph we find that the coordinates of this vertex D are $\left(6 ; 2\frac{1}{2}\right)$.

So, for maximum profit, 6 sets of model X and 2 sets of model Y should be manufactured (we cannot use $2\frac{1}{2}$ so we choose the furthest appropriate point in the feasible region, which is 2).

(ii) Max $P = 500(6) + 250(2) = R3\ 500$

The following exercises can now be attempted:

(a) A carpenter makes two types of tables, X and Y. He has 600 m² floor space available. Table X requires 50 m² floor space and table Y 60 m². He can only afford to invest R1 600 in equipment that amounts to R160 per table X and R100 per table Y. He does not have enough skilled labourers to make more than 6 tables of type X.

(i) Write down the inequalities to illustrate the data above.

(ii) Represent the inequalities graphically and clearly shade the feasible region.

(iii) The profit on each type of table is the same, and the time to make each one, is the same. How many tables of each type can be made simultaneously to ensure maximum profit?

(b) The owner of a clothing factory intends to manufacture shorts and trousers. A maximum of 60 pairs of shorts and 40 pairs of trousers can be manufactured per day. No more than 70 items can be made per day. It takes 5 hours to manufacture a pair of shorts, and 10 hours for a pair of trousers. A total of 500 working hours is available.

(i) Write down a set of inequalities to illustrate the restrictions.

(ii) Draw the necessary graphs to enable you to shade the feasible region.

(iii) If each pair of shorts yields a profit of R15 each and each pair of trousers R25, write down an objective function which will enable you to determine the profit.

(iv) Determine the maximum profit.

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PHYSICAL SCIENCES GRADE 11 LESSON NOTES

ELECTROSTATICS

Electrical charge, force between charges and capacitance

OVERVIEW

This lesson is about charge and the behaviour of charge under various circumstances. Remember that physics is a study of the interactions of inanimate matter in the universe. That's a fancy way of saying "how bodies interact." We aren't immediately interested here in the organic causes of motion, e.g. what makes the muscles work. (Muscle physiology is about the chemistry of living systems and chemistry is simply physics on a microscopic scale so it's all physics in the end – even though biologists and chemists may not agree!). What we are interested in here is forces – the agency of movement – and the origins of forces. In this lesson we will deal, in particular, with electrostatic charge, how it behaves, the nature of fields and what forces arise when charges interact.

FORCES

You have learnt about forces and the effect of forces. One must remember that regardless of the source of the force on a body with mass (any body with mass can be referred to as "massive" without meaning "huge" or "large in size"):

- Any force has the ability to make something move: keep it moving, or change its motion.
- Motion can be in a line, along a curve, or a combination of these two.

CHARGE: A PROPERTY OF MATTER

- Charge can be described as a fundamental property of matter, just as mass is.
- In this lesson we will look at the force involved when charge interacts with charge.
- In the same way that matter interacts with other matter because of its mass (the gravitational interaction), matter can also interact with other matter because of its charge.
- When charges interact, we can get either repulsion or attraction because two kinds of charge exist. By contrast, there is only one kind of mass, which is why there is only a gravitational attraction and no gravitational repulsion.

In the early 1700s, in the town of Leyden in Holland, a device called a Leyden jar was invented. The Leyden jar was used to store what was called "electricity". These days we call it "electrical charge". Although electrical charge is difficult to describe, we can say that (a) it is an accumulation of electrical matter and (b) when this "electrical matter" accumulates it can give rise to electrical effects that we can observe.

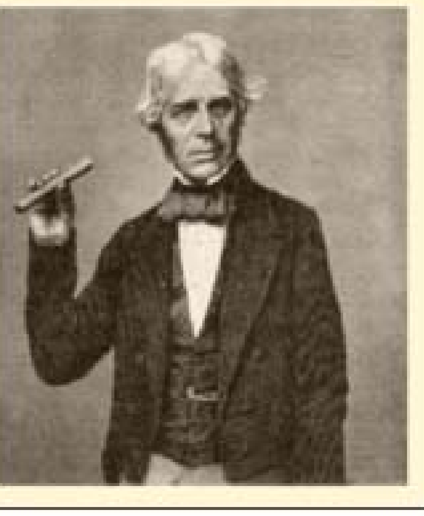
SOME FACTS ABOUT CHARGE

Charge comes in two varieties: positive charge and negative charge.

- Charge is measured in units of coulombs (C). (1C is a very large charge!)
- A body can have both its gravitational mass (m) and an electrostatic charge caused by an excess or deficiency of electrons.
- Unlike (different to each other) charges (+ and –) attract and like (the same as each other) charges (+ & + or – & –) repel.
- Charge can flow from one place to another in materials called conductors. Flowing charge is referred to as current ($q = I \times t$).
- Moving/flowing electrical charge (i.e. current) causes magnetic effects: a current in a wire will cause a magnetic needle on a compass to deflect.
- Atoms are electrically neutral but are known to consist of a positively charged nucleus surrounded by moving, negative charges called electrons.
- Ions are atoms or groups of atoms that have had one or more electrons removed or on which one or more additional electrons have been added. (Ions can exist in solutions, in the air or in a vacuum.)
- The charge on a negative ion is $-1,6 \times 10^{-19}$C for every electron that has been added or $+1,6 \times 10^{-19}$C for every electron that has been removed. So, although we write Fe^{2+} for the iron (II) ion, we know that the charge resting on that ion is actually $(2 \times +1,6 \times 10^{-19}\text{C})$ or $+3,2 \times 10^{-19}$C.

1

Michael Faraday (1791–1867) did much research on electricity. To explain his understanding of charge in the regular lectures he gave to the public and to children, Faraday used special diagrams to illustrate electric fields. The idea of an electrical field arose through the work of Faraday and other scientists. An electric field was defined as a region (or a "space") in which electrical effects can be observed or felt. The words "region" or "space" are used rather than "area" because an area refers to flat surfaces only. Faraday also developed rules for his diagrams to show magnetic fields. These rules are now also used to draw representations of electrical fields around static electrical charges.



