STUDY Mate

Learning channel

Department of Education

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Learning activities for Grades 8–11

Education Minister Naledi Pandor

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Dear Learner

This is a wake-up call for those of you still in winter hibernation mode. Even though many of you are about to have a short break, this is the time of year to start thinking seriously about those end-of-year exams. We know that being on holiday means rest and relaxation, but if you spend only an hour or two a day going over your school work you will be so much better prepared when exams come along. Use this issue of Study Mate to revise and practise your work - we have Maths, Maths Literacy, English writing, Life Sciences and History for you this week. Other subjects will be covered in the next few weeks. Those of you taking a break, enjoy! And all of you, keep focused.

Just think: in less than three months the school year will be over!

ENGLISH WRITING - TOPIC ANALYSIS

A task topic or an essay topic requires you to respond to it. It is necessary to understand what you are being asked to do if you are to do this properly. This section takes you through steps to enable you to analyse and understand such topics.

Let's analyse some topics one at a time.

Copy out the topic so that you can use different coloured pens or pencils to circle or underline its different sections, and the steps you are going to take to analyse them.

TOPIC ONE
Write a letter to your local sports club, cancelling your arrangement to hire their hall for a wedding for the following month. Include all necessary detail.

Step one: What form is the response going to take?
A letter

Step two: What word is used to describe the task?
Cancelling

Step three: What is the task?
Cancel an arrangement to hire a hall
Step four: What are the details?

- Who are you to write to? – the secretary/chairperson of your local sports club.
- What is the purpose of the letter? – to cancel the hiring of the hall.
- What other information will need to be included? – you have already arranged to hire the hall on a certain date. You will have to name the date (the following month) and time. You can briefly give a reason for cancelling the arrangement. You might want to refer to a deposit paid, and that you realize you will forfeit it.

Now that you know what to do, you are ready to plan and write the letter.

TOPIC TWO
Write an essay describing a cultural event you have participated in recently.

Step one: What form is the response going to take?
An essay

Step two: What word is used to describe the task?
Describing

Step three: What do you have to describe?
A cultural event

Step four: What are the details?

It must be a cultural event that you have participated in recently. What is a cultural event? Any celebration or festival or day of remembrance that your culture celebrates. It could simply be a family birthday, such as a 21st, it could be a feast day, a religious day, a national day, any such day. In what way did you participate? You could have attended, or helped to arrange, or simply witnessed or observed in some way. You are to describe the event, what happened, what it celebrated, what things looked like, how the event took place.

Once you have understood all of this, you are ready to plan and write your essay.

TOPIC THREE
The rights of criminals are as important as the rights of their victims. Write an essay in which you take up a position on this statement and argue your point of view.

TOPIC FOUR
The rights of criminals are as important as the rights of their victims. Write an essay in which you discuss this statement.

Step one: What form is the response going to take?
An essay

Step two: What word is used to describe the task?
Discuss

Step three: What do you have to discuss?
Whether the rights of criminals and their victims are equally important.

Step four: Look at the details.
A statement is made. It tells us that the person who made the statement thinks that the rights of criminals are as important as the rights of their victims. You are not asked to agree or disagree with the statement but to discuss or talk about it. You can give both sides of the argument, unlike in the previous essay topic.

Once you have understood the topic you are ready to gather ideas, and then to plan and write your essay.

- When you analyse a topic, follow this method:
- Underline or circle the different parts with different coloured pens.
- Write out your understanding of what it is that you have to do.
- Make sure that you understand what your task and purpose are before you begin to even plan.
- Make sure you understand the terms used.

Here are some topics for you to analyse. Follow the same procedure as above. In addition write a sentence, using your own words as far as possible, explaining what it is you are being asked to do.

Exercise 1
Littering has become a national problem that requires urgent attention. Discuss.

Exercise 2
The fight against HIV/Aids must be carried out on our school playgrounds. Discuss this statement with reference to the Department of Education's plans to extend HIV/Aids education to the primary schools.

Exercise 3
Write an essay in which you argue that caring for the environment is the responsibility of everyone.

Exercise 4
Write a letter to your local newspaper, complaining about the litter in your area.

Exercise 5
Write a letter to your neighbours, complaining about their noisy dogs.

Exercise 6
Terrorism's aim is to spread fear and panic. The only defence is to ignore it. Write an essay in which you argue against this statement.

Exercise 7
Tolerance is not developed through ignoring differences but by acknowledging and accepting them. Write a speech for a headmaster at a multi-cultural high school to make to the students on the first day of the new year.

Exercise 8
You ordered an exercise machine through Verimark. They have sent you the wrong one. Write a letter, returning the product and requesting that it be replaced with the one you ordered.

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Exercise 9
Write a letter to your pen pal in another country, inviting him or her to come and spend the month of December at your home with you. Include details about how you will spend the month, and why he or she will find it enjoyable.

Exercise 10
Choose one of the items below. Write an explanation of how it works. Your explanation will appear in a book for teenagers entitled How Things Work.

- A vacuum cleaner
- A can opener
- A hair dryer
- Any technical apparatus such as a television, computer, cell phone, washing machine, microwave oven.
- Any mechanical apparatus such as a car jack, lawn mower, exercise equipment.

TOPIC ANALYSIS ANSWERS

Exercise 1
Littering has become a national problem that requires urgent attention. Discuss.

Step one: An essay.
Step two: Discuss.
Step three: Littering, that it is a national problem, that it requires urgent attention.
Step four: A statement is made. I will have to define littering, explain why it is a national problem, and why it requires urgent attention.
Step five: I must talk about the way the whole country is being polluted by plastic bags, papers and other litter, the effects this is having, and therefore why something needs to be done about it.

Exercise 2
The fight against HIV/Aids must be carried out on our school playgrounds. Discuss this statement with reference to the Department of Education's plans to extend HIV/Aids education to the primary schools.

Step one: An essay.
Step two: Discuss.
Step three: I must discuss whether the fight against HIV/Aids must include primary school children. HIV/Aids education is to be included in the curriculum.
Step four: A statement is made which supports the new policy of the Education Department. I can talk about the advantages and disadvantages of this new policy.
Step five: I must talk about how HIV/Aids is spreading at such a rate that even primary school children are affected. The Education Department is addressing the problem by teaching the children about the disease. I will give the positive and negative sides to this approach.

Exercise 3
Write an essay in which you argue that caring for the environment is the responsibility of everyone.

Step one: An essay.
Step two: Write an argument.
Step three: I must argue that each person has the responsibility of looking after our environment.
Step four: This topic tells me which side of the argument to take up. I will have to decide what is meant by the environment, and in what way individuals can be responsible for looking after it. The topic assumes that some people think that it is not their responsibility, but that of government or other organizations.
Step five: I am going to give reasons why looking after our countryside and resources such as water, plants, the ozone layer (you can mention other things that you would include in 'environment') is not only the work of governments and environmental protection societies such as Green Peace, but that every human being can do something to contribute.

Exercise 4

Write a letter to your local newspaper, complaining about the litter in your area.

Step one: A letter.
Step two: Complaining.
Step three: Complain about the litter in my area.
Step four: I must write to my local newspaper. I must complain about what my area looks like, that is, the problems caused by litter.
Step five: My purpose is to complain. My letter will be read by readers of the newspaper. I want action to be taken as a result of the letter, so I will have to give details and be specific.

Exercise 5
Write a letter to your neighbours, complaining about their noisy dogs.

Step one: A letter.
Step two: Complaining.
Step three: Complain about the barking of my neighbours' dogs, which disturbs me.
Step four: The letter must be personal. I know my neighbour. His dogs bark and disturb me at night (or at any specific time). I want him to do something about this.
Step five: I am going to write to my next door neighbours, telling them that their dogs keep me awake at night because they bark so much, and asking them to do something such as lock them up, or put them on the other side of the house, so that they no longer bother me.

Exercise 6
Terrorism's aim is to spread fear and panic. The only defence is to ignore it. Write an essay in which you argue against this statement.

Step one: An essay.
Step two :Argue.
Step three: I must argue that to ignore terrorism is not the way to defend oneself or one's country against it.
Step four: I must first discuss the aim of terrorism as being to spread fear and panic. Then I must argue that to ignore terrorism is not the only defence against it. There are better ways of dealing with it.
Step five: I will define terrorism and discuss how it spreads fear and panic, and then show that one can fight it rather than ignore it as a way of defending oneself and one's country.

Exercise 7
Tolerance is not developed through ignoring differences but by acknowledging and accepting them. Write a speech for a headmaster at a multi-cultural high school to make to the students on the first day of the new year, explaining this concept to them.

Step one: A speech.
Step two: Write the speech for a headmaster to give.
Step three: Saying that for tolerance to develop, people have to see that they are different from each other, and accept that difference rather than pretending there are no differences.
Step four: The headmaster will be talking to students from different cultural backgrounds on the first day of school. His purpose will be to promote tolerance among the different race groups. He doesn't want the students to ignore their differences, but to explore them and find them interesting. He wants students to accept each other despite their differences.
Step five: I am going to write a speech that the headmaster of a school with different race groups can give on the first day of school to his students. He wants them to find their differences interesting and to accept them, rather than to pretend they are all the same and ignore their differences. Differences can be enriching.

Exercise 8
You ordered an exercise machine through Verimark. They have sent you the wrong one. Write a letter, returning the product and requesting that it be replaced with the one you ordered.

Step one: A letter.
Step two: Returning and requesting.
Step three: Returning the wrong product and asking them to send the right one.
Step four: My letter must be addressed to Verimark customer services. I ordered an exercise machine through them. They have sent me the wrong one. I am returning it with the letter, and asking them to send me the one I ordered.
Step five: I am letting Verimark know that they sent me the wrong exercise machine, which I have returned with the letter, and I am asking them to send me the right one, which I will identify.

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Exercise 9
Write a letter to your pen pal in another country, inviting him or her to come and spend the month of December at your home with you. Include details about how you will spend the month, and why he or she will find it enjoyable.

Step one: A letter.
Step two: Inviting and including.
Step three: Inviting my pen pal to visit me in South Africa in December and telling him/her about what we will do and why it will be enjoyable.
Step four: My pen pal lives in another country. He/she has never been to South Africa. I would like him/her to spend December, a summer month, with me here. I must tell him/her about all the exciting and interesting things to do that South Africa can offer, and how I will entertain him/her here.
Step five: I am writing a letter to invite my pen pal to visit me in South Africa in December, and telling him/her of all the exciting and interesting things we will do together in my country.

Exercise 10
Choose one of the items below. Write an explanation of how it works. Your explanation will appear in a book for teenagers entitled How Things Work.

- A vacuum cleaner
- A can opener
- A hair dryer
- Any technical apparatus such as a television, computer, cell phone, washing machine, microwave oven.
- Any mechanical apparatus such as a car jack, lawn mower, exercise equipment.

Step one: An explanation.
Step two: How it works.
Step three: I have to choose one of the listed items, and write an explanation of how it works.
Step four: The explanation is for a book for teenage readers. The book describes how different things work. I must choose one of the items listed. I must know exactly how it works in order to be able to describe it. My description will need to be detailed.
Step five: I am going to describe to teenage readers how a certain item works.

