



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

WISKUNDE V2

FEBRUARIE/MAART 2009

MEMORANDUM

Hierdie memorandum bestaan uit 13 bladsye.

VRAAG 1

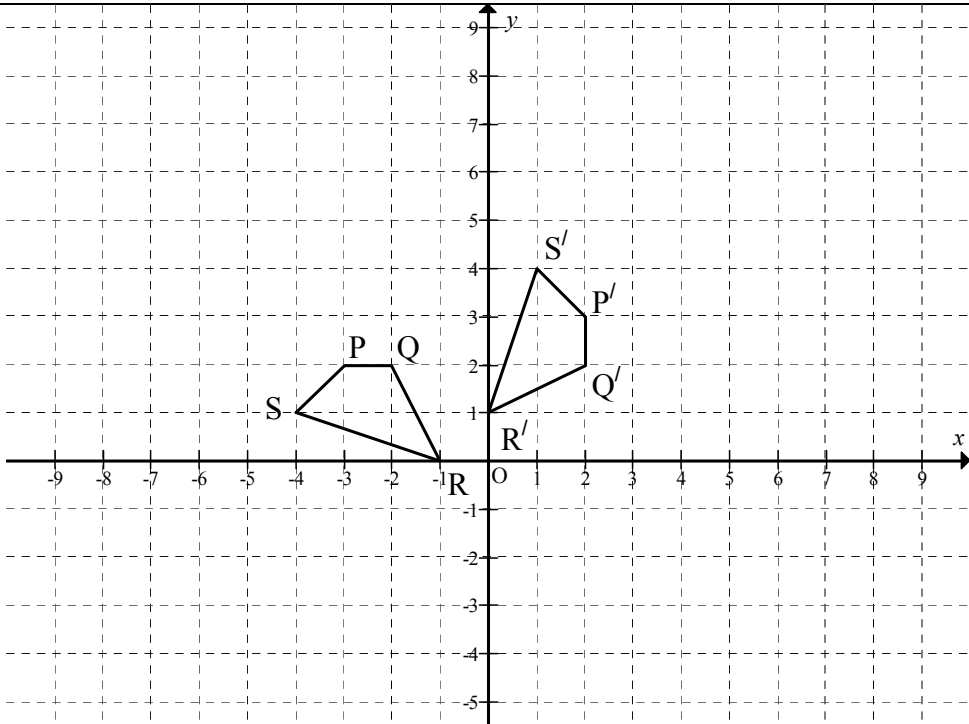
1.1	$m_{BC} = \frac{1 - 0}{6 - 3}$ $m_{BC} = \frac{1}{3}$	✓ substitusie in gradiënt-formule ✓ antwoord (2)
1.2	$m_{AD} = m_{BC}$ $m_{AD} = \frac{1}{3} \quad \text{-----} \quad AB//BC$ ∴ Vergelyking van AD is: $y = \frac{1}{3}x + c$ $6 = \frac{1}{3}(1) + c$ $c = \frac{17}{3}$ $\therefore y = \frac{1}{3}x + \frac{17}{3}$ OF $y - 6 = \frac{1}{3}(x - 1)$ $y - 6 = \frac{1}{3}x - \frac{1}{3}$ $y = \frac{1}{3}x + \frac{17}{3}$	✓ $m_{AC} = \frac{1}{3}$ ✓ substitusie of (1 ; 6) in 'n reguitlynvergelyking ✓ vergelyking (3) ✓ $m_{AC} = \frac{1}{3}$ ✓ substitusie of (1 ; 6) in 'n reguitlynvergelyking ✓ vergelyking (3)
1.3	$y = \frac{1}{3}x + \frac{17}{3}$ $t = \frac{1}{3}(7) + \frac{17}{3}$ $t = 8$ OF $\frac{t - 6}{7 - 1} = \frac{1}{3}$ $t - 6 = 2$ $\therefore t = 8$	✓ ✓ substitusie van x -waarde in 'n reguitlynvergelyking. (2)