THE CONSERVATION OF CHARGE

"**Charge can neither be created nor destroyed**." This statement is known as the **principle of conservation of charge**. Note the similarity of this principle to the principle of conservation of energy. Both are so-called "conservation laws". In physics there are several conservation laws including energy-matter, charge and angular momentum.

When a single positive charge and a single negative charge combine, we get a net charge of zero. But this does not mean that charge has been destroyed! Those charges are still present on the object.

When an electrically **neutral** body becomes positively charged, it does so because some negative charge has left it. That negative charge still exists, i.e. it has not been destroyed. If the negative charge returns to the positively charged body, the body becomes neutral once again.

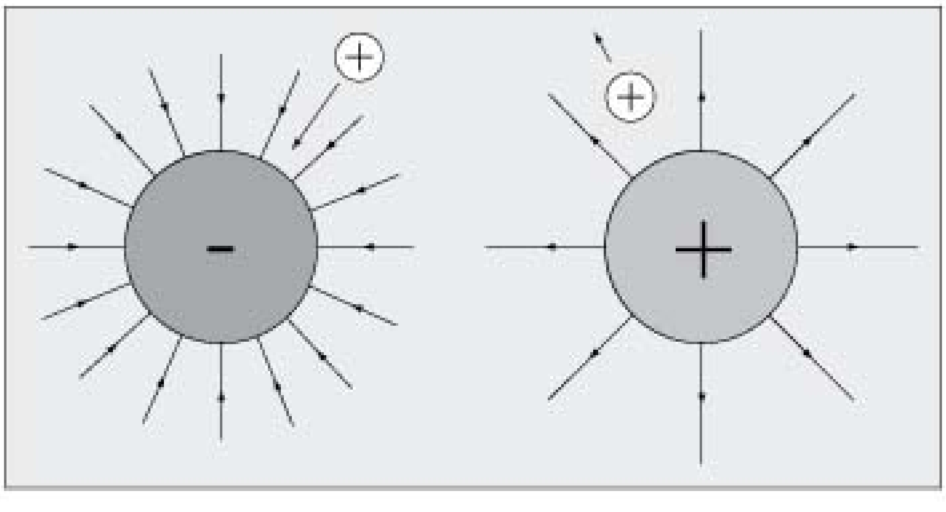
The basic amount of negative charge is the charge carried by one electron, i.e. $-1,6 \times 10^{-19}$C. This means that when an electron leaves a body, the body has a positive charge equal to $+1,6 \times 10^{-19}$C. So the charge on an ion is $-1,6 \times 10^{-19}$C for every electron that is added or $+1,6 \times 10^{-19}$C for every electron that has been removed. Therefore, although we write Fe^{2+} for the iron (II) ion (when two electrons are removed from a neutral iron atom), we know that the charge resting on it is actually $2 \times (+1,6 \times 10^{-19}\text{C})$ or $+3,2 \times 10^{-19}$C.

Part 1

DIAGRAMS OF ELECTROSTATIC FIELDS

When we use field diagrams to represent electric fields, we follow certain rules:

- Field lines have a direction.
- The direction of a field line is given by the path a positive charge would move along in that field, i.e. towards a negative charge and away from a positive charge. [Note: this is also the direction of the force that would act on that positive charge.]
- Field lines are always drawn perpendicular (i.e. at 90°) to a charged surface.
- Field lines are drawn closer together where the field is stronger, e.g. closest to the charged body. [Note: the negatively charged body above has a greater charge on it than the positively charged body. Note the difference in density of the field lines as a result.]
- The strength of the field decreases with the distance away from the charged body.

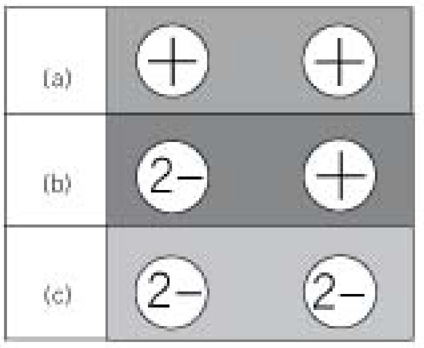


TASK

Apply the rules and draw lines around each of these three pairs of charges. A useful hint is to place an imaginary positive test charge at points around the charges. Imagine the direction and magnitude of the resultant force on the test charge. That will be the direction of the field line at that point.

2

PHYSICAL SCIENCES GRADE 11 LESSON NOTES



[Field diagrams for the pairs of charges can be found in ANSWERS AND ASSESSMENT at the end of these lesson notes.]

THE FORCE BETWEEN TWO CHARGED BODIES

Coulomb's Law states that: ***The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.***

We write Coulomb's Law as: $F = \frac{kQ_1Q_2}{r^2}$

The law speaks of "point charges" and the way we use the formula when we apply the law is to assume that all of the charge on a body acts as though it were concentrated at the centre of the body. So the distance "**r**" is taken as the distance between the centres.

The value of the proportionality constant is taken as $\mathbf{9 \times 10^9\ Nm^2C^{-2}}$. The units of **k** can be seen from the formula. We have seen a similar formula before: when we studied the gravitational force.

The electrostatic and the gravitational forces are both "inverse square laws". Inverse square laws behave in the following way: as we get further from the matter causing the field, the strength of the field gets smaller and smaller by a fixed ratio related to distance. For every doubling of the distance from the charge (or mass), the force is reduced in strength by the square of one half, i.e. the strength is quartered. So, if we move two charges to three times the distance apart, the force will become one-ninth the strength, and so on.

COMPARING G AND k

The difference between the strengths of the electrostatic field and the gravitational field can be seen in the size of **G** and **k**, and in particular by examining the powers of ten involved in the constants. If we ignore the units for the moment, we see that while $\mathbf{G} = 6{,}67 \times \mathbf{10^{-11}}\ Nm^2kg^{-2}$, the value of **k** has a factor of 10^9 in it. 10^{-11} is a very, very small number (one divided by one hundred thousand million, to be exact!) whereas 10^9 is a billion. The difference between the two factors gives us a clue to the relative strengths of the electrostatic and gravitational forces.