GRADE 11 LIFE SCIENCES: ENVIRONMENTAL STUDIES

Diseases and their impact on the Environment

1. INTRODUCTION

Diseases cause illness. Environmental conditions increase the chances of diseases being transmitted by air, water and some are land borne. Diseases are caused by a number of factors like:

- poverty
- malnutrition
- pathogens
- pollution and
- genetic disorders

2. DISEASE-CAUSING ORGANISMS

Infectious diseases are cause by pathogens. When diseases are passed from one organism to another, they are termed contagious or communicable diseases.

Pathogens are:

- viruses,
- bacteria,
- protozoans,
- fungi and
- worms.

Pathogens are located in air, water, soil and food. Virulence is the ability to cause disease.
Resistance is the organism's ability to fight the infection. Pathogens cause damage to the cells and may also produce toxins which effects the normal functioning of the cells. Pathogens go through an incubation period. Once incubation is complete, the infected host will become ill with symptoms like nausea, headaches, temperature and vomiting.

3. DISEASE TRANSMISSION

Direct transmission: This is when a pathogen is transmitted from one host to the next by coughing, sneezing, blood transfusions, and touch.

Physical contact: spreads diseases like leprosy, mumps, chicken pox, small pox, measles and many various skin diseases. Bites from infected dogs or bats will transfer rabies to humans. Sexual contact will transmit sexually transmitted diseases (STDs) e.g.: gonorrhoea, syphilis, HIV/AIDS and genital herpes.

Indirect transmission: pathogens are passed to an intermediate organism called a vector. Vectors like mosquitoes carry malaria, Tsetse flies carry sleeping sickness and rat fleas carry Bubonic Plague. Lifestyle plays a role in the transmission of diseases.

Think of examples of how diseases can be transmitted through lifestyle.

When fighting diseases, social, economic and medical interventions are necessary. Hygiene, sanitation, housing and nutrition can reduce diseases and outbreaks. Direct medical intervention includes prevention, vaccinations and treatment of diseases with the use of antibiotics.

4. AIRBORNE DISEASES

Pneumococcal disease:
This disease results in pneumonia of the lungs. The bacterial pathogen is called Streptococcus Pneumoniae. It is caused by the direct exposure to respiratory droplets when an infected person coughs on you. The incubation period is 1 to 3 days. Symptoms include fever and chills, headache, chest pain, coughing, shortness of breath and weakness (due to lack of oxygen). As the bacteria multiply, the alveoli in the lungs produce much mucus and surfactant. The surfactant builds up and causes excess fluid in the lungs. Double pneumonia means that both lungs become infected.

Meningococcal disease:
This disease results in meningitis in the bloodstream and infects the membranes (the meninges) of the brain and spinal cord. The bacterial pathogen is called Meningococcus. The disease is transmitted by air-borne bacteria and direct exposure to droplets. The incubation period is 2 to 10 days. Symptoms include a severe headache, fever, vomiting, stiffness of the neck and a rash on the skin. Meningitis is treated with antibiotics but 10 to 15% of infected people die. Those that survive, suffer permanent damage to the brain, loss of hearing, loss of sight, problems with the nervous system, and failure of the kidneys.

SARS (Severe Acute Respiratory Syndrome):
This is a viral respiratory illness caused by the Coronavirus. Cases of SARS were first reported in Asia in 2003. Within a few months, the World Health Organization (WHO) reported that the illness had spread to more than 20 countries across the globe. More than 8 000 people were infected with 774 fatalities. SARS is transmitted by air-borne viruses, direct exposure to droplets and contamination of objects by the droplets. The incubation period is 2 to 7 days. Symptoms are fever, headaches, a mild body ache and a dry cough. Then, fever increases and the patient develops pneumonia. Prevention and control: wear masks in public places and avoid touching contaminated surfaces

Person on a respirator

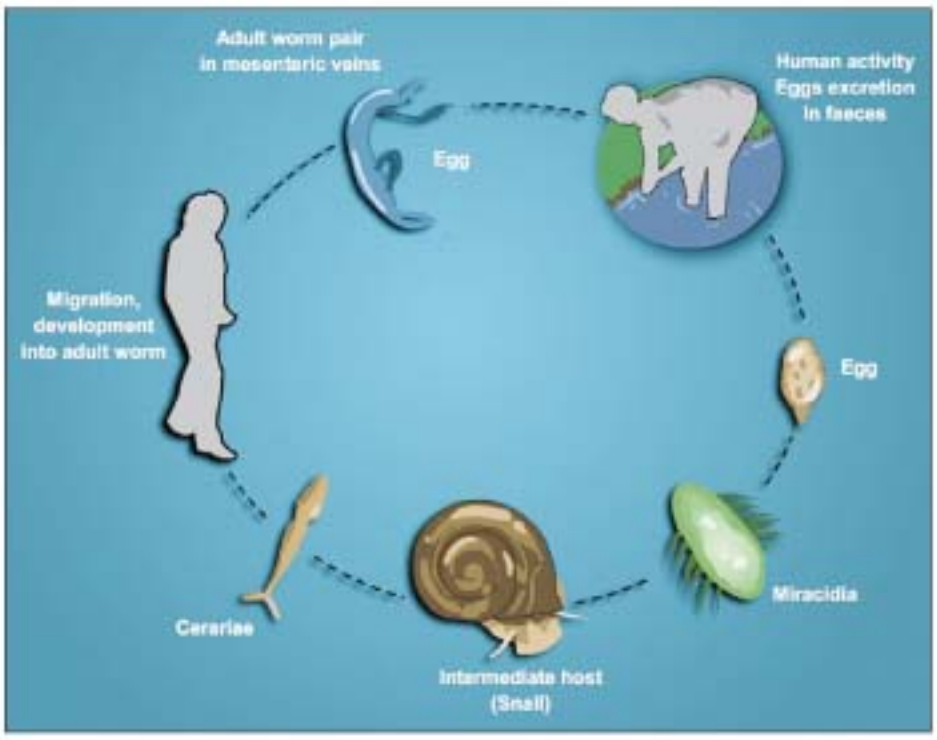
5. WATERBORNE DISEASES

Typhoid fever:
This is a bacterial infection of the intestinal tract caused by Salmonella typhi. Typhoid is water-borne but flies also transmit the disease when they come into contact with human faeces and urine. Bacteria multiply when in the digestive system and no symptoms appear during the incubation period of 1 to 2 weeks. The bacteria invade the bloodstream, causing symptoms like fever, vomiting and diarrhoea, rose-coloured spots on the trunk and an enlarged liver and spleen. Blood vessels may also be damaged which causes bleeding of intestines. Typhoid is treated with antibiotics and approximately 10% of infected people die. Typhoid occurs mainly in areas where sewage disposal is poor with unhygienic sanitation practises.

Cholera:
This is a bacterial infection of the digestive tract caused by Vibrio cholerae, which is transmitted through water supplies, by food contaminated with faeces and urine from infected people and direct contact with faeces and urine. Bacteria multiply when in the digestive system and the incubation period is 1 to 5 days. Symptoms are fever, vomiting and severe diarrhoea, loss of fluids and electrolytes, falling blood pressure and metabolic acidosis. Cholera is treated with I.V. drips and antibiotics and approximately 8% of infected people will die. If untreated, up to 50% of infected people will die. Cholera vaccines are available but are only 60% to 80% effective and last for 3 to 6 months. Cholera occurs mainly in areas where sewage disposal is poor with unhygienic sanitation practises.

Bilharzia:
The pathogen is a parasitic flatworm called Schistosome (blood flukes). There are two hosts: humans and snails - larvae are found in snails and the adult worms develop in humans.

Life cycle: the adult worm lays eggs → passed out through human faeces and urine into the water → eggs hatch into tiny, ciliated larvae → larvae invade host snail ® larvae develop to adult → leave the snail and swim in the water → larvae pierce the skin and enter human body → migrate to liver via the lymphatic system → develop in the liver causing fever and severe abdominal pains → mature larvae migrate from the liver through Hepatic Portal vein to urinary bladder or intestinal walls → adults lay eggs with sharp spines that pierce the walls of the bladder, the intestines and the rectum.



Bilharzia symptoms are fever and abdominal pains with initial infestation. The eggs have spines, which cause irritation, inflammation and blood in the faeces and urine.

Prevention and control:
- Bilharzia and mosquito larva are generally found in the same water.
- Spray stagnant water to kill snails.
- Destroy breeding grounds of snails like vegetation at the banks of ponds, rivers and streams.
- Introduce ducks to the ecosystem as they eat snails and are eco-friendly.
- Boil water before drinking and washing in areas prone to Bilharzia outbreaks.
- Do not swim in rivers with stagnant water

6. LAND-BORNE DISEASES
There are many land-borne diseases carried by vectors like mosquitoes and flies that transmit parasites.

Tetanus (commonly called lockjaw):
This is caused by a bacterium called Chlostridium tetani with transmission occurring when the skin is pierced. The incubation period is 3 to 4 days with symptoms like swelling of the area around the cut, painful spasms of muscles that begin at the jaws and spread all over the body, fever, the patient loses awareness as the nervous system is affected. Tetanus may result in death after 7 to 14 days if left untreated. Precautions and control would be to treat all cuts and wounds early by disinfecting and cleaning the injury properly. See your General Practitioner as soon as possible for a check-up and a tetanus injection that will provide immunity for a period of 5 years. People are usually infected by tetanus when they are bitten by dogs.

Botulism:
This is caused by a nerve toxin produced by the bacterium Clostridium botulinum and is transmitted when food is not properly processed, when spores contaminate a wound, and contaminated honey. The incubation period is 6 to 24 hours with symptoms like double vision, blurred vision, slurred speech, difficulty in swallowing, difficulty in breathing and muscle weakness. Paralysis may result if the toxin concentration is high. Hospitalization is essential and with early diagnosis, it is treated with an antitoxin. The antitoxin blocks the action of the toxin circulating in the blood. In severe cases, patients are placed on a ventilator (breathing machine) because lungs collapse and the body becomes paralysed. Recovery takes 2 to 3 months of intensive medical care. Precautions and control: canned and preserved food must be properly processed. Never eat food from a dented, damaged or bulging can. Ensure that cuts and wounds are properly cared for and covered when working in the soil, to prevent infection.

Traditional healers use antitoxins produced from specific mushrooms, leaves and roots. The recipes are kept secret and handed from healer to trainee. The mushrooms, leaves and roots are ground into a paste and applied to the wound. Some of the paste is also eaten by the patient. Large doses of Epson salts and liquorice root are given to the patient to cleanse and detoxify the blood.

ACTIVITIES

Activity 1 - Diseases and their impact on the environment.

1. In groups research one of three infectious diseases:
   One airborne (pneumonia/ meningitis) / one water-borne (typhoid fever/ bilharzias)/ one landborne (tetanus/ botulism).
   Your information should include:
   - Symptoms
   - Prevention
   - Treatment

2. Prepare a poster on the chosen disease to help make people aware.

HELPFUL TIPS

- Make sure that you are able to separate the various types of diseases and how they are carried. You could make a table indicating the type of disease, how it is transmitted, incubation period, symptoms and special precautions.
- Discuss with your friends how to ensure that the spread of diseases is prevented as this will help to make you aware of your normal behaviour and the level at which you risk exposure. Talk about how to encourage preventative measures among your community members.

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GRADE 11 LIFE SCIENCES: DIVERSITY, CHANGE AND CONTINUITY

Population Studies

1. INTRODUCTION:

Ecosystems are found within the biosphere. Different species depend on one another for survival. The loss of one species will affect the whole balance of an ecosystem. Survival of each individual species depends on the sustainability of the environment, which ensures that the carrying capacity can be maintained.