1.4	$AD = \sqrt{(8-6)^2 + (-1-3)^2}$ $AD = \sqrt{40}$ $AD = 2\sqrt{10}$ $BC = \sqrt{(6-3)^2 + (1-0)^2}$ $BC = \sqrt{10}$ $AB = \sqrt{(6-0)^2 + (1-3)^2}$ $AB = \sqrt{40}$ $AB = 2\sqrt{10}$	✓ gebruik afstandsformule ✓ antwoord vir AD ✓ Antwoord vir BC ✓ antwoord vir AB (4)
1.5	$m_{AB} = \frac{6-0}{1-3}$ $m_{AB} = -3$ $m_{BC} = \frac{1-0}{6-3} = \frac{1}{3}$ $m_{AB} \cdot m_{BC} = \frac{1}{3} \times -3$ $= -1$ $\therefore AB \perp BC$	✓ $m_{AB} = -3$ ✓ $m_{AB} \times m_{BC} = -1$ ✓ gevolgtrekking (3)
1.6	Area van vierh ABCD = area van ΔADC + area van ABC $= \frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ $= 20 + 10$ $= 30 \text{ vierkante eenheid}$ Of Area van ABCD = $\frac{1}{2}(\text{som van parallel sye}) \times h$ $= \frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ $= \sqrt{10}(3\sqrt{10})$ $= 30 \text{ vierkante eenheid}$	✓ formule van area van Δ ✓ $\frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ ✓ antwoord (4) ✓ formule van area van trapezium ✓ $\frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ ✓ antwoord (4)
1.7	Uit 1.1 $m_{BC} = \frac{1}{3}$ $\tan \theta = \frac{1}{3}$ $\therefore \theta = 18,43^\circ$	✓ $\tan \theta = \frac{1}{3}$ ✓ $\theta = 18,43^\circ$ (3) [21]

VRAAG 2

2.1	Middelpunt AB $\left(\frac{-8+0}{2}; \frac{1+5}{2}\right)$ $= (-4; 3)$	✓ substitusie in middelpuntformule (1)
2.2	$M_{AD} = \frac{5+1}{0-3} = \frac{-2}{1}$ $y - y_1 = m(x - x_1)$ $y - 5 = -2(x - 0)$ $y = -2x + 5$	✓ substitusie in gradiënt formule ✓ $M_{AD} = \frac{-2}{1}$ ✓ substitusie van $(0; 5)$ in 'n reguitlynvergelyking ✓ antwoord (4)
2.3	$AM^2 = (5-3)^2 + (0+4)^2$ $AM^2 = 2^2 + 4^2$ $AM = \sqrt{20}$	✓ substitusie in afstands- formule ✓ vereenvoudiging ✓ antwoord (3)
2.4	$(x+4)^2 + (y-3)^2 = (\sqrt{20})^2$ $(x+4)^2 + (y-3)^2 = 20$ $x^2 + y^2 + 8x - 6y + 5 = 0$	✓ $(x+4)^2$ ✓ $(y-3)^2$ ✓ 20 ✓ antwoord (4)
2.5	$AT = TK = 6$ $CD \perp AK$ Dus ACKD is 'n vlieër want diagonaal CD halveer diagonaal AK reghoekig. OF $\hat{CAD} = 90^\circ$ $M_{KC} \cdot M_{KD} = \frac{6}{-12} \cdot \frac{6}{3} = -1$ $\therefore \hat{CKD} = 90^\circ$ ΔCAD & ΔCKD is reghoekig & kongruent ACKD is 'n vlieër	✓ $AT = TK$ ✓ CD loodreg op AK ✓ Vlieër ✓ rede (4) ✓ ✓ $M_{KC} \cdot M_{KD} = \frac{6}{-12} \cdot \frac{6}{3} = -1$ ✓ ΔCAD & ΔCKD is reghoekig & kongruent ✓ ACKD is 'n vlieër (4) [16]

VRAAG 3

3.1.1	$P'(-\sqrt{3}; -2)$	✓ ✓ koördinate P' (2)
3.1.2	$P'(-\sqrt{3}; 2)$	✓ ✓ koördinate P' (2)
3.2.1	$Q'(2; 2)$	✓ ✓ koördinate Q' (2)
3.2.2		✓ koördinate P' ✓ koördinate Q' ✓ koördinate R' ✓ koördinate S' (4)
3.2.3	$P''(4; 6)$	✓ ✓ antwoord (2)
3.2.4	Nie star. Die vorm bly dieselfde, maar die grootte verander.	✓ nie star ✓ verduideliking (2)
3.2.5	$(x; y) \rightarrow (y; -x)$ $(y; -x) \rightarrow (2y; -2x)$ $\therefore (x; y) \rightarrow (2y; -2x)$	✓ $(y; -x)$ ✓ ✓ $(2y; -2x)$ (3)
3.2.6	Area PQRS : area $P''Q''R''S''$ $= 1^2 : 2^2$ $= 1 : 4$	✓ kwadrering ✓ antwoord (2) [19]

