



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

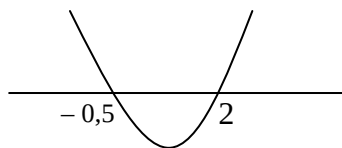
FEBRUARY/MARCH 2010

MEMORANDUM

MARKS: 150

This memorandum consists of 19 pages.

QUESTION 1

1.1.1	$(x-3)(x+5) = 9$ $x^2 + 2x - 15 = 9$ $x^2 + 2x - 24 = 0$ $(x+6)(x-4) = 0$ $x = 4$ or $x = -6$	✓ expansion ✓ standard form ✓ factorisation ✓ answers (4)
1.1.2	$2x^2 - 3x - 2 \leq 0$ $(2x+1)(x-2) \leq 0$ Critical values : $-\frac{1}{2}$ and 2 + 0 - 0 + -0,5 2  $-\frac{1}{2} \leq x \leq 2$	✓ factors ✓ critical values ✓✓ answer (4)
1.2	$y = -2x - 2$(1) $-2x^2 + 8xy + 42 = y$(2) $-2x^2 + 8x(-2x - 2) + 42 = -2x - 2$ $-2x^2 - 16x^2 - 16x + 42 + 2x + 2 = 0$ $-18x^2 - 14x + 44 = 0$ $9x^2 + 7x - 22 = 0$ $(9x - 11)(x + 2) = 0$ $\therefore x = \frac{11}{9}$ or $x = -2$ $\therefore y = -2\left(\frac{11}{9}\right) - 2$ $\therefore y = -2(-2) - 2$ $\therefore y = -\frac{40}{9}$ $\therefore y = 2$ OR	✓ $y = -2x - 2$ ✓ substitution ✓ simplification ✓ factors ✓ answers for x ✓✓ answers for y (7)

$y = -2x - 2 \dots\dots\dots(1)$ $-2x^2 + 8xy + 42 = y \dots\dots\dots(2)$ $y = -2(x + 1) = \frac{-2(x^2 - 21)}{1 - 8x}$ $\therefore (x + 1)(1 - 8x) = x^2 - 21$ $x - 8x^2 + 1 - 8x = x^2 - 21$ $9x^2 + 7x - 22 = 0$ $(9x - 11)(x + 2) = 0$ $\therefore x = \frac{11}{9} \quad \text{or} \quad x = -2$ $\therefore y = -2\left(\frac{11}{9}\right) - 2 \quad \therefore y = -2(-2) - 2$ $\therefore y = -\frac{40}{9} \quad \therefore y = 2$ OR $y = -2x - 2 \dots\dots\dots(1)$ $-2x^2 + 8xy + 42 = y \dots\dots\dots(2)$ $x = \frac{(-y - 2)}{2}$ $-2\left(\frac{(-y - 2)}{2}\right)^2 + 8y\left(\frac{(-y - 2)}{2}\right) + 42 - y = 0$ $-2\left(\frac{y^2 + 4y + 4}{4}\right) + 4y(-y - 2) + 42 - y = 0$ $y^2 + 4y + 4 + 8y^2 + 16y - 84 + 2y = 0$ $9y^2 + 22y - 80 = 0$ $(y - 2)(9y + 40) = 0$ $y = 2 \quad \text{or} \quad y = -\frac{40}{9}$ $x = -2 \quad \text{or} \quad x = \frac{11}{9}$	✓ equating ✓ simplification ✓ standard form ✓ factors ✓ answers for x ✓✓ answers for y (7) ✓ $x = \frac{(-y - 2)}{2}$ ✓ substitution ✓ simplification ✓ factors ✓ answers for y ✓✓ answers for x (7)
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1.3	$g(x) = x^2$ $g(9) = 81$ $f(x) = \sqrt{4x}$ $f(g(9)) = f(81) = \sqrt{4(81)}$ $= 2(9)$ $= 18$ OR $g(9) = 9^2$ $\therefore f(g(9)) = \sqrt{2^2 \cdot 9^2} = 18$ OR $f(g(x)) = \sqrt{4g(x)}$ $= \sqrt{4x^2}$ $= 2x$ $f(g(9)) = 2(9)$ $= 18$	$\checkmark g(9) = 81$ $\checkmark$ substitution $\checkmark$ answer (3) $\checkmark g(9) = 9^2$ $\checkmark$ substitution $\checkmark$ answer (3) $\checkmark$ $f(g(x)) = \sqrt{4g(x)}$ $\checkmark$ substitution $\checkmark$ answer (3)
1.4	$\frac{14}{\sqrt{63} - \sqrt{28}}$ $= \frac{14}{3\sqrt{7} - 2\sqrt{7}}$ $= \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ $= 2\sqrt{7}$ $a = 2$ and $b = 7$ <i>But</i> $2\sqrt{7} = \sqrt{28}$ So $a = 1$ and $b = 28$ is also a solution.	$\checkmark$ simplification $\checkmark$ simplification $\checkmark$ rationalising the denominator $\checkmark$ answer (4) [22]