2. CHARACTERISTICS OF POPULATIONS

Population dynamics means the study of the changes in numbers of a population and the factors that affect the population numbers. The growth, maintenance and decline of populations will depend on:

- behaviour,
- food needs,
- preferred habits,
- distribution within the environment,
- diseases and
- evolutionary history.

Definition of a population: A population is a group of individuals of all ages, belonging to the same species, living in the same area, that interbreed.

3. NEO-DARWINIAN / MODERN THEORY OF EVOLUTION

This theory is based on Darwin's principles of variation, natural selection and over-production of offspring. Individuals in a species are not all the same. There are variations such as genetic differences, strength, dominance and fertility. Organisms change to adapt to the environment by genes mutating or by accident, like in the case of albinos. If mutations are good, the organism survives. If mutations are detrimental, death will result.

4. POPULATION SIZE

Population size is expressed in numbers or biomass.

Numbers:
This is the number of organisms in a specific ecosystem.

Biomass:
Multiply the number of organisms with the average mass of the individuals in the population;
e.g: 10 rats x 100g (average weight of each rat)
– 1000g (1 kg) biomass

Population increase:
Birth rate / natality - the natural ability of a population to increase in numbers. Immigration - new individuals move from one area to another permanently. However, the birth rate is dependant on the female fertility. Fecundity is the ability of a female to produce fertile egg cells.

Population decrease:
Death rate / mortality causes population decrease. Factors such as lack of food, shelter, space, increase in disease and predation will cause death. Decrease in population participation results because individuals are not replaced at the same rate as the death rate. Emigration results as conditions become unfavourable so individuals leave the habitat permanently in favour of a better one.

Environmental resistance:
When organisms compete for food, water, shelter and space, the abundance or lack of resources regulates the populations. The carrying capacity of an area is the number of organisms that can survive on the natural resources. Environmental resistance is a lack of resources that prevents the population from increasing when the carrying capacity of the habitat has been reached. This carrying capacity will change according to food supply, competition, the seasonal weather patterns and impact of disease. The key factors responsible for fluctuations in populations are rainfall, snow and drought. If the population density varies a lot from one year to the next year, then the population is regarded as unstable. If the population density does not change much from year to year, then we say the population is stable.

Population stability:
When the population is stable then Natality – Mortality. Stability is accessed over a period of time because the environment is never constant and is affected by:

- food, shelter, disease and predation changes
- seasonal changes - animals may die in winter (decrease in numbers) and in spring the new babies are born (increase in numbers)

5. GROWTH PATTERNS

Geometric growth pattern:
Exponential growth results in the shape of a "J" shaped graph. There is a rapid increase in the population and then the population growth stops abruptly. Environmental resistance results because of a lack of resources, seasonal changes and/or pollution by faeces and toxins. This growth pattern is characteristic of organisms which reproduce by binary fission / asexually by mitosis. For bacteria, an antibiotic is used and the population dies resulting in a rapid halt. If only one bacterium survives, the whole infection will begin again.

Logistic growth pattern:
This pattern is also called the Sigmoid graph and "S" shaped graph. There are five distinct stages/phases:

- **Initial lag phase** – acclimatizing and finding mates, gestation period.
- **Accelerated growth phase (Geometric growth phase)** – population growth is at a maximum. Little or no environmental resistance is present. The natality rate is high.
- **Deceleration growth phase** – the birth rate slows down - environmental resistance like competition for food, space, increased predation and/or parasitism.
- **Equilibrium phase** – the population is stable. Natality – Mortality. The carrying capacity of the environment is reached at this phase
- **Death Phase** – graph shows a sharp downward slope after the Equilibrium phase has been reached - extreme environmental resistance due to disease, pollution, drought or human impact.

6. DENSITY PARAMETERS

There are fluctuations within a population near the equilibrium phase. To remain stable each pair of sexually reproducing individuals must produce 2 surviving offspring per lifetime. These two offspring must survive to sexual maturity. This will keep most species at a constant density year after year and ensure that there are enough resources. Density is the number of individuals of a population, located in the same area, within a specific time period. Factors that regulate population growth when the population density is high are called density-dependent factors. Factors that regulate population size and that are NOT related to population density are called density-independent factors.

Density-dependent factors
Density-dependent factors prevent over-population and maintain a stable population. The larger the population number, the more the population will be affected as mortality increases, causing the population density to decrease. The population stabilises and the carrying capacity is maintained.

Density-dependent factors are:

- **Food and water:** the more individuals, the more food and water is needed
- **Competition:** the more individuals, the more competition for resources to survive - ensuring survival of the fittest
- **Space and shelter:** the more individuals, the less space and shelter available, resulting in high levels of stress. Stress causes a decrease in the fertility and natality rate of the population
- **Predation:** the more prey, the better the chances of survival of the predator. Less prey results in the predator having to work harder to catch its food.
- **Disease:** diseases spread quickly when overcrowding occurs

Density-independent factors
Density-independent factors are not related to the population density. These factors are independent of the numbers within the population. The population will be effected regardless of the number of individuals and a causes a drastic decreases in the population size. The population is able to recover after the event and stabilise at the carrying capacity.

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Density-independent factors are:

- Natural catastrophes: these will include droughts, hurricanes, earthquakes and floods. The population density is reduced non-biologically.
- Climate: severe temperatures affect population numbers.
  - very cold winters - individuals die even though food is available - the strongest survive - natural selection occurs
  - adverse climates - food shortage - weaker individuals die of starvation

Limiting factors

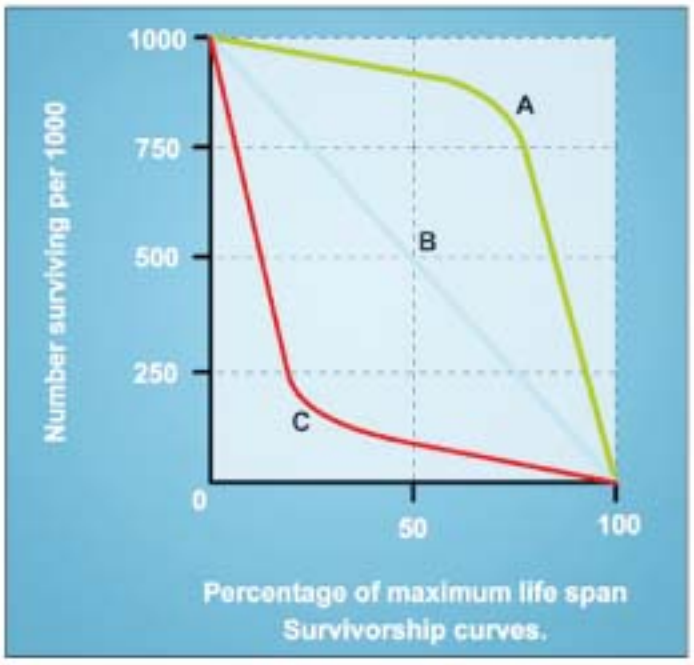
Remember that a population consists of individuals in various stages of their life cycle.

- **Misdistribution of resources:** Individuals are not evenly spread in the habitat. This can be overcome with education in correct farming methods, pest control and conservation of natural resources.

- **Abiotic factors:** rainfall, temperature, light intensity and fertility of the soil will affect animal, plant and human populations

- **Biotic factors:** plants are affected when leaves are eaten by animals and caterpillars, because there is a reduction in the ability to photosynthesise. Plants are also affected by parasites, fungal infections and diseases. Animals are affected by the availability of food, shelter, predation, parasitism and diseases – only the fittest and hardiest of the species survive.

- **Potential life span:** In higher species, mortality is high among the very young and the very old, resulting from predation, availability of food, competition, and accidents/ natural disasters like earthquakes, forest fires and floods. The potential life span of an individual is usually longer than the actual life span. Humans are supposed to live to about the age of 70 years, but some live to be more than 100, while others die before the age of 30. HIV/AIDS has made an impact on the actual lifespan of many people.



Survivorship Curve Graph explained:

Type A: Many of the individuals are born - survive to old age. The curve represents the typical survivorship of K-strategy species, i.e.: animals with extended parental care and humans living in first world countries like the United Kingdom, United States of America and Germany.

Type B: There are losses throughout the life span of the individual, but the mortality rate is evenly distributed between the old and the young. It is typical of animals with selective parental care and humans living in second world countries like China, Russia and Romania.

Type C: Many organisms die during the first stage of life and few survive to old age. The curve represents the typical survivorship of r-strategy species, i.e.: animals with no parental care (eggs are laid and left to hatch) and humans living in third world countries like Ethiopia, Zimbabwe and Kenya.

Survival:

- **Agricultural, medical and economic developments:** As populations increase, natural resources such as food, water and space become a problem.
- **Competition** within the population increases and environmental resistance results in a high death rate.
- **Poverty and hunger** - overcrowding, lack of hygienic conditions and the spread of infectious diseases.
- The mortality rate **increases** when the size of the population exceeds the food supply (carrying capacity and environmental resistance).

Survival rates are improved by using agricultural, medical and economic developments.

- Increased food supply, proper housing, good sanitation and clean water supplies
- Spread of infectious diseases reduced by the use of vaccines, modern drugs and proper education and improved primary health care
- Pesticides used correctly - soil erosion reduced – educate farmers
- Economic growth = natality rate decreases
- Education and career focus – job creation, birth control and sex education

ACTIVITIES

ACTIVITY 1: Population growth forms

A farmer has an enclosure of 5 km2 in which he has kept some springbok since 1970. Predators are kept out of this area, but organized hunting was allowed on two occasions. The number of buck at the end of the year is shown in the table below.

YEAR	No. of buck
1970	10
1975	127
1980	160
1985	153
1990	150

1. Draw a graph to show the changes in population size since 1970.
2. Determine the number of buck population in the year 1983.
3. Explain the reason/s for the decrease in the population of the buck in 1985 and 1990.
4. Give one reason for the high increase of buck in 1975.
5. During which years did hunting take place?

ACTIVITY 2: Density parameters

Draw a concept map where you are able to distinguish clearly between density-dependent and density-independent parameters. Use examples to enhance your understanding of these concepts.

ACTIVITY 3: Survivorship curves

Draw the different survivorship curves and give detailed explanations what each curve denotes.

HELPFUL TIPS

- Ensure that you know the meaning of the terms, especially density-dependent and density-independent factors as these two terms are often confused.
- You must be able to apply the various growth patterns to data that is provided and be able to graph this data.
- When you study population studies, think in terms of animals and plants and not humans. Human population factors will only apply when reference is specifically made, like in the case of HIV infections.