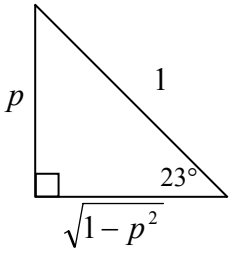
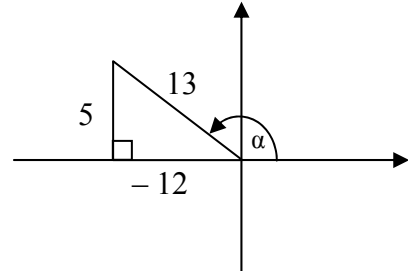
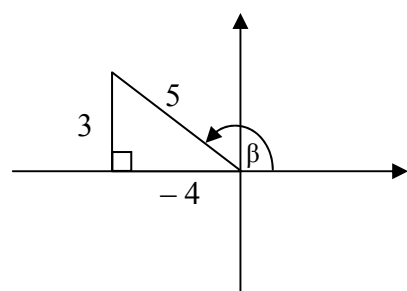
VRAAG 4

4. 1	$x' = x \cos(135^\circ) - y \sin(135^\circ)$ $x' = -x \cos 45^\circ - y \sin 45^\circ$ $x' = x \left(\frac{-\sqrt{2}}{2} \right) - y \left(\frac{\sqrt{2}}{2} \right)$ $x' = -\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y$ en $y' = y \cos(135^\circ) + x \sin(135^\circ)$ $y' = -y \cos 45^\circ + x \sin 45^\circ$ $y' = y \left(-\frac{\sqrt{2}}{2} \right) + x \left(\frac{\sqrt{2}}{2} \right)$ $y' = -\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}x$	✓ 135° ✓ substitusie ✓ antwoord vir x ✓ antwoord vir y (4)
4.2	$x' = -\frac{\sqrt{2}}{2}(2) - \frac{\sqrt{2}}{2}(4)$ $x' = -\sqrt{2} - 2\sqrt{2}$ $x' = -3\sqrt{2}$ $y' = -\frac{\sqrt{2}}{2}(4) + \frac{\sqrt{2}}{2}(2)$ $y' = -\sqrt{2}$ $\therefore M(-3\sqrt{2}; -\sqrt{2})$	✓ x koördinate ✓ y koördinate (2) [6]

VRAAG 5

5.1	$\frac{\tan(180^\circ + x) \cos(360^\circ - x)}{\sin(180^\circ - x) \cos(90^\circ + x) + \cos(540^\circ + x) \cos(-x)}$ $= \frac{\tan x \cdot (\cos x)}{(\sin x) \cdot (-\sin x) - \cos x \cdot \cos x}$ $= \frac{\frac{\sin x}{\cos x} \cos x}{-\sin^2 x - \cos^2 x}$ $= \frac{\sin x}{-(\sin^2 x + \cos^2 x)}$ $= -\sin x$	✓ $\tan x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ $\cos x$ ✓ vereenvoudiging ✓ antwoord (8)
5.2	$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2\sin^2 x) - \sin x}{2\sin x \cdot \cos x - \cos x}$ $= \frac{2\sin^2 x - \sin x}{2\sin x \cdot \cos x - \cos x}$ $= \frac{\sin x(2\sin x - 1)}{\cos x(2\sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	✓ $1 - 2\sin^2 x$ ✓ $2\sin x \cdot \cos x$ ✓✓ faktorisering ✓ antwoord (5) [13]