	The sequence is $20^2 - 1 ; 19^2 - 1 ; 18^2 - 1 ; 17^2 - 1 ; \dots\dots$ $T_1 = 20^2 = (20 - 0)^2 - 1$ $T_2 = 19^2 = (20 - 1)^2 - 1$ $T_3 = 18^2 = (20 - 2)^2 - 1$ $T_n = (20 - (n - 1))^2 - 1 = (21 - n)^2 - 1$	✓✓ rewriting terms as squares ✓✓✓ establishing that $T_n = (20 - (n - 1))^2$ ✓ $T_n = (21 - n)^2$ (6)
2.2	$n^2 - 42n + 440 = 0$ $(n - 22)(n - 20) = 0$ $n = 22$ and $n = 20$ both terms 22 and 20 have values of 0. OR $(21 - n)^2 - 1 = 0$ $21 - n = 1$ or -1 $n = 20$ or $n = 22$	✓ equation ✓✓ answers (3) ✓ equation ✓✓ answers (3)
2.3	$n = \frac{-(-42)}{2(1)}$ $n = 21$ At the 21st term, the lowest value is obtained. OR $2n - 42 = 0$ $2n = 42$ $n = 21$ $\therefore$ At the 21st term, the lowest value is obtained. OR $T_n = (21 - n)^2 - 1 \therefore$ For $n = 21$, $T_n = (21 - n)^2 - 1 = (21 - 21)^2 - 1 = -1$ For $n = 21$, the lowest value ($= -1$) is obtained.	✓ answer (1) ✓ answer (1) ✓ answer (1) [10]

QUESTION 3

3.1	Let $S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} \quad (1)$ Then $r \times S_n = r(a + ar + ar^2 + ar^3 + \dots + ar^{n-1})$ $= ar + ar^2 + ar^3 + \dots + ar^n + ar^n \quad (2)$ (2) – (1) gives: $rS_n - S_n = ar^n - a$ $S_n(r - 1) = a(r^n - 1)$ $S_n = \frac{a(r^n - 1)}{(r - 1)}$	✓ writing S_n as a series ✓ writing $r.S_n$ as a series ✓ subtracting ✓ removing common factors (4)
3.2	$a = 3; r = \frac{1}{3}$ $S_\infty = \frac{a}{1-r}$ $= \frac{3}{1-\frac{1}{3}}$ $= \frac{9}{2}$	✓ $r = \frac{1}{3}$ ✓ substitution ✓ answer (3) [7]