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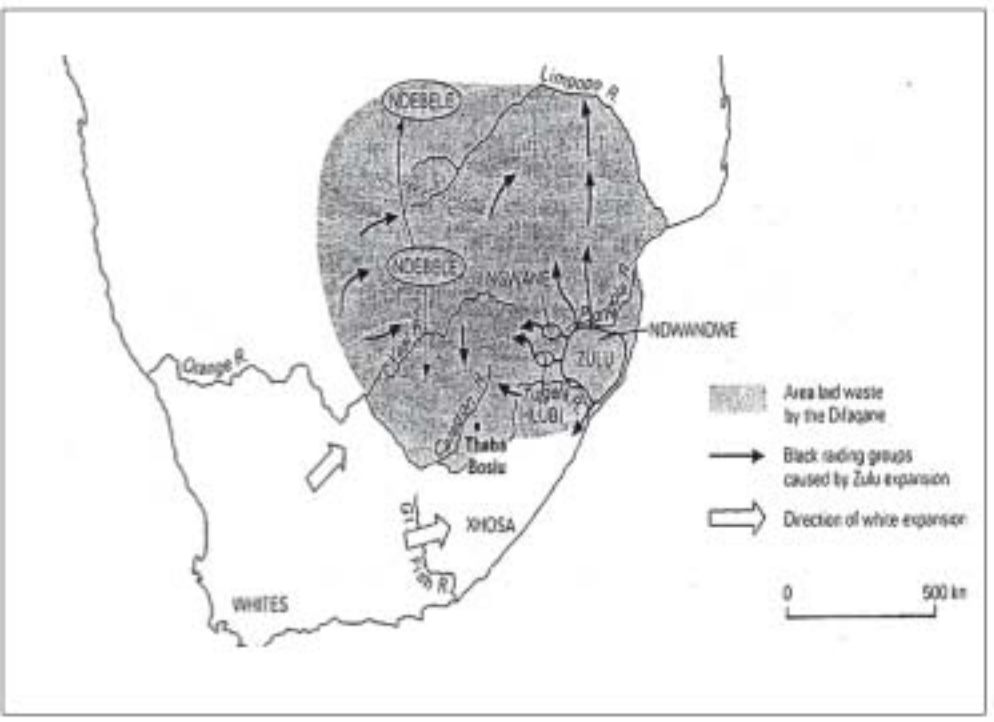
HISTORY GRADE 10 EXAMINATION QUESTIONS

QUESTION 1: TRANSFORMATIONS BETWEEN 1750 AND 1850

HOW DID SHAKA TRANSFORM SOUTHERN AFRICA IN THE EARLY 1800s?
Study Sources 1A, 1B and 1C to answer the following questions:

SOURCE 1A

This is a historical map of Southern Africa that shows the movement of people during the Mfecane (Difaqane) in the early 1800s. Taken from M Roberts, *A History of South Africa.*



SOURCE 1B

This is an excerpt from HF Fynn's diary (Fynn an English trader who supplied Shaka with guns and medicines). He describes the attack that was launched by Shaka and his impis (army) against the Ndwandwe tribe in 1824.

> [The Ndwandwe warriors placed their women and children at the top of a hill and took up a defensive position and surrounded themselves by cattle, in anticipation of an attack by Shaka's army.]
> 'Shaka's forces marched slowly and with caution .... till about twenty yards from the enemy they made a halt.... The enemies were not inclined to move, till Jacob [the Zulu interpreter with Fynn] fired at them three times [using one of Fynn's muskets]. The first and second shots seemed to make no impression on them, for they only hissed and cried in reply, 'That is a dog.' After the third shot, both parties, with a tremendous yell, clashed together and continued stabbing for about three minutes, when both fell back a few paces. Seeing their losses about equal, both armies raised a cry, and this was followed by another rush, and they continued closely engaged about twice as long as in the first onset, when both parties again drew off. But the enemy's losses were now more severe. This urged the Zulus to a final charge ... The remnant of the enemy's army sought shelter in a nearby wood from which they were soon driven. Then began the slaughter of the women and children. They were all put to death. The cattle ... were taken to the kraal lately occupied by Sikonyana (the Ndwandwe chief). The battle ... did not last more than an hour and a half. The number of the hostile tribe, including women and children, could not have been less than 40,000 ...
> Early next morning Shaka arrived and each regiment, previous to its inspection by him, had picked out its 'cowards' and put them to death. Many of these, no doubt, forfeited their lives because their chiefs were in fear that, if they did not condemn some as being guilty, they would incur the resentment of Shaka ...'

SOURCE 1C

This is extract from Alan Mountain's book The Rise and Fall of the Zulu Empire which focuses on Shaka, the leader of the Zulus.

> The name of Shaka stands out in the minds of many as the beacon from which Zulu history radiates. For many, particularly those who are persuaded by the theory of the 'great man', the creator of history, he is seen as the creator or founder of the Zulu nation – an almost supernatural being who virtually single-handedly carved a great nation and empire out of a loosely composed amalgam of small independent or semi-independent polities. Shaka was not a supernatural being, but instead an actor of great brilliance, determination, ruthlessness, perception and persuasion who, for twelve short years strode the stage of southern African politics like Colossus and who, during this time, was able to use his fellow actors and indeed the audience, for whom the actors exist, to choreograph and redirect the human affairs of his time.
>
> It is difficult to be objective about Shaka; so much of what we know about the man is wrapped in legend and is unreliable. The only written records we have of those times were written by traders – all of whom had their own agendas and none of whom could remotely purport to be objective historians.
> Oral tradition, which is the principal source of information about Shaka – particularly in the formative years of Shaka's accession to power – is, by its nature, untested and unscientific and subject to wrong interpretation, exaggeration and misconstruction by both the narrator and the listener – depending on their personal perspective and memory.

1.1 Refer to Source 1A.

1.1 1.1.1 With reference to the geographic position of Shaka's kingdom, explain why you think this was strategic (tactical). (2 x 1) (2)

1.1.2 Name TWO African tribes that fled their homelands during Shaka's consolidation of power. (2 x 1) (2)

1.1.3 In your own words, explain how this movement of people affected other parts of South Africa. (1 x 2) (2)

1.1.4 Using the evidence from the map, what effect do you think the movement of white people had on the indigenous black South Africans? (1 x 2) (2)

1.2 Consult Source 1B.

1.2.1 Using your own knowledge and the evidence from the source, explain the limitations of Fynn's account to a historian researching the early 1800s. (1 x 1) (1)

1.2.2 Why do you think the Zulus won the battle against the Ndwandwe? (1 x 2) (2)

1.2.3 What impression do you get about Shaka from the methods he used against Ndwandwe? (2 x 2) (4)

1.3 Study Source 1C.

1.3.1 Why, according to the source, is Shaka regarded as the 'founder' of the Zulu nation? (1 x 1) (1)

1.3.2 Explain whether you think the comparison between Shaka and Colossus is justified and fair. (1 x 2) (2)

1.3.3 Using the information from the source and your own knowledge, explain why oral evidence became necessary to historians studying the history of Shaka. (2 x 2) (4)

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1.4 Compare Sources 1B and 1C.

1.4.1 What are the differences in the way Shaka is portrayed in both sources? (2 x 2) (4)

1.4.2 Which of the two views regarding Shaka do you support? Substantiate your choice. (2 x 2) (4)

1.5 EXTENDED WRITING

You may choose EITHER QUESTION 1.5.1 OR QUESTION 1.5.2 and your answer should be at least 1½ to 2 pages (500 words).

1.5.1 Using the information from ALL the sources and your own knowledge, write an account on how Shaka transformed Southern Africa in the early 1800s. (20)

OR

1.5.2 Using ALL the sources and your knowledge, write a report for your local history newspaper on whether you consider Shaka to be a military genius. (20)

[50]

ANSWERS

QUESTION 1: TRANSFORMATIONS BETWEEN 1750 AND 1850

HOW DID SHAKA TRANSFORM SOUTHERN AFRICA IN THE EARLY 1800s?

1.1.1
- Shaka situated his kingdom between the Pongola and Tugela rivers, this was a tactical move when launching attacks on neighbouring tribes
- It insulated him from his attackers (2 x 1) (2)

1.1.2
- Ndebele
- Ngwane
- Hlubi
- Ndwandwe

(any 2 x 1) (2)

1.1.3
- Left large tracts of land vacant
- Destroyed and dislocated indigenous tribes
- Led to the destruction of homesteads and traditional family life

(any 2 x 2) (4)

1.1.4
- Led to the war
- Destruction of indigenous family life
- Dispossession and occupation of tribal land (any 2 x 2) (4)

1.2.1
- Fynn's view seems unreliable and biased
- Gives only one side of the story
- A historian needs to get both sides of the story which is based on evidence to make an informed decision and report objectively
- Any other relevant (any 2 x 2) (4)

1.2.2
- They had superior weapons (guns and short fighting spears)
- Determined and brave impi's (soldiers)
- Better organised fighting style (any 1 x 2) (2)

1.2.3
- Intelligent leader
- Strategic and highly organised leader
- Ruthless and manipulative
- Fearless and brutal (any 2 x 2) (4)

1.3.1
- Was innovative, strategic and fearless
- Responsible for the creation the Zulu nation (any 1 x 2) (2)

1.3.2
- Colossus and Shaka were both great leaders
- They played a significant role in establishing great kingdoms (2 x 2) (4)

1.3.3
- Oral tradition has often been the only source of evidence that was used by traditional African societies to pass on information from one generation to another
- Written sources on Shaka were often unreliable, bias and suspect
- To ensure sources containing differing viewpoints are used to study the history of Shaka (2 x 2) (4)

3

1.4.1
- Source 1B gives a bias and unreliable view of Shaka (portrays him as a villain and war monger)
- Source 1C gives a more balanced perspective of Shaka (gives both views to Shaka) (2 x 2) (4)

1.4.2
- Source 1B – it gives a balanced view of Shaka

Candidates may disagree – a relevant point of view should be credited (2 x 2) (4)

1.5 EXTENDED WRITING

1.5.1 You should include the following aspects in your response:

SYNOPSIS

Discuss the role played by Shaka in transforming Southern Africa in the 1880s. Discussion should focus on Shaka's leadership role and military skills and how he managed to unite the Zulus into a mighty kingdom.

MAIN ASPECTS
- Introduction: Focus on Shaka's early years and the role he played.

ELABORATION
- Shaka's coming to power
- Leadership and military skills
- Military strategist (training of soldiers in the use of weapons and fighting techniques)
- Consolidation of power by defeating neighbouring tribes
- Created and established the powerful Zulu kingdom
- Impact of Shaka's rule
- Contribution of Shaka to the Mfecane

- Conclusion: You should tie up your argument with a relevant conclusion, showing how Shaka was able to unite the Zulus into a powerful kingdom in the early 1800s. (20)

Use the matrix below to assess this essay

1.5.2 If you state YES, the following points should be included in your response.
- Shaka was an excellent soldier
- Was a military strategist and ardent task master
- Developed new and innovative fighting strategies such as the short fight spear and the 'bull horn' formation
- Military genius ensured his consolidation of power
- Increased the influence of Zulus by defeating weaker tribes and co-opting them
- Increased his sphere of influence – occupied more land
- Proved indestructible for about 12 years – the influence of his military genius is still felt today.
- Any other relevant point

If you state NO, the following points may be included in your response.
- Shaka was a ruthless and brutal leader
- The circumstances that prevailed during the early 1800s allowed him to rise to power
- Shaka's brutality ensured his success as leader
- Many surrounding tribes feared his methods and hence dispersed
- Any other relevant point (20)

Use the following holistic rubric to allocate a mark

LEVEL 3	• The information selected from the sources is relevant • Information is coherent and is presented in a logical and structured manner • Discussion is well planned and constructed • Discussion is based on the evidence from all the sources provided • Conclusion is well structured based on the evidence from the sources	**Marks: 14 – 20**
LEVEL 2	• Some of the evidence from the sources is used • Discussion contains some structure • Discussion has evidence of some evidence of coherency and logical flow • Evidence extracted relates to some extent to the question. • Can draw a conclusion from the sources given.	**Marks: 7 – 13**
LEVEL 1	• Has not been able to select and use the information from the sources • Discussion lacks a clear plan and lacks coherency • Uses information in a very basic or elementary manner to construct an argument • Lacks a conclusion	**Marks: 0 – 6**

3

GRADE 11 - MATHEMATICAL LITERACY

PROPORTION
Direct Proportion

INTRODUCTION
In this lesson we continue working with ratio and begin to focus on a related concept – proportion. In particular we will focus on direct proportion.