VRAAG 6

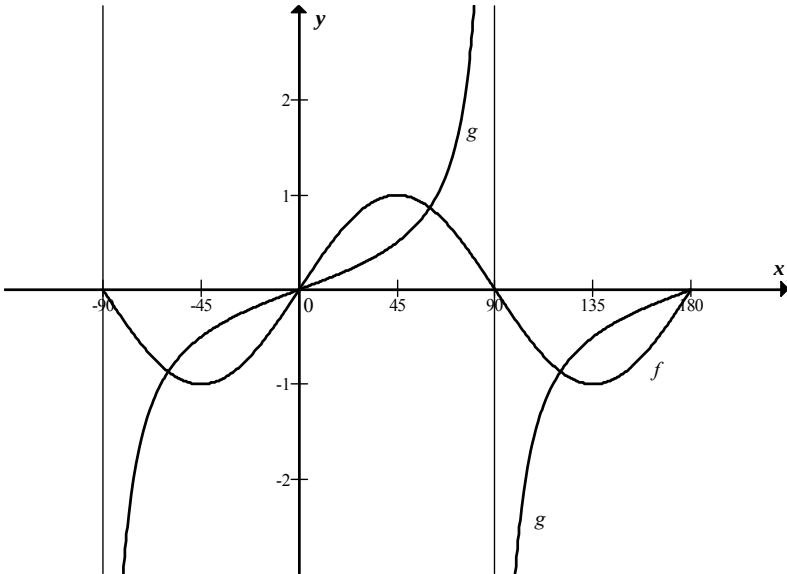
6.1.1	$\cos 113^\circ$ $= \cos (90^\circ + 23^\circ)$ $= -\sin 23^\circ$ $= -p$	✓ reduksie ✓ antwoord (2)
6.1.2	$\cos 23^\circ$ $= \sqrt{1 - p^2}$ OF $\cos^2 23^\circ + \sin^2 23^\circ = 1$ $\cos^2 23^\circ = 1 - p^2$ $\cos 23^\circ = \sqrt{1 - p^2}$	 ✓ diagram ✓ antwoord OF ✓ identiteit ✓ antwoord (2)
6.1.3	$\sin 46^\circ$ $= 2\sin 23^\circ \cdot \cos 23^\circ$ $= 2p\sqrt{1 - p^2}$	✓ uitbreiding ✓ antwoord (2)
6.2.1	$\sin \alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos \alpha = -\frac{12}{13}$	 ✓ vereenvoudiging ✓ diagram ✓ antwoord (3)
6.2.2	$\tan \beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$ $\cos(\alpha + \beta)$ $= \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$ $= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $= \frac{48 - 15}{65}$ $= \frac{33}{65}$	 ✓ diagram ✓ uitbreiding ✓ $-\frac{4}{5}$ ✓ $\frac{3}{5}$ ✓ antwoord (5)

6.3	$\frac{1}{2} \cos x = 0,435$ $\cos x = 0,87$ $x = 29,54^\circ$ or $x = 330,46^\circ$	✓ vereenvoudiging ✓✓ antwoorde (3) [17]
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VRAAG 7

7.1	$\frac{7}{PB} = \sin 18^\circ$ $PB = \frac{7}{\sin 18^\circ}$ $PB = 22,65 \text{ m}$ (22,65247584...)	✓ verhouding ✓ antwoord (2)
7.2	$\frac{18}{PA} = \cos 23^\circ$ $PA = \frac{18}{\cos 23^\circ}$ $PA = 19,55 \text{ m}$ (19,55448679..)	✓ verhouding ✓ antwoord (2)
7.3	$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55) \cdot \cos 42^\circ$ $= 237,0847954...$ $AB = 15,40 \text{ m}$ (15,3975581...)	✓ gebruik van cos-reel ✓ substitusie ✓ 237,0847... ✓ antwoord (4) [8]