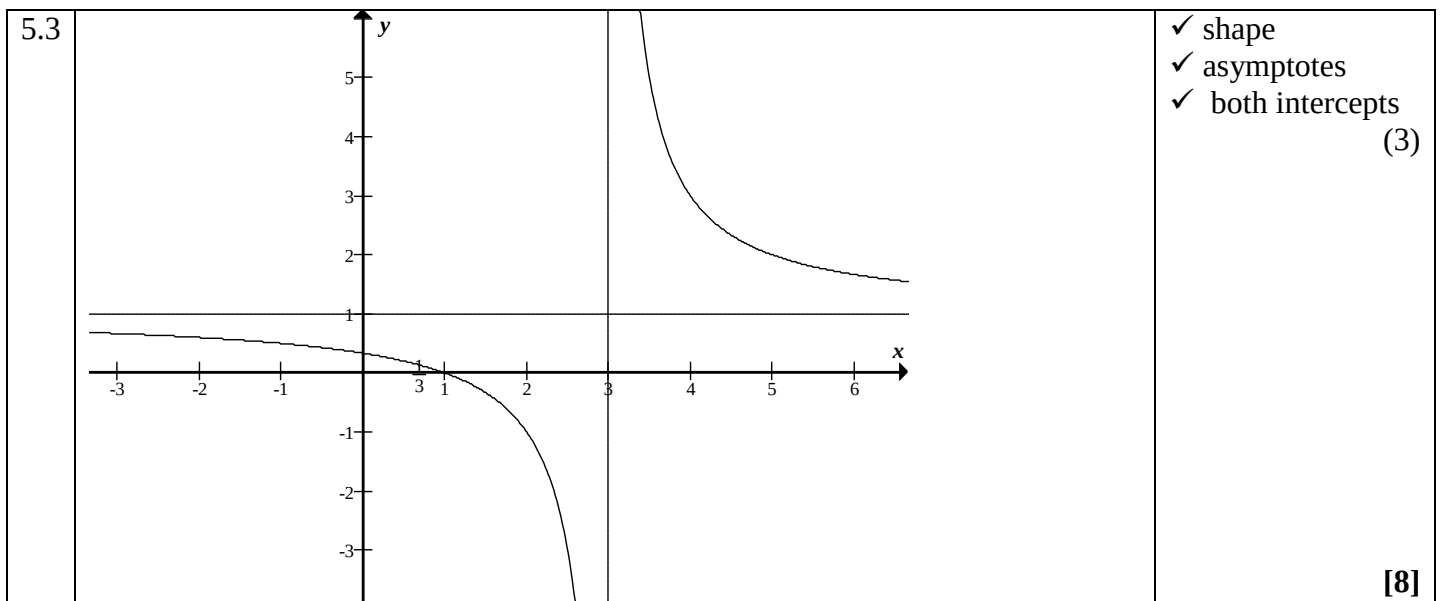
QUESTION 4

4.1	Term	Income	Expenses	Savings	✓ 30 000 ✓ 27 000 ✓ 24 000
	1	120 000	90 000	30 000	
	2	132 000	105 000	27 000	
	3	144 000	120 000	24 000	
	30 000 + 27 000 + 24 000 + ...+ 0.				
4.2	Savings = Income – Expenses Income in year $n = 120\,000 + 12\,000(n - 1)$ Expenses in year $n = 90\,000 + 15\,000(n - 1)$ $120000 + 12000(n - 1) = 90000 + 15000(n - 1)$ $30000 + 12000n - 12000 = 15000n - 15000$ $33000 = 3000n$ $n = 11$ ∴ After 11 years. OR				✓✓ equating ✓ answer (3)

	$a = 30\,000$ $d = -3000$ $T_n = 30000 + (n-1)(-3000)$ $0 = 30000 - 3000n + 3000$ $3000n = 33000$ $\therefore n = 11$ $\therefore$ After 11 years	✓✓ equation ✓ answer (3)
4.3	$120000 + 12000(25-1) = 90000 + x(25-1)$ $x = 13250$	✓ equating ✓ answer (2) [9]