The reasons we usually see the gravitational force as being so impressive (the weight of a heavy object makes it difficult to lift) while static electricity causes a tiny spark and just a brief moment of discomfort is this: the Earth has a mass of approximately 6×10^{24}kg and this more than makes up for the fact that the gravitational force is so weak. The charge on an electron, on the other hand, is incredibly tiny ($1{,}6 \times 10^{-19}$C) so, even though the value og **k** is large and there are a few billion electrons in a spark of static electricity, we are still dealing with a very small force indeed.

USING COULOMB'S LAW TO CALCULATE ELECTROSTATIC FORCE

One coulomb is a very large charge. Problems of this kind, therefore, usually speak of **nano**coulombs (nC) or 10^{-9} coulombs or **micro**coulombs (μC) or 10^{-6}C because these charges are more realistic. (*One nC of charge still represents many, many electrons. If the charge on a single electron is $1{,}6 \times 10^{-19}$C,* ***can you show that, when there is one nC of negative charge on a body there must be an excess of about six billion electrons?)***

PROBLEM

Two identical, charged bodies are brought into contact (i.e. they touch) and are then moved apart to a distance of 20 cm. If the bodies have initial charges of +10μC and -2μC, what is the size of the force on each body after they have touched and been separated?

Answer: In problems like this, we must first determine the charges on the two bodies AFTER they have touched. When they touch the electrons from the negatively charged body will move onto the positively charged body. Because the bodies are identical, the negative charge will continue to flow until the charges on the two bodies are equal.

To calculate how much charge is on each body when they are separated we do two simple calculations: (a) How much charge is available altogether? (b) What is half of the total charge?

$$Q_{TOT} = +10\mu C + -2\mu C$$
$$= +8\mu C$$

Half of this charge will be left on each body i.e. $+4\mu C = +4 \times 10^{-6}C$

When the two bodies holding this amount of charge are separated by 20cm, the force between them is:

$$F = \frac{kQ_1Q_2}{r^2} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 4 \times 10^{-6}}{(20 \times 10^{-2})^2}$$
$$= 3{,}6\ N$$

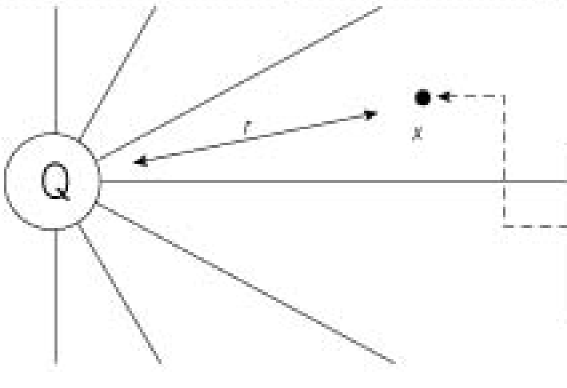
(This force is equal to the gravitational force, i.e. the weight of a 360g mass.)

THE STRENGTH OF A FIELD

A field's strength can be found by measuring the size of the force acting at that point. This could be done by measuring the force on a mass placed at a point in a gravitational field or a charge at a point in an electric field. To measure either of these forces in a consistent way, we **must measure the force on one unit of either mass or charge**.

As we know, a unit of mass is the kilogram. Therefore, a convenient way to measure the strength of the field at a point in that gravitational field is by measuring the force per unit mass, i.e. the force of gravity per kilogram at that point.

The principle is exactly the same when we measure **electric field strength** (also called **electric field intensity**) at a point in an electric field. The symbol is **E** and obviously this is "the force per unit charge" or the force per coulomb.



To find the field strength at the point X, we place a charge

$\mathbf{q_t = 1C}$ *at that point.*

Having placed the 1C charge at **X**, we define the strength of the field at that point as:

$$E = \frac{F}{q_t}$$

But we know, from Coulomb's Law, the force that both **Q** and $\mathbf{q_t}$ will experience. (Remember that the force of repulsion or attraction – we don't know which because we have not specified the signs on the two charges – is the same on both charges as we have an action-reaction pair.) So we know that the electric field strength at **X** is:

$$E = \frac{\frac{kQq_t}{r^2}}{q_t} = \frac{kQ}{r^2}$$

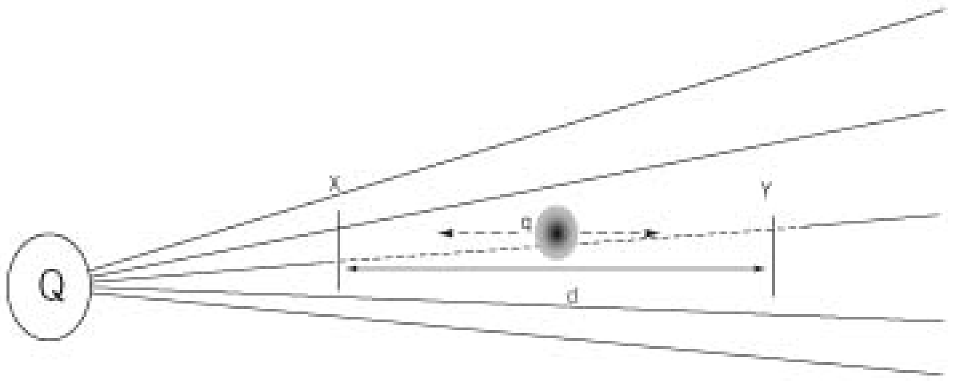
Now, as you can readily see, the electric field strength (**E**) at point **X** (which can be a point anywhere in the field at a distance or **r** from **Q**) depends only on the size of **Q**, the charge whose field we are interested in, and $\mathbf{r^2}$, the square of the distance from the charge **Q**. Again we note that there is an inverse square relationship so the electric field strength or intensity decreases rapidly as **r** increases.