When two ratios are equal, for example if a : b = c : d, then we say that the four quantities, a, b, c and d form a proportion.

In addition, we say that two variables are in direct proportion to one another if they are always in the same ratio.

In direct proportion situations one quantity increases as the other quantity increases and any pair of the two quantities is always in the same ratio as any other pair of the same quantities.

Examples of direct proportion include:
- Scales on plans or maps;
- Buying recipe ingredients for a certain number of people;
- The relationship between travelling time at constant speed and distance; and
- The relationship between the length of time that your electric lights are on and the amount of electricity you use.

METHODS AND WORKED EXAMPLES
Consider the scale on a house plan which is given as 1 : 50. This means that every 1 unit on the plan corresponds to 50 units on the ground. 1cm on the plan corresponds to 50cm on the ground, similarly 1mm on the plan corresponds to 50mm in the ground, and 3cm in the plan corresponds to 3 × 50cm = 150cm on the ground. The length on the plan is in direct proportion to the length on the ground.

Note how:
- As the length on the plan increases, the length on the ground also increases.
- The graph is a straight line and passes through the origin (0;0).

These are characteristics of direct proportion relationships. Direct proportion relationships can be represented in a number of ways.

(1) In symbols:
distance (ground) α distance (plan)

(2) As an equation:
distance (ground) = 50 × distance (plan)
This is called the proportionality constant

(3) As a ratio:
distance (ground) : distance (plan) = 50 : 1

Consider the following example:
If 5 bags of sweets contain 120 sweets in total, how many sweets will there be in 7 bags?

Solution:

	bags		sweets	
÷5 ↓	5	:	120	↓ ÷5
×7 ↓	1	:	24	↓ ×7
	7	:	168	

These are all equivalent ratios (a characteristic of direct proportion relationships):
5 : 120 = 1 : 24 = 7 : 168
There will be a total of 168 sweets in 7 bags of sweets.

Worked example:
If the width of a bedroom on the plan is 6cm, calculate the actual width of the bedroom (i.e. the width of the bedroom on the ground).

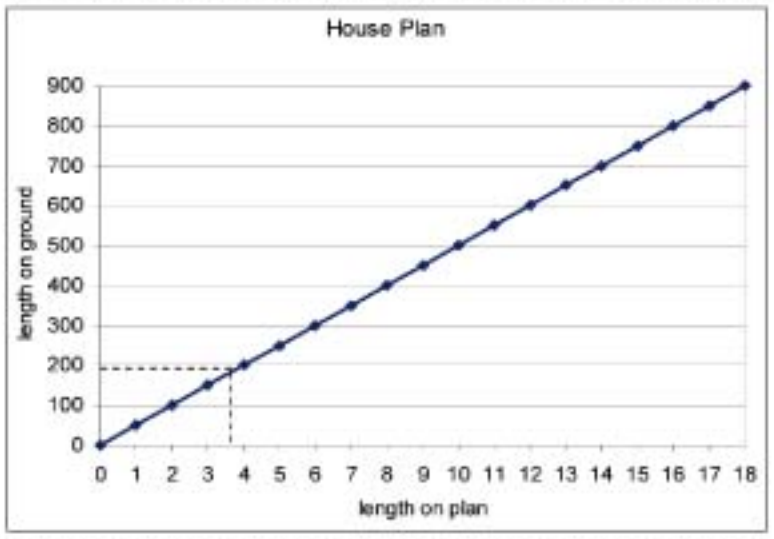
	plan		ground	
x6 ↓	1	:	50	↓ x6
	6	:	300	

The width of the bedroom is 300cm or 3m wide.

You could calculate all the dimensions on a plan in this way or you could draw a table of values and read off the answers. See below.

Length on plan (cm)	1	2	3	4	5	**6**	7	8	9	10
Length on ground *(cm)*	50	100	150	200	250	**300**	350	400	450	500
(m)	0,5	1,0	1,5	2,0	2,5	**3,0**	3,5	4,0	4,5	5,0

If you drew up a table like the one above, then you could just read off the answer. 6cm on the plan corresponds to 300cm on the ground. However, this table would not be much use if the distance on the plan was, say, 3,7cm. In this case, a graph of the situation might be more useful allowing you to read off the value from the graph. See below.



ACTIVITIES

1 The following recipe caters for 6 people:
600g chicken fillets
250g mushrooms
3 tomatoes
½ cup chicken stock
30ml flour

1.1 Calculate the quantity of each of the ingredients if you want to make the recipe for 12 people.
1.2. Calculate the quantity of each of the ingredients if you want to make the recipe for 8 people.

2. A map of South Africa has a scale 1:50 000.
2.1 What will the distance on the ground (in cm) be if the distance on the map is 2cm?
2.2 What is this distance in kilometres?
2.2 What will the distance on the map be if the distance on the ground is 6km?

3. Three chocolate bars cost R7,50.
3.1 Draw a table to show how the price of chocolate bars increase from one bar to eight bars.
3.2 Using the values in the table above, plot a graph of the situation and determine how much 12 bars will cost.

4. You need 6,28m of fencing to enclose a circular flower bed with a diameter of 2m. How much fencing do you need to enclose a flower bed with a diameter of 1,5m?

5. If it typically takes you 6 hours to travel 540km, how long will it take you to travel 700km if you assume that you can travelling at a constant speed.

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6. My watch gains 5 minutes every 8 hours. How many minutes will it gain in 24 hours?

7. A builder uses 5 pockets of cement to lay a floor of area 12m². How many pockets of cement will he need to lay a floor of 20m²?

8. My car can travel 285km on 30l of petrol. Estimate approximately how much petrol l will need to travel 650km?

ANSWERS AND ASSESSMENT

1.1 1 200g chicken; 500g mushrooms; 6 tomatoes; 1 cup stock; 60ml flour
1.2 800g chicken; 330g mushrooms; 4 tomatoes; $\frac{2}{3}$ cup stock; 40ml flour

2.1 100 000cm
2.2 1km
2.3 12cm

3.1

bars	1	2	3	4	5	6	7	8
cost	2,50	5	7,50	10	12,50	15	17,50	20

3.2 R30,00

4. 4,71m

5. 7,8 hours

6. 15 minutes

7. 9 packets. (you need to round up)

8. 68,4l

HELPFUL TIPS

1. Note that if any one quantity increases as another quantity increases, it does not necessarily mean that the quantities are in direct proportion. For the quantities to be in direct proportion the ratio formed by the two quantities must be constant.
2. It is important to relate the word problem to a table and to a graph as these are all methods that can be used to solve direct proportion problems. Try to see the link between the different methods.
3. A graph of a direct proportion problem will always be a straight line graph with the gradient representing the proportionality constant. The graph goes through the origin.

NOTE FOR MATHS OLYMPIAD

The questions that follow will help to prepare you for the Maths Olympiad. You should at least be able to solve 25% of the questions to enter the first round of the SA Mathematics Olympiad (grades 8 to 12). Entry forms are posted to all secondary schools on the address list.

Contact the SA Mathematics Foundation to register your school or update your school's contact details:
SA Mathematics Foundation
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Pretoria 0001
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MATHEMATICS OLYMPIAD QUESTIONS

CHAPTER 3
Exponents, Logarithms and Surds

1. The value of $(2^0 - 2^{-1/2})$ is
(A) 16 **(B)** 0 **(C)** $4(\sqrt{2}-1)$ **(D)** $\sqrt{2}$ **(E)** 2

2. If $\log_{10} 2 = a$, then $\log_5 10$ is equal to
(A) $\frac{1}{a}$ **(B)** $\frac{5a}{2}$ **(C)** $\frac{2}{5a}$ **(D)** $\frac{1}{2a-1}$ **(E)** $\frac{1}{1-a}$

3. Which one of the following numbers is the largest?
(A) $\left(\frac{1}{2}\right)^{\frac{1}{2}}$ **(B)** $\left(\frac{1}{3}\right)^{\frac{1}{3}}$ **(C)** $\left(\frac{1}{4}\right)^{\frac{1}{4}}$ **(D)** $\left(\frac{1}{6}\right)^{\frac{1}{6}}$ **(E)** $\left(\frac{1}{12}\right)^{\frac{1}{12}}$

4. $\frac{10^7 - 5^{10}}{5^7}$ is equal to
(A) $\frac{1}{5}$ **(B)** $-\frac{1}{125}$ **(C)** 5 **(D)** $-\frac{3}{25}$ **(E)** 3

5. $2^n - 2^{n-1}$ equals
(A) 2^{n-1} **(B)** 2^{2n-1} **(C)** 2 **(D)** 2^{2n} **(E)** $\frac{1}{2}$

6. $\sqrt{\sqrt{36} - \sqrt{4}}$ equals
(A) $\sqrt{2}$ **(B)** $\sqrt{6} - \sqrt{2}$ **(C)** 2 **(D)** $\sqrt{32}$ **(E)** 4

7. If $2^x + 3^y = 41$ where x and y are natural numbers, then the value of $x + y$ is
(A) 9 **(B)** 8 **(C)** 7 **(D)** 6 **(E)** 5

8. $\frac{2^4 + 2^0 + 2^{-1}}{2^{-4} + 2^{-5} + 2^{-6}}$ is equal to
(A) 6 **(B)** 8 **(C)** $\frac{31}{2}$ **(D)** 24 **(E)** 512

9. The sum of the digits of $10^{20} - 2$ when expressed as a single number is
(A) 180 **(B)** 3 **(C)** 171 **(D)** 170 **(E)** 179

10. When written in full 2^{2000} has m digits and 5^{2000} has n digits. The value of $m + n$ is
(A) Less than 1999 **(B)** 1999 **(C)** 2000
(D) 2001 **(E)** More than 2001

11. Which of the following is the closest approximation to $\frac{3}{\sqrt{\sqrt{24}-1}}$?
(A) 1,5 **(B)** 0,8 **(C)** 1,7 **(D)** 1,3 **(E)** 0,75

12. $\sqrt{2000^{2000}}$ is the same as
(A) 1000^{1000} **(B)** 1000^{2000} **(C)** $(20\sqrt{5})^{2000}$
(D) $2000^{2000\sqrt{5}}$ **(E)** none of these

13. Which one of the following numbers is the largest?
(A) 333 **(B)** 33^3 **(C)** $(3^3)^3$ **(D)** $3^{(3^3)}$ **(E)** 3^{33}

14. Given $a = 2^{30}$ and $b = 3^{20}$, which one of the following is true?
(A) $a > b$ **(B)** $2a = 3b$ **(C)** $3a = 2b$ **(D)** $a < b$ **(E)** $a = b$

15. The remainder when 7^{2000} is divided by 10 is
(A) 1 **(B)** 3 **(C)** 7 **(D)** 9 **(E)** 5

16. Of the numbers below, the largest one that divides every term of the sequence $2^5 - 2^3$, $3, \ldots - n^3 n, \ldots$, is
(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 5

17. $(1-2)^{2003}$ is equal to
(A) 2 **(B)** 1 **(C)** 1 **(D)** 2003 **(E)** 2

18. If $2^x = 2003$, then the closest integer to x is
(A) 11 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 9

With acknowledgement to the South African Maths Foundation. Content taken from '*The Love of Mathematics*' by Dr Sudan Hansraj and Prof. Poobhalan Pillay.