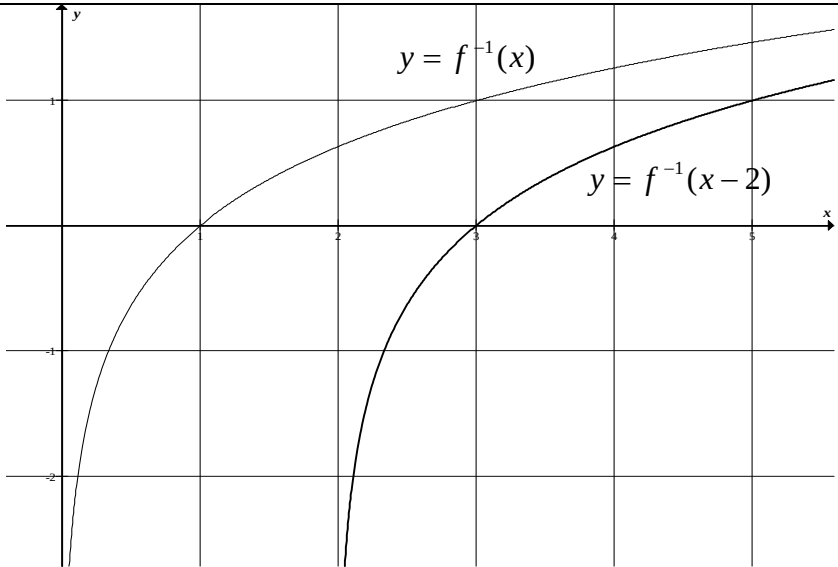
QUESTION 5

5.1	$y = 1$ $x = 3$	✓ answer ✓ answer (2)
5.2	$\frac{2}{0-3} + 1$ $= \frac{1}{3}$ $y\text{-int} \left(0; \frac{1}{3} \right)$ $x\text{-int: } 0 = \frac{2}{x-3} + 1$ $0 = 2 + (x-3)$ $1 = x$ $(1 ; 0)$ OR $y = \frac{x-1}{x-3}$ $\therefore f(0) = \frac{1}{3}$ $f(x) = 0$ $\Rightarrow x-1 = 0$ $\Rightarrow x = 1$ OR	✓ answer ✓ substitution $y = 0$ ✓ answer (3)