ELECTRIC POTENTIAL AT A POINT IN AN ELECTRIC FIELD

When we hear the word "potential" mentioned in physics we are immediately on the alert for something to do with work and energy. Here is the test: **Was work done in order to move something from one point to another?** If the answer is "yes" then we have a field. If it is a gravitational field then somewhere there is a mass that is causing the field. And if it is an electric field then somewhere there is a charge causing the field. (If we place a "test mass" or a "test charge" at a point, then if there is a gravitational or an electric field there, the test objects will experience a force.)

Now if we are dealing with an electric field and we move a charge from one point to another in the field like this:

PHYSICAL SCIENCES GRADE 11 LESSON NOTES



Study the following table and make sure you understand the information it gives and how the conclusions are arrived at. The **three important facts** you must remember are:

1. If an outside agent (like you) does work on a charge (by exerting a force on the charge and moving it some distance (**d**), then the work done is stored IN or BY THE ELECTRIC FIELD as electrical potential energy.
2. If a charge moves in a field without an outside agent applying a force, then WORK IS DONE ON the charge BY THE FIELD (and the charge will probably gain in kinetic energy (E_k) because it will increase its velocity).
3. If the field does work on a charge then the field loses an amount of energy equal to the work it has done.

	q goes from X to Y	q goes from Y to X
q positive Q positive	**q** experiences a force away from **Q** (in the direction **X to Y**) so work is done **BY** the field on **q** as its kinetic energy increases.	Work is done **ON q** against the repulsive force (in the direction **X to Y**) and the work done is stored **IN** the field as potential energy.
q positive Q negative	Work must be done **ON q** to move it in the direction **X to Y**, and the work done is stored **IN** the field as potential energy.	**q** experiences a force towards **Q** (in the direction **Y to X**) so work is done **BY** the field on **q** and its kinetic energy increases.
q negative Q positive	**q** experiences a force towards **Q** (in the direction **Y to X**) so work is done **ON q** and this is stored **IN** the field as potential energy.	**q** experiences a force towards **Q** (in the direction **Y to X**) so work is done **BY** the field on **q** and its kinetic energy increases.
q negative Q negative	**q** experiences a force away from **Q** (in the direction **X to Y**) so work is done **BY** the field on **q** as its kinetic energy increases.	Work is done **ON q** against the repulsive force (in the direction **X to Y**) and work done is stored **IN** the field as potential energy.

Make sure that you can follow the logic in each case and understand the difference between work done ON the field (in which case, energy is stored IN the field as potential energy) and work done BY the field, in which case the field loses potential energy and the charge gains kinetic energy.

	q goes from X to Y	q goes from Y to X
q positive Q negative	Work must be done **ON q** to move it in the direction **X to Y**, and the work done is stored **IN** the field as potential energy.	
q negative Q positive	**q** experiences a force towards **Q** (in the direction **Y to X**) so work is done **ON q** and this is stored **IN** the field as potential energy.	

Read the two shaded blocks again. The wording is different. Are they saying the same or different things? Which wording do you think is preferable?

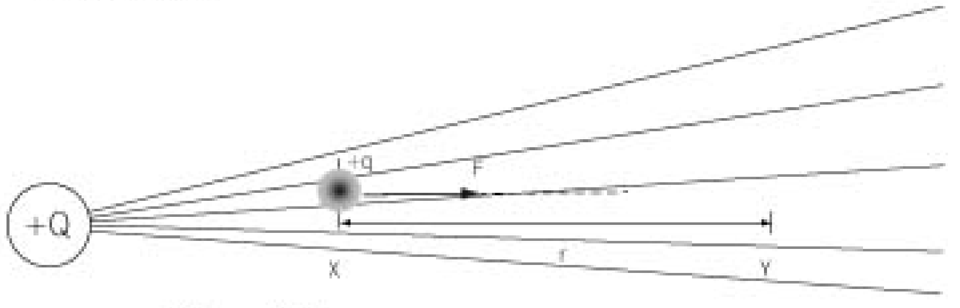
POTENTIAL DIFFERENCE IN AN ELECTRIC FIELD

The term "potential difference" in physics is just as simple as it seems. If work is done on a charge to move it from **X** to **Y** then the work done per unit charge in getting it from **X** to **Y** is the **potential difference**. In fact, this is how we define potential difference.

$$P \cdot d = \frac{\text{Word done}}{\text{Charge}}$$

$$V = \frac{W}{Q}$$

- We say that if two points in the field are at different potentials then there is a potential difference between them.
- Work must be done to move charge from a low potential to a high potential – in other words, if we do work on a charge (as in a battery) we will increase its potential.
- When a charged body has had work done on it we say that potential energy has been stored in the field.



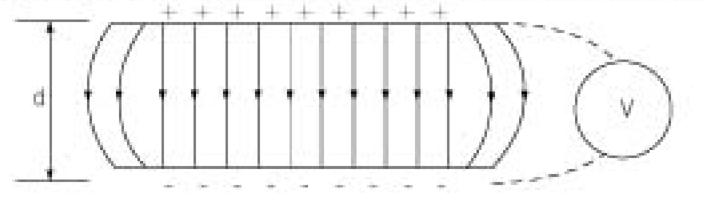
$$W = F \cdot r = \frac{kQq}{r^2} \times r = \frac{kQq}{r}$$

The potential energy of a charge **(q)** because of other charges **(Q)** can be given as $U = \frac{kQq}{r}$ and can be used to calculate the potential energy of a charge because of other charges. We can also say that "the electric potential at a point is the electrical potential energy per unit charge", i.e. **the potential energy a positive test charge would have if it were placed at that point**.

UNIFORM FIELDS

So far we have only looked at fields around single charges or pairs of charges. Those fields are called "radial fields" because the field lines "radiate" outwards from the charge. If we look at the field between charged, parallel plates, we get a different and surprising picture. Instead of the field getting weaker as we get further from the charge, we get a field of uniform strength. This means that wherever we go in the field between the plates, the field strength is the same. (The exception is that when we are close to the edges of the plates, the field becomes "non-uniform".)