VRAAG 8

8.1		tan-grafiek ✓ vorm ✓ asymptote ✓ snypunte Sin-grafiek ✓ vorm ✓ snypunte ✓ periode (6)
8.2	$\sin 2x = \frac{1}{2} \tan x$ $2 \sin x \cdot \cos x = \frac{\sin x}{2 \cos x}$ $4 \sin x \cdot \cos^2 x - \sin x = 0$ $\sin x (4 \cos^2 x - 1) = 0$ $\sin x = 0 \quad \cos^2 x = \frac{1}{4}$ $x = 0^\circ \text{ or } 180^\circ \text{ of } \cos x = \pm \frac{1}{2}$ $x = 60^\circ ; -60^\circ \text{ or } 120^\circ$	✓ gelykstelling ✓ $2 \cdot \sin x \cdot \cos x$ ✓ $\frac{\sin x}{2 \cos x}$ ✓ vereenvoudiging ✓ faktorisering ✓ $\sin x = 0$ ✓ $x = 0^\circ \text{ of } 180^\circ$ ✓ $\cos^2 x = \frac{1}{4}$ ✓ $\cos x = \pm \frac{1}{2}$ ✓ antwoorde (10)
8.3	$\{x \mid -60^\circ < x < 0^\circ\} \cup \{x \mid 60^\circ < x < 90^\circ\} \cup \{x \mid 120^\circ < x < 180^\circ\}$ OF $x \in (-60^\circ ; 0^\circ) \cup (60^\circ ; 90^\circ) \cup (120^\circ ; 180^\circ)$ OF $-60^\circ < x < 0^\circ \text{ or } 60^\circ < x < 90^\circ \text{ or } 120^\circ < x < 180^\circ$	✓✓✓ antwoorde (3) [19]

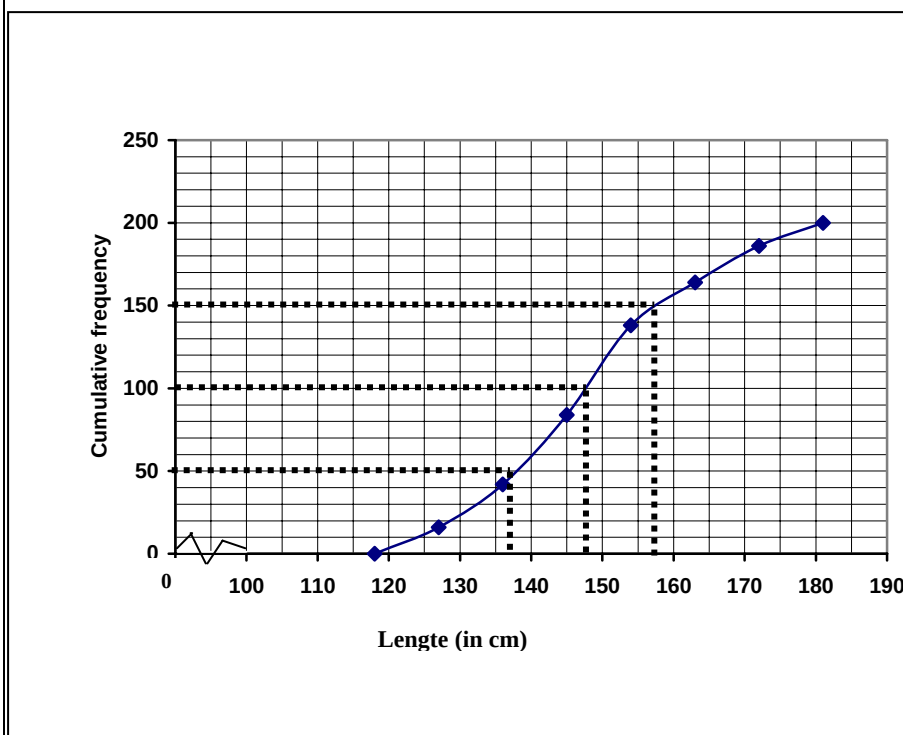
VRAAG 11

11.1

LENGTE (IN CM)	FREKWENSIE	KUMULATIEWE FREKWENSIE
$118 \leq h < 127$	16	16
$127 \leq h < 136$	26	42
$136 \leq h < 145$	42	84
$145 \leq h < 154$	54	138
$154 \leq h < 163$	26	164
$163 \leq h < 172$	22	186
$172 \leq h < 181$	14	200

✓✓ antwoorde in kumulatiewe frekwensiekolom
(2)

11.2



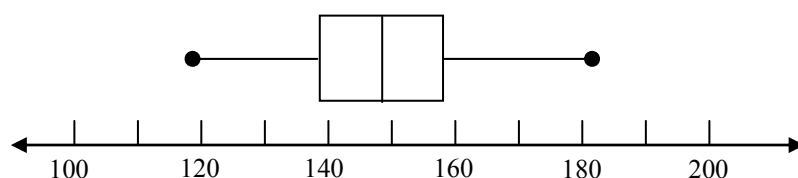
✓ kumulatiewe totale
✓ punte by boonste limiete van intervale
✓ kurwes
(3)

11.3

Onderste kwartiel ≈ 138 cm
Mediaan ≈ 148 cm
Boonste kwartiel ≈ 158 cm

✓✓✓ Lees korrek vanaf ogief
(3)

11.4



✓ minimum en maksimum waardes
✓ kwartiele en mediaan
✓ snorbaarde
(3)

11.5

Die lengtes van die leerders is redelik eweredig versprei.