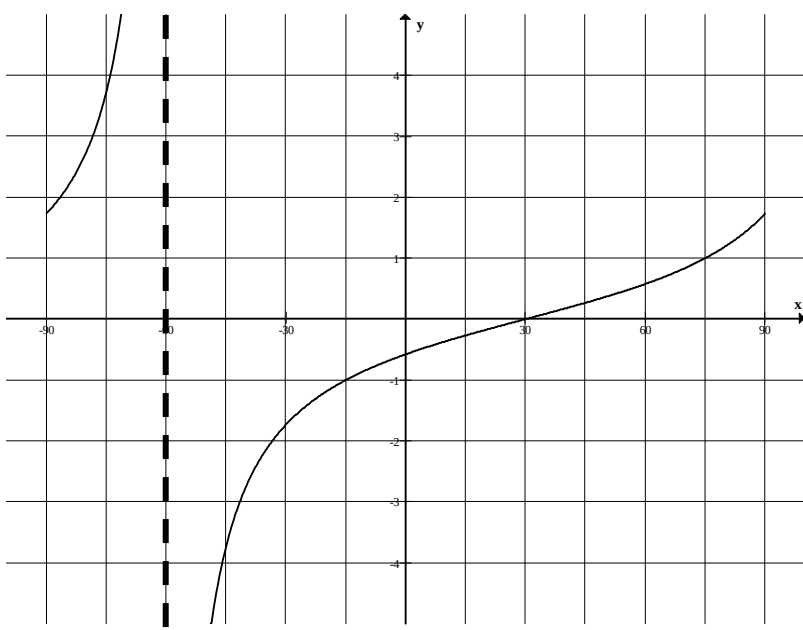
**QUESTION 6**

6.1	$-x^2 + 7x + 8 = 0$ $x^2 - 7x - 8 = 0$ $(x - 8)(x + 1) = 0$ $x = 8$ or $x = -1$ $A(-1; 0)$ $B(8; 0)$	✓ = 0 ✓ factors ✓ answer A ✓ answer B (4)
6.2	$-x^2 + 7x + 8 = -3x + 24$ $-x^2 + 10x - 16 = 0$ $x^2 - 10x + 16 = 0$ $(x - 8)(x - 2) = 0$ $x = 8$ or $x = 2$ x -value of D is 2. (i.e. $a = 2$)	✓ equating ✓ standard form ✓ factors ✓ answer (4)
6.3	$ST = (-x^2 + 7x + 8) - (-3x + 24)$ $= -x^2 + 10x - 16$	✓ subtraction ✓ answer (2)
6.4	Maximum length of ST is at $x = \frac{-10}{2(-1)} = 5$. Maximum length of ST is $-5^2 + 50 - 16 = 9$. OR Maximum length of ST is $\frac{4(-1)(-16) - 10^2}{4(-1)} = 9$	✓ method ✓ answer (2) ✓ method ✓ answer (2) [12]

QUESTION 7

7.1	$y = \log_3 x$	✓ answer (1)
7.2		$y = f^{-1}(x)$ ✓ x-intercept ✓ shape $y = f^{-1}(x - 2)$ ✓ x-intercept ✓ shape (4)
7.3	$2 < x < 5$	✓✓ answer (2) [7]

QUESTION 8

8.1		✓ shape ✓ x-intercept ✓ vertical asymptote (3)
8.2	$x = -60^\circ$	✓ answer (1)
8.3	$\tan(30^\circ - x) = -\tan(x - 30^\circ)$ Reflection about the x-axis	✓ reflection ✓ x-axis (2) [6]

QUESTION 9

9.1.1	Total amount = $P(1 + in)$ $= 55\,000(1 + 0,1275(4))$ $= 83\,050$ Monthly instalment = $\frac{83050}{4 \times 12}$ $= R\,1730,21$	✓ substitution into simple interest formula ✓ answer ✓ $\div 48$ ✓ answer (4)
9.1.2	$55000 = \frac{x \left[1 - \left(1 + \frac{0,2}{12} \right)^{-12 \times 4} \right]}{\frac{0,2}{12}}$ $x = R\,1\,673,67$ a better option because monthly repayments are less.	✓ substitution into formula ✓ $i = \frac{0,2}{12}$ ✓ $n = 48$ ✓ answer (4)
9.1.3	$1673,67 \times 48$ $= 80336,16$ $80336,16 = 55000(1 + 4i)$ $1,460657455... = 1 + 4i$ $i = 0,11516436...$ Rate = 11,52%	✓ 80336,16 ✓ $80336,16 = 55000(1 + 4i)$ ✓ answer (3)
9.2	$80000 = \frac{25000[1 - (1 + 0,1375)^{-n}]}{0,1375}$ $\frac{11}{25} = 1 - (1 + 0,1375)^{-n}$ $\frac{14}{25} = (1 + 0,1375)^{-n}$ $\log \frac{14}{25} = -n \log(1,1375)$ $n = 4,50054779$ The money will last for 4 full years OR Candidate guesses 4 years. Then balance available at the end of 4 years (after the 4th withdrawal) is $80000(1 + 0,1375)^4 - 25000 \left(\frac{(1 + 0,1375)^4 - 1}{0,1375} \right) = R11354,86.$ At the end of the 5th year cannot have grown to R25000.	✓ substitution into correct formula ✓ simplification ✓ taking log of both sides ✓ answer (4) ✓ guesses 4 years ✓✓ calculates balance at end of 4th year ✓ conclusion about balance. (4)

QUESTION 10

10.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ $= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$ $= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h}$ $= \lim_{h \rightarrow 0} -\frac{1}{x(x+h)}$ $= -\frac{1}{x^2}$	✓ substitution into correct formula ✓ expansion ✓ simplification ✓ answer (4)
10.2	$y = (2 - 5x)^2$ $y = 4 - 20x + 25x^2$ $\frac{dy}{dx} = -20 + 50x$ OR $y = (2 - 5x)^2$ By the chain rule $\frac{dy}{dx} = (2)(2 - 5x)(-5)$ $= -20 + 50x$	✓ simplification ✓✓ answers (3) ✓ simplification ✓✓ answers (3) [7]