(Note that we use the word "constant" to denote something that does not vary with time, i.e. as time elapses the quantity or characteristic remains unchanging and invariable. The word uniform is better used when we are not dealing with time elapsing. A uniform field is one in which the field strength is the same ***at every point*** *in that field. A constant field is one that is the same no matter* ***when*** *you measure it. In the next lesson you will see that transformers work because of changing electrical and magnetic fields, i.e. field that are neither constant nor uniform.)*



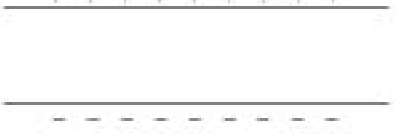
ELECTRIC FIELD STRENGTH IN A UNIFORM FIELD

It is a little more difficult to derive an equation for the strength of a uniform electric field. But the equation is a simple one:

$E = \frac{V}{d}$ where **V** is the potential difference across a pair of parallel plates and **d** is the separation of the plates (the distance between them), as one sees in the diagram above. The unit of electric field strength when calculated in this way is "volts per metre" or $\mathbf{Vm^{-1}}$. This is the official, SI unit but NC^{-1} is still acceptable, especially when using $\mathbf{E = F/q}$.

CAPACITANCE

As we know, two parallel, conducting plates can store charge. We represent this in the familiar way:



You know from experience that we can get small electric shocks during dry weather. Static charge builds up on your body from the friction of cloth on cloth or cloth on plastic surfaces, etc. When one gets close enough to a conductor (say a door handle), the static charge will move off the body suddenly as a spark.

The important thing to realise here is that the spark jumps when there is a combination of two factors: a sufficient amount of charge and a small enough gap to spark (or "arc") across. It is also important to remember that when two plates hold charge (as in the diagram above), there will be a measurable potential difference across the plates. This also means that by applying a potential difference across the plates it is possible to maintain charge on the plates.

Question: If we carry on increasing the potential difference across to parallel plates, will the charge build up just keep on increasing and increasing?

The answer to the question lies in ...

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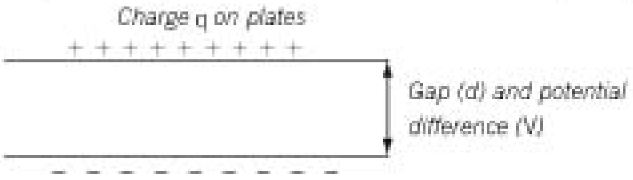
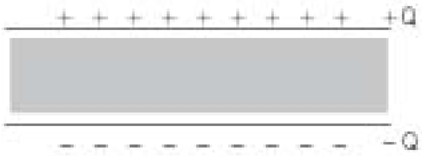
CAPACITANCE

Capacitance is a measure of the amount of electric charge that can be stored or kept separate (without sparking or arcing across the separating gap) for a given electric potential (or potential difference). (*Obviously, the word "capacitance" is related to "capacity".*)



When the potential difference (**V**) increases, the strength of the field increases. This makes it easier for the charge (**q**) to move. Similarly, if **V** remains constant and the separation (**d**) decreases, electric field strength increases and charge will move.



It is also obvious that any non-conducting material (it could be an insulator) between the plates will affect the amount of charge that can be held on the plates. We call any material that is placed between two charged plates a **dielectric**. The type of material can be anything from air to carbon to leather to paper. Every type of material will have a different effect on capacitance because the molecular structure of every type of material is different and hence its effect on charge is different. *(We would not place a metal between the plates. Why not?)*

A set of parallel plates which can store electrical charge is called a **capacitor**. In capacitors there are two electrodes or conductors which are insulated from one another. The charge on the conductors (they could be plates or concentric cylinders, for example) is +Q and −Q. **V** represents the potential difference between the electrodes/conductors.

The relationship between **capacitance (C)**, **charge (Q)** and **potential difference (V)** is given as:

$$C = \frac{Q}{V}$$

Capacitance, therefore, can be defined as the charge stored per volt. The unit of capacitance is **one farad** (1F). 1 farad = 1 coulomb per 1 volt.

Remember that capacitance is telling us about the amount of electrical energy that can be stored because that is the amount of energy that is released when we have a **dis**charge of charge.

Capacitance tells us the maximum charge that can be contained on two plates for a given potential difference. **Q = CV** (from the formula for the definition of capacitance) tells us that if the capacitance of a pair of plates is C, then for a potential difference V, the maximum charge the plates can hold is Q.

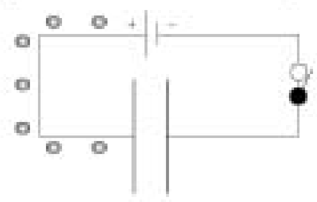
CAPACITORS

The capacitance of the majority of capacitors used in electronic circuits is much smaller than one farad, mainly because the size of one coulomb of charge is ENORMOUS. (It works out to $6{,}25 \times 10^{18}$ electrons, i.e. 6 250 000 000 000 000 000!). The most common units of capacitance in use in the electronics industry are the millifarad (**mF**), microfarad (**µF**), the nanofarad (**nF**) and the picofarad (**pF**). A capacitor's capacitance can be calculated if the dielectric properties of the insulator between the conductors are known.

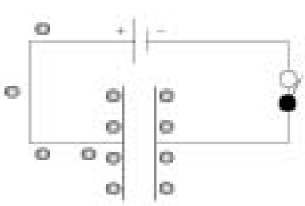
A CAPACITOR IN A CIRCUIT

As we have seen, a capacitor in a circuit will store energy and when a capacitor is used as a circuit device that will be its main function. When the value of Q/V for a capacitor is exceeded, it discharges. That means there is a sudden surge of charge in the circuit, i.e. a sudden of current. You will see that as the charge builds up and discharges we get a current that pulses. The amount of resistance in the circuit (**R**) and the size of the capacitance(**C**) can be used to determine the length of time it takes for the capacitor to discharge (**t = RC**). By adjusting **RxC** to the desired value, one can build a simple circuit that "pulses" at very regular intervals.

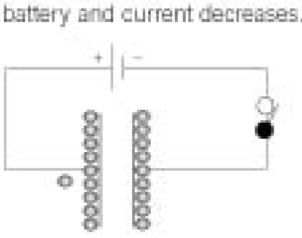
Study these diagrams to get a picture of the flow of charge in a capacitive circuit.