19. If $x > 0$, then $\sqrt{x\sqrt{x}\sqrt{x}}$ equals

(A) $x^{\frac{1}{2}}$ **(B)** $x^{\frac{3}{4}}$ **(C)** $x^{\frac{7}{8}}$ **(D)** $x^{\frac{1}{8}}$ **(E)** $x^{\frac{5}{8}}$

20. When $33333^2 + 22222$ is written as a single decimal number, the sum of its digits is

(A) 15 **(B)** 25 **(C)** 22 **(D)** 10 **(E)** 20

21. $\sqrt{2003}$ is approximately equal to

(A) 39 **(B)** 41 **(C)** 45 **(D)** 43 **(E)** 47

22. A calculator displays 5.123456789 20 to represent the number $5.123456789 \times 10^{20}$. If 30 is added to the number, then the calculator displays

(A) 5.123456819 20 **(B)** 5.123456789 20 **(C)** 5.123456789 50
(D) 35.12345679 **(E)** none of the above.

23. $(0.2)^4$ equals

(A) 0.8 **(B)** 0.16 **(C)** 0.0016
(D) 0.000016 **(E)** 0.0008

24. $\frac{37^2 + 111}{37}$ equals

(A) 4 **(B)** 40 **(C)** 113 **(D)** 148 **(E)** 37

25. $\frac{4444^4}{2222^4}$ equals

(A) 8888 **(B)** 2222^4 **(C)** 2 **(D)** 2222 **(E)** 16

26. $1 - \frac{1 - \frac{1-\frac{1}{2}}{1+\frac{1}{2}}}{1 + \frac{1+\frac{1}{2}}{1-\frac{1}{2}}}$ equals

(A) $\frac{7}{9}$ **(B)** $\frac{5}{6}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{4}$

27. If $2^2 \times 3^3 \times 4^4 \times 5^{11}$ is multiplied out, then the sum of the digits is

(A) 9 **(B)** 207 **(C)** 14 **(D)** 135 **(E)** 814

28. $\sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}}$ equals

(A) 2 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** 1 **(E)** $12^{\frac{1}{2}}$

29. Half of 2^{20} equals

(A) 1^{10} **(B)** 1^{20} **(C)** 20 **(D)** 2^{10} **(E)** 2^{19}

30. How many of the following numbers are greater than 10?

$3\sqrt{11}$; $4\sqrt{7}$; $5\sqrt{5}$; $6\sqrt{3}$; $7\sqrt{2}$

(A) 1 **(B)** 3 **(C)** 5 **(D)** 4 **(E)** 2

31. $\frac{1}{\frac{3}{4}+\frac{1}{4}}$ equals

(A) $\frac{7}{12}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{7}{2}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{12}{7}$

32. If $2^a 2^b 2^c = 256$, then the average of a, b and c is

(A) $\frac{256}{3}$ **(B)** $\frac{8}{3}$ **(C)** $\frac{4}{3}$ **(D)** 8
(E) impossible to determine

33. Which one of the following has the largest value?

(A) 5×5^5 **(B)** 5^{25} **(C)** $(5^5)^5$ **(D)** 5^{55} **(E)** $(5 \times 5)^5$

34. $2^{26} - 1$ is exactly divisible by two numbers between 120 and 130.

The sum of these two numbers is

(A) 255 **(B)** 256 **(C)** 257 **(D)** 248 **(E)** 251

35. If $w = 2^{129} \times 3^{81} \times 5^{128}$, $x = 2^{127} \times 3^{81} \times 5^{128}$, $y = 2^{126} \times 3^{82} \times 5^{128}$
$z = 2^{125} \times 3^{82} \times 5^{129}$

then the order from the smallest to largest is

(A) w, x, y, z **(B)** x, w, y, z **(C)** x, y, z, w
(D) z, y, x, w **(E)** x, w, y, z

MATHEMATICS OLYMPIAD QUESTIONS

Solutions

1. Answer **(E)**. $(2^{9-8\cdot 1/2})^8 - \frac{1}{2}[2^8 \frac{1}{2}1)] - [2] - 2$

2. Answer **(E)**. $\log_5 10 = \frac{\log 10}{\log 5} = \frac{1}{\log 5} = \frac{1}{\log 10 - \log 2} = \frac{1}{1-a}$

3. Answer **(E)**. Raise all five numbers to the 12^{th} power.
$(\frac{1}{2})^6 = \frac{1}{64}$; $(\frac{1}{3})^4 = \frac{1}{81}$; $(\frac{1}{4})^3 = \frac{1}{64}$; $(\frac{1}{6})^2 = \frac{1}{36}$; $\frac{1}{12}$
The largest of these is $\frac{1}{12}$ so the answer is **(E)**.

Remark: We are assuming here that if $x^{12} < y^{12}$, where x and y are positive, then $x < y$. This is quite in order since the function $f(x) = x^{12}$ is increasing for $x > 0$ (its derivative, namely, $12x^{11}$, is > 0 for $x > 0$). Note that $(\frac{1}{4})^{\frac{1}{4}} = (\frac{1}{2})^{\frac{2}{4}} = (\frac{1}{2})^{\frac{1}{2}}$.

4. Answer **(E)**. $\frac{10^7 - 5^{10}}{5^7} = \frac{10^7}{5^7} - \frac{5^{10}}{5^7} = 2^7 - 5^3 = 128 - 125 = 3$

5. Answer **(A)**. $2^n - 2^{n-1} = 2 \times 2^{n-1} - 2^{n-1} = 2^{n-1}$

6. Answer **(C)**. $\sqrt{\sqrt{36} - \sqrt{4}} = \sqrt{6-2} = \sqrt{4} = 2$

7. Answer **(C)**. $2^x = 41 - 3^y$ where 2^x has to be positive. Therefore $41 - 3^y$ has to be positive. This only happens for $y = 1$, 2 and 3.
If $y = 1, 2x = 41 - 3 = 38$ which is not a power of 2.
If $y = 2, 2x = 41 - 9 = 32 = 2^5$, $x = 5$ i.e.
If $y =$ which is also not a power of two.
Therefore $y = 2$, $x = 5$, $x + y = 7$

8. Answer **(B)**. It can be noted that each term in the denominator is 8 times that in the numerator or the long approach would be as follows:
$\frac{2^1 + 2^0 + 2^{-1}}{2^{-2} + 2^{-3} + 2^{-4}} = \frac{2 + 1 + \frac{1}{2}}{\frac{1}{4} + \frac{1}{8} + \frac{1}{16}} = \frac{\frac{7}{2}}{\frac{7}{16}} = \frac{7}{2} \times \frac{16}{7} = 8$

9. Answer **(E)**. 10^{20} when written out in full consists of a one followed by 20 zeroes. Therefore 10^{20} 2 consists of 19 nines and ends in an eight. The sum of these digits is $19 \times 9 + 8 = 20 \times 9$ 1 = 179.

10. Answer **(D)**. 6275 is a 4-digit number which stands for:
$6 \times 10^3 + 2 \times 10^2 + 7 \times 10 + 5$
5^{2000} has m digits. Therefore $2^{2000} = a \times 10^{m-1}$ plus lower powers of 10, where a is a nonzero digit. 5^{1000} has n digits. Therefore $5^{2000} = b \times 10^{n-1}$ plus lower powers of 10, where b is also a nonzero digit. Notice that $2^{2000} \times 5^{2000} = 10^{2000}$. But is also $(a \times 10^{m-2} + \ldots)(b \times 10^{n-1} + \ldots)$ Hence it follows that ab can only be equal to 10. So: $10^{2000} = 10 \times 10^{m+n-2} = 10^{m+n-1}$
and $m + n = 2001$.

11. Answer **(C)**.
$\sqrt{24} \approx \sqrt{25} - 5$, so $\frac{3}{\sqrt{\sqrt{24} - 1}} \approx \frac{3}{\sqrt{5-1}} = \frac{3}{\sqrt{4}} = \frac{3}{2} = 1{,}5$

12. Answer **(C)**.
$\sqrt{2000^{2000}} = (2000^{2000})^{\frac{1}{2}} = 2000^{(2000 \times \frac{1}{2})}$
$= 2000^{(\frac{1}{2} \times 2000)} = (2000^{\frac{1}{2}})^{2000}$
$= (\sqrt{2000})^{2000} = (20\sqrt{5})^{2000}$

13. Answer **(E)**. The cube roots of **(A)** to **(E)** are 7 (approximately), 33, $3^2 = 27$, 3^3 and 3^{11}, and it is clear that 3^{11} is the greatest.

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14. Answer **(D)**. while (Again only **(A)**, **(D)** or **(E)** need to be considered.)

15. Answer **(D)**. The last digit of any number is its remainder when divided by 10, so we must look at the last digits of the powers of 7. These are 7 (from 7 itself) ,then 9 (from 49) ,then 3, then 1 , then 7 again. Thus the pattern repeats itself after every four powers. Since $2002 = 4 \times 50 + 2$, the last digit of 7^{2002} will be the same as the last of 7^2, which is 9.

16. Answer **(D)**. Factorise $n^5 - n = n(n^4 - 1) = n(n^2 - 1)(n^2 + 1) = n(n - 1)(n + 1)(n^2 + 1)$. This is divisible by the product of three successive integers $(n - 1)(n)(n + 1)$. Any such product is divisible by 6, since one must be divisible by 2 and another by 3. Thus $n^5 - n$ is divisible by at least 6 for every natural number $n \geq 2$. (In fact, it can be shown that $n^5 - n$ is always divisible by 30. Try it using an induction argument!).

17. Answer **(B)**. First evaluate the bracket $(1 - 2)^{2003} = (-1)^{2003}$. Next, note that -1 raised to any odd power is equal to -1. Since 2003 is an odd number, it follows that $(-1)^{2003} = -1$.

18. Answer **(A)**. List the powers of two 2: $2^1 = 2$, $2^3 = 8$, ..., $2^9 = 512$, $2^{10} = 1024$, $2^{11} = 2048$. This is the closest to 2003, so the closest integer to x is 11.

19. Answer **(E)**. If we put $y = \sqrt{x\sqrt{x\sqrt{x}}}$, then $y = x\sqrt{x\sqrt{x^2}}$, $y = x^2x\sqrt{x^8}$, and $x = x^4x^2x = x^7$, so $y = x^{\frac{7}{8}}$

20. Answer **(D)**. $33333^2 + 22222 = (11111.3)^2 + (11111.2)$
$= 11111\,(99999 + 1)$
$= 11111\,(100001)$
$= 1111111111$

which has a digit sum of 10.

21. Answer **(C)**. Note that $44^2 = 1936$ and $45^2 = 2025$. Since 2003 lies between these two numbers we see that its square root must be between 44 and 45.

22. Answer **(B)**. If $5.123456789 \times 10^{20}$ is written without using scientific notation, then the decimal point moves 20 places to the right, giving 512 345 678 900 000 000. If now 30 is added to this number, then the last three digits on the right become 030, but the significant digits (which are on the left) are unchanged. Thus the display on the calculator is also unchanged.