QUESTION 11

11.1	$0 = x - 2$ $x = 2$ A(2 ; 0)	✓ answer (1)
11.2	$f(-1) = 0: -a + c = 2$ $f(2) = 0: 8a - 2c = 2$ $a = 1, c = 3$ OR $a(x+1)(x+1)(x-2) = 0$ $a(0+1)(0+1)(0-2) = -2$ $-2a = -2$ $a = 1$ $f(x) = (x^2 + 2x + 1)(x - 2)$ $= x^3 - 3x - 2$ $c = -3$	✓ $-a + c = 2$ ✓ $8a - 2c = 2$ ✓ $a = 1$ ✓ $c = 3$ ✓ factors ✓ substitution ✓ a ✓ $c = -3$ (4)
11.3	$f'(x) = 0$ $3x^2 - 3 = 0$ $x^2 - 1 = 0$ $(x+1)(x-1) = 0$ B(1 ; -4)	✓ $f'(x) = 0$ ✓ $x^2 - 1$ ✓ answer (3)
11.4	$x - 2 = x^3 - 3x - 2$ $0 = x^3 - 4x$ $0 = x(x^2 - 4)$ $0 = x(x-2)(x+2)$ $x_C = -2, y_C = (-2)^2 - 3(-2) - 2 = -4$ C(-2 ; -4) $m_{BC} = \frac{-4 - (-4)}{1 - (-2)}$ $= 0$ BC is parallel to the x-axis. OR Following from C(-2 ; -4), B and C have the same y – coordinate, viz. -4. So BC is parallel to the x-axis. OR	✓ equating f and g ✓ standard form ✓ factors ✓ $x_C = -2$ ✓ $y_C = -4$ ✓ $m = 0$ ✓ conclusion (7) (7)

	$(x-2) = (x-2)(x+1)^2$ $\therefore (x+1)^2 = 1 \text{ for } x \neq 2$ $\therefore x+1 = \pm 1$ $\therefore x = 0 \text{ or } x = -2$ $y = -4$	(7)
11.5	$f''(x) = 0$ $6x = 0$ $x = 0$	$\checkmark f''(x) = 0$ $\checkmark$ answer (2)
11.6	$k < -4 \text{ or } k > 0$	$\checkmark\checkmark$ answer $\checkmark$ or (3)
11.7	$f'(x) < 0$ $-1 < x < 1$ OR $3(x^2 - 1) < 0$ if $(x+1)(x-1) < 0$ $-1 < x < 1$	$\checkmark\checkmark$ answer (2) $\checkmark\checkmark$ answer (2) [22]

This alternative memo must be used for students who follow through using $c = -3$. This marking memo must be used independently of the one provided in the existing memorandum.

QUESTION 11

11.1	$0 = x - 2$ $x = 2$ $A(2 : 0)$	$\checkmark$ answer (1)
11.2	$f'(-1) = 0; -a + c = 2$ $f(2) = 0; 8a - 2c = 2$	$\checkmark\checkmark f'(-1) = 0; -a + c = 2$ $\checkmark\checkmark f(2) = 0; 8a - 2c = 2$ (4)
11.3	$f(x) = x^2 + 3x - 2$ $f'(x) = 0$ $3x^2 + 3 = 0$	$\checkmark\checkmark f'(x) = 0$ $\checkmark 3x^2 + 3 = 0$ (3)