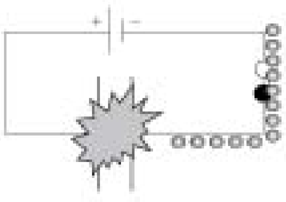
The switch is closed and positive charge begins to flow from the battery towards the positive plate of the capacitor.



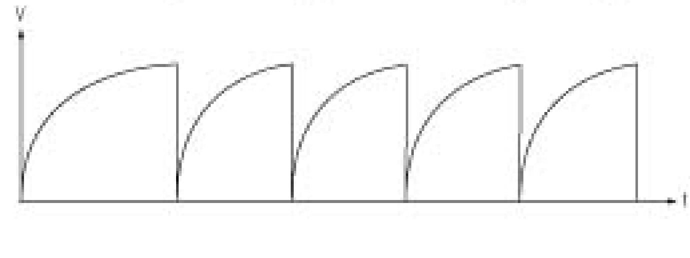
As charge builds up on the red plate, opposite charge will build up on the blue plate. As this happens, the potential difference across the capacitor (red and blue plates) builds up. As charge builds up on the red plate, it becomes more and more difficult for positive charge to leave the positive terminal of the battery and current decreases.



At some point, the capacity of the capacitance to hold charge is reached and the capacitor discharges, releasing a large amount of charge very suddenly. This can be measured by an ammeter as a surge of current.



A graph of potential difference across the capacitor plates vs time can be drawn. (Make sure that you can match the parts of the graph with the build up of charge.)



SOMETHING INTERESTING ABOUT MECHANICAL FORCES

When you sit on a chair, why don't you fall through it? After all, Earth's gravity attracts you towards its centre so there is a contact force acting downwards on the chair seat. The simple answer is that the chair is able to exert an equal and opposite reaction to the action that you exert on the chair. The reason the chair can exert an action is that it has the strength to do so.

Now all of this is pretty obvious so far. But as we look at the contact forces acting downwards on the chair's seat (and upwards on the sitter's seat!), we come to a startling realisation. All the contact forces are electrostatic in nature! What we refer to as the strength of the chair comes down to the strength of the bonds between molecules in the wood or the plastic of the chair seat. (Think about this statement: *All chemistry is about the forces between charged bodies.*)

MILLIKAN'S OIL DROP EXPERIMENT

This work, sometimes jokingly referred to as "Millidrop's oil can", is not included in the Grade 11 Physical Sciences curriculum. Yet it is an interesting and important application of the electrostatic theory that you have been studying. Robert Millikan is regarded as one of the finest experimental physicists of the 20th century. He made contributions in many areas of physics but is most famous for his experimentation that gave us the accurate value of the charge on an electron. As far as we can tell about the accuracy of the experimental methods used, the charge on electrons and protons are identical.

In recent years, there has been some suspicion cast on the unerring accuracy of Millikan's results and the fact that his determinations of the electronic charge varied so little from experiment to experiment.

A suggestion has been made – though certainly ***not*** proved – that Millikan may have made accurate estimates of the correct value and then made sure that any experimental measurements outside of a desired range of values were excluded from his calculations. This kind of suspicion arises from time to time and is taken very seriously by the scientific community. The reason the community is so sensitive to the issue is that the integrity of the entire community is damaged when the accusations – be they true or false – surface into the public domain. The scientific community polices itself very strictly on matters of integrity and transparency.

TASK: SCIENTIFIC ETHICS

Debate or write an essay on the issue of honest reporting of scientific results and scrupulous care in adhering to accepted scientific methods and procedures. Why should the scientific community be so careful to avoid this kind of behaviour by scientists that it leads to such accusations? What effect do scientific scandals have on (a) the scientific community and (b) the public perception of scientific inquiry?

PHYSICAL SCIENCES GRADE 11 LESSON NOTES

CHARGED PARTICLES

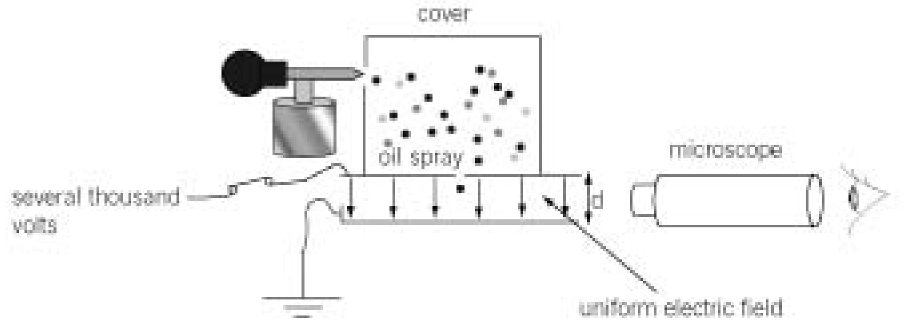
By the end of the 19th century, the English scientist, JJ Thompson, had proposed a model of the atom involving negative charge stuck into a positively charged mass. This was his famous "plum pudding model" of the atom. Early in the 20th century, Ernest Rutherford had advanced the model to that of a tiny, positive nucleus surrounded by negative charge. The negatively charged particle, called an electron, was known from so-called cathode rays: negatively charged particles that flowed from a negatively charged electrode. It was guessed that the negative particles in the atom were electrons, but it remained for Robert Millikan to measure their charge.

THE EXPERIMENT

Millikan performed this in 1909 to determine the size of the charge on a single electron. What he did was to charge tiny drops of oil and measure how strong an applied electric field had to be in order to stop the oil drop from falling. Millikan was able to determine the mass of the oil drop. From this he could calculate the force of gravity on the drop and hence the electric charge that the drop must have. By varying the charge on different drops, he noticed that the charge was always a multiple of about -1.6×10^{-19}C. He correctly deduced that this was the charge on a single electron. We now realise what this meant: electrons carrying this unit of charge.