23. Answer **(C)**.
$0.2^4 = \left(\frac{2}{10}\right)^4 = \frac{2^4}{10^4} = \frac{16}{10000} = 0{,}0016$

24. Answer **(B)**.
$\frac{37^2 + 111}{37} = \frac{37 \times 37 + 37 \times 3}{37} = \frac{37(37 + 3)}{37} = 37 + 3 = 40$

25. Answer **(E)**.
$\frac{4444^4}{2222^4} = \left(\frac{4444}{2222}\right)^4 = 2^4 = 16$

26. Answer **(B)**.
The expression equals $1 - \dfrac{1 - \frac{1}{1 + \frac{1}{2}}}{1 + \frac{1}{1 + \frac{1}{2}}} = 1 - \frac{\frac{2}{3}}{1 + \frac{2}{3}} = 1 - \frac{2}{3} = 1 - \frac{2}{4} = 1 - \frac{1}{6} = \frac{5}{6}$.

27. Answer **(A)**.
$2^2 \times 3^3 \times 4^4 \times 5^{11} = 2^2 \times 27 \times 2^8 \times 5 \times 5^{10}$
$= 2^{10} \times 27 \times 5 \times 5^{10}$
$= (2 \times 5)^{10} \times 5 \times 27$
$= 135 \times 10^{10}$

which is 135 followed by ten zeros. Thus the sum of the digits is $1 + 3 + 5 = 9$.

28. Answer **(B)**.
Let $p = \sqrt{2 + \sqrt{3}}$, $q = \sqrt{2 - \sqrt{3}}$, and so $pq = \sqrt{2^2 - 3} = 1$.
Then $(p - q)^2 = p^2 - 2pq + q^2 = (2 + \sqrt{3}) - 2 + (2 - \sqrt{3}) = 2$.
Since $p - q$ is clearly positive, it follows that $p - q = \sqrt{2}$.

MATHEMATICS OLYMPIAD QUESTIONS

29. Answer **(E)**.
Half of 2^{20} equals $\frac{1}{2} \times 2^{20} = 2^{-1} \times 2^{20} = 2^{-1+20} = 2^{19}$.

30. Answer **(B)**.
Since the numbers are all positive, it is sufficient to determine how many of their squar are greater than 100. Their squares are, respectively, $9 \times 11 = 99$; $16 \times 7 = 112$; $25 \times 5 = 125$; $36 \times 3 = 108$; $49 \times 2 = 98$ of which the three middle numbers are greater than 100.

31. Answer **(E)**.
$\frac{1}{\frac{1}{3} + \frac{1}{4}} = \frac{1}{\left(\frac{4 + 3}{12}\right)} = \frac{1}{\left(\frac{7}{12}\right)} = \frac{12}{7}$.

32. Answer **(B)**.
If $2^a \cdot 2^b \cdot 2^c = 256$, then $2^{a+b+c} = 2^8$, so $a + b + c = 8$.
The average is $\frac{a + b + c}{3} = \frac{8}{3}$.

33. Answer **(B)**.
If the expressions are all simplified in exponent form, then they are:
(A) $5^{1+5} = 5^6$ **(B)** 5^{2125} **(C)** $5^{5+5} = 5^{25}$
(D) 5^{55} **(E)** $(5^2)^5 = 5^{10}$
and the largest exponent is in B.

34. Answer **(B)**.
Factorise as the difference of two squares:
$2^{28} - 1 = (2^{14} - 1)(2^{14} + 1) = (2^7 - 1)(2^7 + 1)(2^{14} + 1)$
It may or may not factorise further, but since $2^7 = 128$ it follows that the first two factors are 127 and 129 and both lie between 120 and 130, and their sum $= 127 + 129 = 256$.

35. Answer **(C)**.
Let $a = 2^{125} \times 3^{81} \times 5^{119}$. Then $w = a \times 2^4 = 16a$ while $x = a \times 2^2 = 4a$ and $y = a \times 2 \times 3 = 6a$ and $z = a \times 3 \times 5 = 15a$. Since $4 < 6 < 12 < 15$, it follows that $x < y < z < w$.

CHAPTER 4
Ratio, Proportion and Average

1. Which one of the following is the best approximation to the positive square root of $\frac{1995}{100000}$?
(A) 0,044676 **(B)** 0,1412 **(C)** 0,4467
(D) 0,01412 **(E)** 0,004467.

2. Before his last Maths test John's average mark was 33 exactly. In his last test he scored 40 marks which increased his average to exactly 34 marks. What must he score in his next test to increase his average to exactly 35 marks?
(A) 35 **(B)** 36 **(C)** 40 **(D)** 41 **(E)** 42

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3. Every year a farmer cuts down all the trees on his farm. He finds that 60% die and the rest grow again from the stumps which remain. After three years the percentage of the original number of trees that have died is:

(A) 21,6 **(B)** 78,4 **(C)** 6,4 **(D)** 93,6 **(E)** 80

4. A pizza restaurant sells two sizes of pizza. The smaller size is cut into four pieces and the larger size is cut into six pieces. If each piece has the same area then the radii of the two sizes are in the ratio:

(A) $\sqrt{2} : \sqrt{3}$ **(B)** $2 : 3$ **(C)** $1 : 2$
(D) $1 : \sqrt{2}$ **(E)** $\sqrt{3} : 2$

5. The scale on a map is $1 : x$. What is the area, in m^2, of a plot of land represented by y cm^2 on the map?

(A) $10000yx^2$ **(B)** yx^2 **(C)** $\frac{yx}{10000}$
(D) $\frac{yx^2}{10000}$ **(E)** yx

6. In a group of men and women, the average age is x years. If the men's ages average $(x + 4)$ years, and the women's ages average $(x - 6)$ years, then the ratio of the number of men to the number of women is:

(A) 5 : 7 **(B)** 7 : 5 **(C)** 2 : 1 **(D)** 4 : 3 **(E)** 3 : 2

7. In a maths test the average mark obtained by the pupils who passed is 65, while the average mark of the pupils who failed is 35. If the average mark of all the pupils who wrote the test is 53, what percentage of pupils passed?

(A) 35 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 65

8. An aircraft is flying a return journey of a distance of L km in both directions. The aircraft is flying at a speed of 160 km / h against head winds on the forward journey and at 240 km / h with tail winds on the return journey. Determine the average speed of the total flight in km / h. (Takeoff and landing times ignored.)

(A) 184 **(B)** 192 **(C)** 200 **(D)** 208 **(E)** 216

9. Successive discounts of 10% and 20% are equivalent to a single discount of:

(A) 30% **(B)** 15% **(C)** 72% **(D)** 28% **(E)** None of these.

10. The value of $\frac{0,1 + 0,01}{1 + 0,1}$ is:

(A) $\frac{1}{11}$ **(B)** 0,1 **(C)** 0,2 **(D)** $\frac{1}{9}$ **(E)** 0,11

11. Three parties contested an election for 400 seats in parliament. Party A won 35% of the seats and party B won 20% of the seats. How many seats did party C win?

(A) 345 **(B)** 200 **(C)** 350 **(D)** 250 **(E)** 180

12. For long distance telephone calls, Telkom charges 30, 9 cents per metering unit of 13, 6 seconds. A long distance call of 3 minutes will cost you about:

(A) R3,00 **(B)** R4,00 **(C)** R5,00 **(D)** R6,00 **(E)** R7,00

13. A given set of 20 numbers has an average of 20. Nine of these numbers have an average of 9. The average of the remaining 11 numbers is:

(A) 20 **(B)** 31 **(C)** 29 **(D)** 10 **(E)** 11

14. Which of the following numbers are the largest?
(Remember $6,\overline{967}$ means 6, 967967967...)

(A) 3,1416 **(B)** $3,\overline{1416}$ **(C)** $3,1\overline{416}$ **(D)** $3,14\overline{16}$ **(E)** $3,141\overline{6}$

15. The average of three numbers is 18. If the largest number is replaced by the number 38, then the average of the three numbers is 23. The original that was replaced, is:

(A) 38 **(B)** 23 **(C)** 15 **(D)** 18 **(E)** 33

16. A question paper consists of twenty questions. Section A (questions 1 to 5) are worth three marks each, Section B (questions 6 to 15) are worth 5 marks each and Section C (questions 16 to 20) are worth 7 marks each. Ellie scores 100% in Section A and 90% in Section B. The least number of questions she should answer correctly in Section C to score more than 80% overall is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

17. In a school there are two grade 11 classes with the same number learners in each. The ratio of girls to boys in one class is 1 : 2 and in the other class it is 3 : 2. The ratio of girls to boys in the entire grade 11 is:

(A) 1 : 1 **(B)** 5 : 3 **(C)** 7 : 8 **(D)** 8 : 7 **(E)** 3 : 5

MATHEMATICS OLYMPIAD QUESTIONS

18. In a science experiment, you are trying to separate sugar from a solution of water and sugar by heating and evaporating the water. The mass of the solution is 2 kg which contains 90% water and 10% sugar by mass. What will be the mass of the solution if after some time you find it to contain 85% water?

(A) 1,9 kg **(B)** 1 kg **(C)** 1,7 kg **(D)** 1,3 kg **(E)** 1.5 kg.

19. A farmer has both sheep and chickens. The average number of legs per animal is l. The ratio of the number of sheep to the number of chickens is

(A) $\frac{l}{3(4 - l)}$ **(B)** $\frac{l - 2}{4 - l}$ **(C)** $\frac{3(l - 2)}{l}$ **(D)** $\frac{l(l - 2)^2}{16 - l^2}$ **(E)** $\frac{7(l^2 - 4)}{5(16 - l^2)}$

20. If 1 ton = 1 000 kg then Aamout the fully grown hippo weighs about

(A) 12 kg **(B)** 120 kg **(C)** 1,2 ton **(D)** 120 ton **(E)** 1 200 ton

21. A one-litre bag of milk costs R3, 00 and a two-litre bottle costs R6, 60.
What percentage do you pay extra if you buy one bottle instead of two bags?

(A) 60 **(B)** 50 **(C)** 40 **(D)** 30 **(E)** 10

22. When the repeating decimal 0,45454545 . . . is written in the simplest fractional form, the sum of the numerator and denominator is

(A) 144 **(B)** 5 **(C)** 11 **(D)** 55 **(E)** 16

23. If x is x% of y and y is y% of z, where x is a positive real number, what is the value of z?
(A) 100 **(B)** 200 **(C)** 10 000
(D) It does not exist **(E)** It cannot be determined

24. If each number in a set of ten numbers is increased by 20, then the average of the set
(A) remains the same **(B)** is increased by 20
(C) is increased by 200 **(D)** is increased by 10
(E) is increased by 2

25. After five games, a rugby team has an average of 28 points per game. In order to increase their average by 2 points, how many points must they score in the sixth game?
(A) 2 **(B)** 12 **(C)** 56 **(D)** 30 **(E)** 40

26. A black and white photograph is 80% black and 20% white. If it is enlarged three times then the percentage of white in the enlargement is

(A) 20 **(B)** 180 **(C)** 60 **(D)** 80 **(E)** 30

27. In a supermarket items are always priced at so many rands and 99 cents. If a shopper pays a total of R41.71, then the number of items bought is

(A) 29 **(B)** 30 **(C)** 71 **(D)** 9
(E) impossible to determine

28. Nine numbers are written in ascending order. The middle number is also the average of the nine numbers. The average of the five largest numbers is 68 and the average of the five smallest numbers is 44. The sum of all the numbers is

(A) 560 **(B)** 504 **(C)** 112 **(D)** 56 **(E)** 70

29. Two jars contain an equal numbers of marbles. The marbles are either red or white. The ratio of red to white marbles is 7:1 in the first jar and 9:1 in the second jar. If the total number of white marbles is 90, then the number of red marbles in the second jar is
(A) 360 **(B)** 400 **(C)** 450 **(D)** 36 **(E)** 40

30. 0.001 ÷ 0.05 equals

(A) $\frac{1}{55}$ **(B)** $\frac{1}{500}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{50}$ **(E)** 2

31. Statistics show that 10% of smokers get lung cancer, and 90% of lung cancer patients are smokers. If 20% of the population smokes, then percentage of the population having lung cancer is

(A) $\frac{9}{5}$ **(B)** 2 **(C)** 3 **(D)** 9 **(E)** $\frac{20}{9}$

Department of Education

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Solutions

1. Answer **(B)**.

   1995 is close to 2000. Now 2000 ÷ 100 000 is the same as $\frac{1}{50}$ and $\sqrt[3]{50}$ is roughly $\frac{1}{7}$ and is close to 0,14.