11.4	If not attempted (3 marks) OR To calculate C $x - 2 = x^3 + 3x - 2$ $x^3 + 2x = 0$ $x(x^2 + 2) = 0$ $x = 0$ $y = -2$	$\checkmark \checkmark \checkmark x - 2 = x^3 + 3x - 2$ $\checkmark x^3 + 2x = 0$ $\checkmark x(x^2 + 2) = 0$ $\checkmark x = 0$ $\checkmark y = -2$ (7)
11.5	$x = 0$ (any method used)	$\checkmark \checkmark$ answer (2)
11.6	Not Attempted : 0 marks OR $k > 0$	$\checkmark \checkmark \checkmark$ answer (3)
11.7	If $x > -1$, Maximum of 1 Mark OR x is between -1 and (x – value) of B $f(x) = x^3 + 3x - 2$ $f'(x) = 3x^2 + 3 < 0$	$\checkmark \checkmark f'(x) = 3x^2 + 3 < 0$ (2) [22]

QUESTION 12

12.1	Length of sides of square = $\frac{4-x}{4} = 1 - \frac{x}{4}$	✓ answer (1)
12.2	$x = 2\pi r$ $r = \frac{x}{2\pi}$ Areas = $\left(\frac{4-x}{4}\right)^2 + \pi\left(\frac{x}{2\pi}\right)^2$ $= \frac{16-8x+x^2}{16} + \frac{x^2}{4\pi}$ $= 1 - \frac{1}{2}x + \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2$ OR $x = 2\pi r$ $r = \frac{x}{2\pi}$ $\left(1 - \frac{x}{4}\right)^2 + \pi\left(\frac{x}{2\pi}\right)^2$ $1 - \frac{1}{2}x + \frac{x^2}{16} + \frac{x^2}{4\pi}$ $1 - \frac{1}{2}x + \left(\frac{\pi+4}{16\pi}\right)x^2$	✓ $r = \frac{x}{2\pi}$ ✓ sum of areas ✓ simplification ✓ simplification (4) ✓ r ✓ sum of areas ✓ simplification ✓ simplification (4)
12.3	$x = \frac{-b}{2a}$ $= \frac{1}{2\left(\frac{\pi+4}{16\pi}\right)}$ $= 1,76 \text{ meter}$ OR	✓✓ substitution ✓ answer (3)

	$f(x) = 1 - \frac{1}{2}x + \frac{x^2(\pi + 4)}{16\pi}$ $f'(x) = 0 = -\frac{1}{2} + \frac{\pi + 4}{8\pi}x$ $4\pi = (\pi + 4)x$ $x = \frac{4\pi}{\pi + 4}$ $x = 1,76 \text{ m for the circle and } 2,24 \text{ m for the square}$	$\checkmark f'(x) = \frac{1}{2} + \frac{\pi + 4}{8\pi}x$ $\checkmark f'(x) = 0$ $\checkmark \text{ answer}$ [8]
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QUESTION 13

13.1	$4x + 2y \leq 32 \quad \therefore y \leq -2x + 16$ $2x + 4y \leq 36 \quad \therefore y \leq -\frac{x}{2} + 9$ $x + y \leq 10 \quad \therefore y \leq -x + 10$	$\checkmark$ answer $\checkmark$ answer $\checkmark$ answer (3)
13.2	Attached graph	$\checkmark y = -2x + 16$ $\checkmark y = -\frac{x}{2} + 9$ $\checkmark y = -x + 10$ $\checkmark$ feasible region (4)
13.3	$P = 60x + 80y$	$\checkmark$ answer (1)
13.4	$80y = -60x + P$ $y = -\frac{3}{4}x + \frac{P}{80}$ Maximum profit at (2; 8) $\therefore$ Grade 10: 2 learners must be trained to give a maximum profit Grade 11: 8 learners must be trained to give a maximum profit	13.4 $\checkmark$ search line $\checkmark\checkmark (2 ; 8)$ (3)
13.5	$m = -\frac{4}{3}$ Since the gradient of the new profit function is not equal to the gradient of the initial profit function, the new maximum point is (6 ; 4) that gives an optimal solution.	$\checkmark m = -\frac{4}{3}$ $\checkmark (6 ; 4)$ (2)

[13]

QUESTION 13.2 & 13.4