MILLIKAN'S OIL DROP APPARATUS

Here's how the apparatus worked:

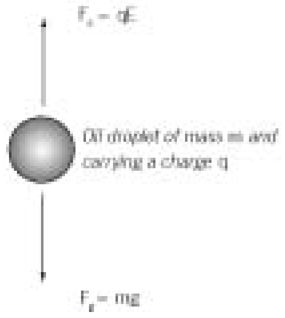


(The diagram is taken from wikipedia)

1. An atomiser sprays a fine mist of tiny oil droplets into a chamber.
2. Some of the tiny droplets fall through a hole in the upper of two, parallel, charged plates which were a distance (**d**).
3. As the droplets fall, they reach terminal velocity. Using a microscope, their terminal velocity is measured and by using a suitable formula, the mass (**m**) of an oil drop can be calculated.
4. Next, Millikan applied a charge to the falling drops by illuminating the space between the parallel plates with x-rays. X-rays can ionise the air and electrons attach themselves to the oil drops.
5. By attaching a battery to the parallel plates, he was able to apply an electric potential difference (**V**).
6. The electric field produced between the plates by the **pd** acted on the charged oil drops. When the **pd** was just right, the electromagnetic force just balanced the force of gravity on a drop, and the drop would remain stationary.

HOW MILLIKAN CALCULATED THE ELECTRONIC CHARGE

When a drop is suspended (zero velocity) or moving with a constant velocity, the weight (**mg**) of the oil droplet is exactly equal to the applied electric force (**qE**).



The value of **E**, the applied electric field, can be calculated because the uniform field strength between two parallel plates is **E** = **V**/**d** and both **V** and **d** can be measured accurately; **m**, the mass of the oil droplet, can be calculated by measuring its density and its diameter (and hence its volume); and **g**, the acceleration caused by gravity at the location of the laboratory, can be determined very accurately. Thus one can solve, for **q**, the charge on the drop:

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$$q \times E = m \times g$$

$$\therefore q = \frac{mg}{E}$$

$$\therefore q = \frac{mgd}{V}$$

Using this equation, Millikan determined the charge on a drop of oil. He repeated the experiment many times. By varying the strength of the x-rays used to ionise the air, differing numbers of electrons would arrive on the oil droplets. But, although he obtained different values for **q**, the value was always very close to a multiple of $\mathbf{-1.6 \times 10^{-19}}$C. He deduced that this was the charge on a single electron.

Activities

QUESTIONS

1.1 *Which of the following is a vector quantity?*

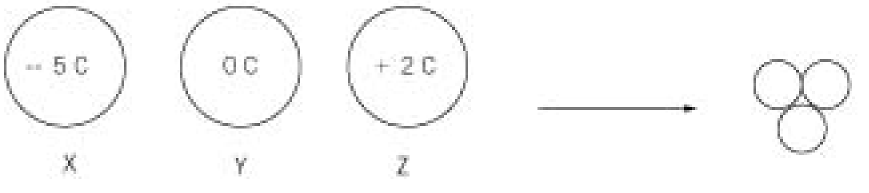
(a) Charge (b) Field strength

(c) Potential difference (d) Current

1.2 *The electrostatic force on a charge between two charged, parallel plates, where the symbols have their normal meanings and units, is given by:*

(a) ma (b) $\frac{q}{E}$ (c) $\frac{QV}{d}$ (d) $\frac{k \cdot Q_1 Q_2}{r^2}$

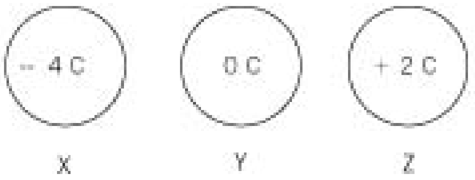
1.3



If three identical charged spheres, X, Y and Z, are placed together in a triangular arrangement (i.e. each sphere touches the other two) and are then separated, the charge on each sphere, in coulombs, after contact has been broken is:

(a) –1 (b) +1 (c) –2 (d) –3

1.4



X, Y and Z are three identical charged spheres, as shown. First X and Y are brought into contact and then separated. Then Y and Z are brought into contact and separated. The charge on Y will now be:

(a) –1 (b) 0 (c) +1 (d) +2

1.5 *The electrostatic force, in newtons, on a positively charged particle (Q = +5 mC) when placed in a field with an electric field strength of 20 $V.m^{-1}$ is:*

(a) 0,1 (b) 0,25 (c) 100 (d) 20

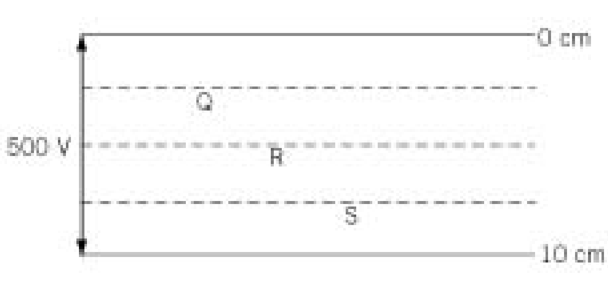
1.6 *An α-particle (He^{2+}) is a Helium atom with two electrons removed, i.e. it is an ion. The repulsive force between two α-particles is **F** when they are at a distance **r** metres apart. When an α-particle and an electron are **0,5 r** apart, the magnitude of the force between them is:*

(a) 8 F (b) 4 F (c) 2 F (d) 1 F

1.7 *When an electron and a proton are at a distance **r** metres apart, the force between them is **F**. When the distance between the two particles is increased to **2 r**, the magnitude of the force between them is:*

(a) 4 F (b) 2 F (c) 1 F (d) 0,25 F

1.8



Two parallel metal plates (shown as dark lines) are exactly 10cm apart. If the potential difference across the plates is a steady 500V, which of the following statements concerning electric field strength is the most accurate?