2. Answer **(E)**.

   Say he wrote x tests at an average of 33. He would have a total of $33x$ marks. When he writes the next test, his number of tests is $x + 1$ and with an average of 34 his total is $34(x + 1)$. This is the same as $33x + 40$. Solving for x in $34(x + 1) = 33x + 40$,gives $x = 6$. Now in his next test (that is test number 8) he wants to average 35. His total would be $8 \times 35 = 280$. After 7 tests his total was $34 \times 7 = 238$. Thus he needs $280 - 238 = 42$ marks.

3. Answer **(D)**.

   For convenience say there are 1 000 trees at the start. After 1 year 600 die; after 2 years 60% of the 400 left die, that is 240 die. Finally after 3 years 60% of the 240 left die, that is 96. Total number of trees that die = 600 + 240 + 96 = 936. As a percentage 936 out of 1 000 is 93,6%.

4. Answer **(A)**.

   Let each pizza piece have area A. Then for the small pizza of radius r we have $r^2 = 4A$ and for the big pizza $R^2 = 6A$.

   Dividing these two gives $\frac{\pi r^2}{\pi R^2} = \frac{4A}{6A}$

   $\frac{\pi r^2}{R^2} = \frac{2}{3}$

   $\frac{r}{R} = \frac{\sqrt{2}}{\sqrt{3}}$

5. Answer **(D)**.

   1cm on map = x cm on land. 1 cm² on map = x^2 cm on land. y cm² on map = yx^2 cm² on land. 100 cm = 1 metre.

   1 cm = 100^1 m. 1 cm² = 100^{-2} m². Hence y cm² on map

   = $yx^2(100^{-2})$ m² = $\frac{yx^2}{10\,000}$ square metres.

6. Answer **(E)**.

   Let m be the number of men, an w be the number of women. Since Average = sum divided by number, the sum is equal to number times average. Thus: sum of all ages is $x(m + n)$. Sum of mens' ages is $m(x + 4)$ ans sum of womens' ages is $w(x\ 6)$. Hence $x(m + w) = m(x + 4) + w(x\ 6)$. Simplify: $4m = 6w$ and $m : w = 6 : 4 = 3 : 2$.

7. Answer **(D)**.

   Average = sum divided by number of terms. So sum = average times the number of terms. Let x be the number of pupils who passed. Total mark of passing pupils is $65x$. Let y be the number of pupils who failed. Total mark of failing pupils is $35y$. There are $x + y$ pupils altogether, so the total mark altogether is $53(x + y)$. Conclusion: $53(x + y) = 65x + 35y$. Simplifying, $3y = 2x$, that is $y = \frac{2}{3}x$ The percentage of pupils who passed is $\frac{\text{number passing}}{\text{total who wrote}} \times \frac{100}{1} = \frac{100x}{x + y} = \frac{100x}{x + \frac{2}{3}x} = \frac{100}{(\frac{5}{3})} = 60$.

8. Answer **(B)**.

   Outward journey: 160 km in 1 hour, so L km in $(\frac{L}{160})$ hours.

   Return journey: 240 km in 1 hour, so L km in $(\frac{L}{240})$ hours.

   Both together: $2L$ km in $(\frac{L}{160}) + (\frac{L}{240})$ hours so in one hour distance covered will be $\frac{2L}{\frac{L}{160} + \frac{L}{240}}$ km. That is, average speed is 192 km/hr.

9. Answer **(D)**. $\frac{90}{100} \times \frac{80}{100} = \frac{72}{100} = 1 - \frac{28}{100}$

   Therefore equivalent to a 28% discount.

MATHEMATICS OLYMPIAD QUESTIONS

10. Answer **(B)**. $\frac{0,1 + 0,01}{1 + 0,1} = \frac{0,11}{1,1} = 0,1$

11. Answer **(E)**. Party C won 100 − (35 + 20) = 45% of the seats, i.e. $\frac{45}{100} \times 400 = 180$

12. Answer **(B)**.

    A 3 minute call will cost you $\frac{30,9}{13,6} \times 3 \times 60$ cents. This is approximately $\frac{31 \times 3 \times 60}{13,6} = \frac{5580}{13,6}$ cents. It is now easy to check that $13,6 \times 4 = 54,4$ cents is the closest. So since 5440 is close to 5580, the closest answer is R4.

13. Answer **(C)**.

    The sum of the twenty numbers is $20 \times 20 = 20^2$. Therefore the average of the remaining 11 is $\frac{20^2 - 9^2}{11} = \frac{29 \times 11}{11} = 29$

14. Answer **(E)**.

    The numbers are **(A)** 3,14160000 ... **(B)** 3,14161416 ...

    (C) 3,1416416 ... **(D)** 3,14161616 ... **(E)** 3,141666 ...

    The first four decimal places are the same in all five of these numbers, but the fifth decimal place is the highest in **(E)**.

15. Answer **(B)**.

    The sum of the original three numbers is $3 \times 18 = 54$, and the sum of the set of three new numbers is $3 \times 23 = 69$. Therefore the largest number was increased by $69 - 54 = 15$. Therefore the number that was replaced is $38 - 15 = 23$.

16. Answer **(C)**.

    Notice that the marks add up to 100. From sections A and B Ellie obtains $5 \times 3 + \frac{90}{100}(10 \times 5) = 60$ points. In order to obtain another 20 points from section C she needs to answer at least three questions correctly.

17. Answer **(C)**.

    Suppose that there are x learners in each class. Then the number of girls in the entire grade 11 is $\frac{1}{3}x + \frac{3}{5}x - \frac{14}{15}x$. Therefore the number of boys is $2x - \frac{14}{15}x = \frac{16}{15}x$. Hence the required ratio is $7\left(\frac{2x}{15}\right)$ to $8\left(\frac{2x}{15}\right)$ which is 7 : 8.

    Alternatively, choose a number which is divisible by 3 and by 5, for example 15. Let $x = 15$. Then there are 14 girls and 16 boys. Again the ratio is 7 : 8.

18. Answer **(D)**.

    The result of the heating process is that water evaporates, the sugar stays behind. So originally there are $\frac{10}{100} \times 2 = 0,2$ kg of sugar in the solution. This mass of sugar stays the same. When the solution is 85% water then 15% is sugar. So 15% of the solution weighs 0,2 kg. Therefore 100% of the solution weighs $\frac{100}{15} \times 0,2\ \frac{4}{3}$ kg

19. Answer **(B)**.

    If there are s sheep and c chickens, then the number of legs is $4s + 2c$ and the number of animals is $s + c$. Thus giving $l = (4s + 2c)/(s + c)$ from which $(4 - l)s = (l - 2)c$, g can be found.

20. Answer **(C)**.

    (This problem tests estimation.) 12 kg is about the weight of a small child or a dog. 120 kg is about the weight of a very big man, 1200 kg (or 1,2 ton) is the weight of a hippo or a small car, 120 ton is about the weight of a medium-size airliner, and 1 200 ton is about the weight of two railway engines.

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21. Answer **(E)**.

Two bags cost R6, 00 and one bottle costs 60 cents more, so+ the percentage extra is $0{,}60 \div 6 \times 100$.

22. Answer **(E)**.

If $x = 0{,}454545\ldots$, then $100x = 45{,}454545\ldots = 45 + x$. Therefore $x = \frac{45}{99} = \frac{5}{11}$

23. Answer **(A)**. $x = \frac{x}{100} \times y$ and $y = \frac{y}{100} \times z$. It follows immediately from the second equation, on cancelling y, that $z = 0$.

24. Answer **(B)**.

If you cannot see immediately that the average increases by 20, suppose the original numbers were $n_1, n_2, \ldots, n_{10}$, so the old average was $\frac{1}{10}(n_1, n_2, \ldots, n_{10})$ The new average is $\frac{1}{10}((n_1 + 20) + (n_2 + 20) \ldots + (n_{10} + 20))$ which is 20 more than the old average.

25. Answer **(E)**.

If the team has an average of 28 points per game after five games, then the total points scored is $28 \times 5 = 140$. If the average is to increase by two points per game after six games, then the total will have to be $(28 + 2) \times 6 = 180$. Thus the number of points they must score in the sixth game is $180 - 140 = 40$.

26. Answer **(A)**.

If the photograph is enlarged three times, then all lengths are multiplied by $3^2 = 9$. However, the percentages of black and white are unchanged, since all areas are multiplied by the same factor.

27. Answer **(A)**.

Each item is exactly one cent short of a whole number of rands. Since the total cost is 29 cents short of a whole number of rands, he must have bought 29 items. (The other possible answers, 129 and 229 and so on, are ruled out because then the total cost would be much higher.)

28. Answer **(B)**.

Let the sum of all the numbers be S, so the middle number is $\frac{1}{9}S$. The sum of the five largest numbers is $5 \times 68 = 340$ and the sum of the five smallest numbers is $5 \times 44 = 220$. If we add these sums together, then we have counted the middle number twice, that is $S + \frac{1}{9}S = 340 + 220$ $\frac{9}{10} \times 560 = 504$.

29. Answer **(A)**.

Suppose there are n marbles in each jar. Then the number of white marbles is $\frac{1}{8}n + \frac{1}{10}n = \frac{9}{40}n$, which equals 90, so $n = 400$. The number of red marbles in the second jar is $\frac{9}{10}n = 360$.

30. Answer **(D)**. $0.001 + 0.05 = \frac{0{,}001}{0{,}05} = \frac{1}{50}$.

Alternatively,

$0.001 + 0.05 = \frac{1}{1\,000} + \frac{5}{100} = \frac{1}{1\,000} \times \frac{100}{5} = \frac{1}{50}$

31. Answer **(E)**.

Suppose l% of the population are lung cancer sufferers, s% of the population are smokers, and x% are both.

We are given that $x = 10\%$ of s and $x = 90\%$ of l, so $s = 9l$. Since $s = 20$ it follows that $l = \frac{20}{9}$.

The questions that follow will help to prepare you for the Maths Olympiad. You should at least be able to solve 25% of the questions to enter the first round of the SA Mathematics Olympiad (grades 8 to 12): Entry forms are posted to all secondary schools on the address list.

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