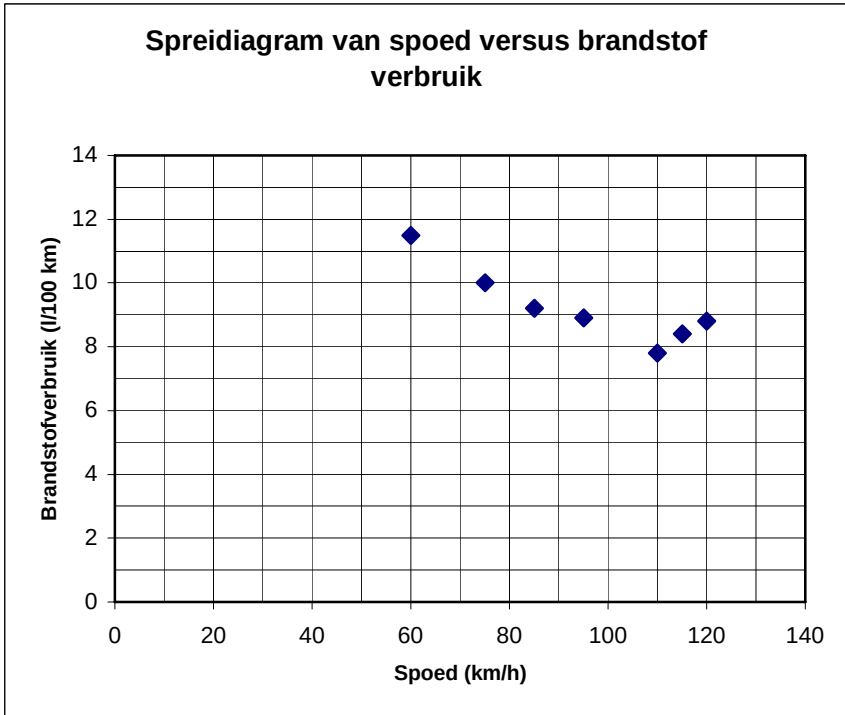
11.6

100 leerders val in hierdie interval.

✓ eweredig versprei
✓ 100
(1)

(1)
[13]

VRAAG 12

12.1	Spreidiagram van spoed versus brandstof verbruik <table><caption>Data points from the scatter plot</caption><thead><tr><th>Spoed (km/h)</th><th>Brandstofverbruik (l/100 km)</th></tr></thead><tbody><tr><td>60</td><td>11.5</td></tr><tr><td>75</td><td>10.0</td></tr><tr><td>85</td><td>9.2</td></tr><tr><td>95</td><td>8.8</td></tr><tr><td>110</td><td>7.8</td></tr><tr><td>115</td><td>8.5</td></tr><tr><td>120</td><td>8.8</td></tr></tbody></table>	Spoed (km/h)	Brandstofverbruik (l/100 km)	60	11.5	75	10.0	85	9.2	95	8.8	110	7.8	115	8.5	120	8.8	✓✓plot van punte ✓ benaminge (3)
Spoed (km/h)	Brandstofverbruik (l/100 km)																	
60	11.5																	
75	10.0																	
85	9.2																	
95	8.8																	
110	7.8																	
115	8.5																	
120	8.8																	
12.2	Kwadratiese	✓ kwadratiese (1)																
12.3	Die kwadratiese patroon wys dat die beste brandstofverbruik plaasvind wanneer die motor teen 110 km/h bestuur word. Om die brandstofrekening van die maatskappy minimaal te hou, moet die bestuurders aangemoedig word om sover moontlik teen die genoemde spoed te ry.	✓✓ antwoord (2) [6]																

TOTAAL: 150