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PHYSICAL SCIENCES GRADE 11 LESSON NOTES

***E**, in appropriate units, at ...*

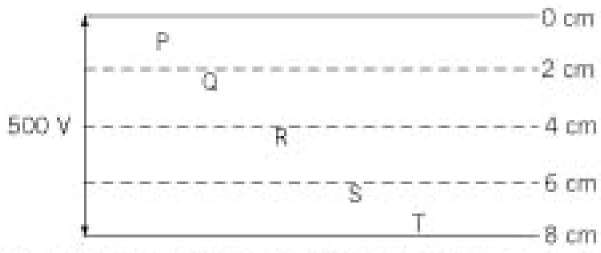
(a) point Q has a lower value than at S

(b) point R is $250 Vm^{-1}$

(c) point S is $5\,000 Vm^{-1}$

(d) points Q, R and S has the same magnitude

1.9



Two parallel metal plates (shown as dark lines) are exactly 8cm apart. If the potential difference across the plates is a steady 500V, which of the following statements concerning electric field strength is the most accurate?

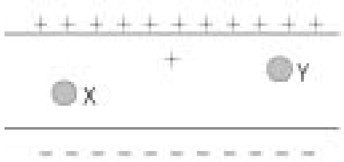
(a) ***E*** at point P is less than ***E*** at point T

(b) ***E*** at point R is $12\,500 Vm^{-1}$

(c) ***E*** at point S is $6\,333,33 Vm^{-1}$

(d) ***E*** at points P and T is $6\,250 Vm^{-1}$

1.10



In this diagram, two charged particles, X and Y, are pictured between two charged parallel plates. X is stationary and Y moves upwards at a constant velocity. What are the charges on X and Y respectively?

(a) positive, negative (b) pos, pos

(c) neg, neg d) neg, pos

1.11 *The potential difference between two conducting plates, A (at a positive potential) and B (at a negative potential), is 2 volts. If an electron (charge = $-1,6 \times 10^{-19}$C) is moved from A to B at a constant speed, its potential energy will:.*

(a) remain unchanged (b) decrease by 2J

(c) increase by $1,6 \times 10^{-19}$J (d) increase by $3,2 \times 10^{-19}$J

1.12 *The potential difference between two charged plates, A (positive) and B (negative), is 2 volts. If a proton (charge = $+1,6 \times 10^{-19}$C) is moved from A to B, its potential energy will:.*

(a) remain unchanged (b) decrease (c) increase

1.13 *Two vertical, parallel plates are 20 mm apart.The electric field strength between the plates is $2,5 \times 10^{-4} Vm^{-1}$.*

Calculate the horizontal and vertical forces acting on the proton.

1.14 *Two point charges of $+ 8 \times 10^{-2}$μC and -4×10^{-2}μC are placed 120 mm apart.*

(a) What is the resultant field intensity at a point midway between the two charges?

(b) If a charge of 4μC is placed at the midpoint, what force will it experience?

1.15 *Two parallel, charged plates have a separation of 40 mm and a potential difference of 400 V between them.*

(a) What is the electric field strength between the two plates?

(b) An electron starts from rest at the negative plate. What is its speed when it reaches the positive plate? (Use the formula for kinetic energy gained.)

$[m_e = 9,0 \times 10^{-31}$kg ; $q_e = 1,6 \times 10^{-19}$C]

1.16 *The flat base of a thunder cloud is 150 m above the ground. Lightning originating from the cloud base strikes the ground. The charge being discharged from the cloud is estimated to be 20C and the duration of the strike (i.e. the time taken for all the charge to move from the cloud to Earth) is measured at 0,5 s.*

a) What is the average size of the electrical current between the cloud and Earth during the strike?

b) If the potential difference between the cloud base and Earth is estimated to be 1MV (one million volts), what is the approximate average electric field strength in the space between the cloud base and the ground?

c) How much energy is released as a result of the lightning strike?

1.17

Two small, identical metallic spheres are placed 20 cm apart. They are then charged with +8nC and –4nC of charge, respectively.

a) Calculate the magnitude and direction of the force exerted by X on Y.

b) i) Calculate the magnitude and direction of the electric field intensity at X.

ii) What is the origin of the electric field at X?

c) Calculate the magnitude of the force between the spheres if they are brought into contact and then removed to a distance of 60 cm apart.

1.18 *A capacitor discharges when a charge of $2,4 \times 10^{-10}$C accumulates on one of its plates at a potential difference of 36 V.*

What is the capacitance of the capacitor?

1.19 *A 1,5 A current in a capacitive circuit flows for 2ms before the 12μF capacitor in the circuit discharges. Assuming that all the charge that flowed in the circuit is on one of the capacitor plates when it discharges:*

a) How much charge accumulates on the plates?

b) What is the pd across the plates of the capacitor when it discharges?

Answers and assessment

1.1 b
1.2 c
1.3 a
1.4 b
1.5 a
1.6 c
1.7 d
1.8 d
1.9 d
1.10 c
1.11 a
1.12 a
1.13 horizontal 0N
vertical 0,0125N

1.14 a) $3 \times 10^5 Vm^{-1}$ (or NC^{-1}) towards the –4μC charge
b) 1,2N
1.15 a) $10^4 Vm^{-1}$
b) $1,2 \times 10^7 ms^{-1}$
1.16 a) 40A
b) $6,67 \times 10^3 Vm^{-1}$
c) 2×10^7J
1.17 a) $7,2 \times 10^{-6}$N towards X
b) i) $900 NC^{-1}$ towards Y
b) ii) Charge at Y
1.18 6,67pF
1.19 a) $3,0 \times 10^{-3}$C
b) $2,5 \times 10^2$V

TEACHER TIPS

Formulae for electrostatics

$F = \frac{kQ_1Q_2}{r^2}$	$(k = 9 \times 10^9 Nm^2C^{-2})$	$V = \frac{W}{Q}$
$E = \frac{F}{q}$		$W = QEs$
$E = \frac{kQ}{r^2}$	$(k = 9 \times 10^9 Nm^2C^{-2})$	$E = \frac{V}{d}$
$C = \frac{Q}{V}$		$Q = It$

Make sure that learners understand the meaning and context in which the equations are used. In general, learners will benefit from knowing how equations are derived even though they are not required to do those derivations.