



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2009

MEMORANDUM

This memorandum consists of 13 pages.

QUESTION 1

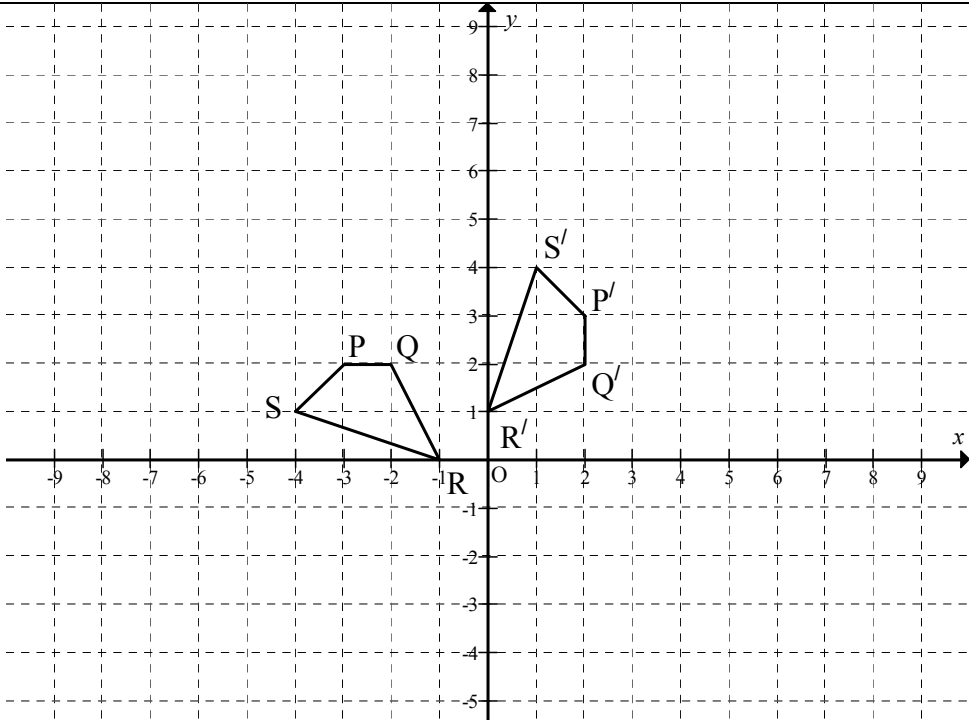
1.1	$m_{BC} = \frac{1 - 0}{6 - 3}$ $m_{BC} = \frac{1}{3}$	✓ substitution into gradient formula ✓ answer (2)
1.2	$m_{AD} = m_{BC}$ $m_{AD} = \frac{1}{3} \quad \text{-----} \quad AB//BC$ ∴ Equation of AD is: $y = \frac{1}{3}x + c$ $6 = \frac{1}{3}(1) + c$ $c = \frac{17}{3}$ $\therefore y = \frac{1}{3}x + \frac{17}{3}$ OR $y - 6 = \frac{1}{3}(x - 1)$ $y - 6 = \frac{1}{3}x - \frac{1}{3}$ $y = \frac{1}{3}x + \frac{17}{3}$	$\checkmark m_{AC} = \frac{1}{3}$ ✓ substitution of (1 ; 6) into a straight line equation ✓ equation (3)
1.3	$y = \frac{1}{3}x + \frac{17}{3}$ $t = \frac{1}{3}(7) + \frac{17}{3}$ $t = 8$ OR $\frac{t - 6}{7 - 1} = \frac{1}{3}$ $t - 6 = 2$ $\therefore t = 8$	✓ ✓ substitution of x value into a straight line equation. (2)

1.4	$AD = \sqrt{(8-6)^2 + (-1-3)^2}$ $AD = \sqrt{40}$ $AD = 2\sqrt{10}$ $BC = \sqrt{(6-3)^2 + (1-0)^2}$ $BC = \sqrt{10}$ $AB = \sqrt{(6-0)^2 + (1-3)^2}$ $AB = \sqrt{40}$ $AB = 2\sqrt{10}$	✓ using distance formula ✓ answer for AD ✓ answer for BC ✓ answer for AB (4)
1.5	$m_{AB} = \frac{6-0}{1-3}$ $m_{AB} = -3$ $m_{BC} = \frac{1-0}{6-3} = \frac{1}{3}$ $m_{AB} \cdot m_{BC} = \frac{1}{3} \times -3$ $= -1$ $\therefore AB \perp BC$	✓ $m_{AB} = -3$ ✓ $m_{AB} \times m_{BC} = -1$ ✓ conclusion (3)
1.6	Area of Quad ABCD = area of $\triangle ADC$ + area of $\triangle ABC$ $= \frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ $= 20 + 10$ $= 30 \text{ square units}$ Or Area of ABCD = $\frac{1}{2}(\text{sum of parallel sides}) \times h$ $= \frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ $= \sqrt{10}(3\sqrt{10})$ $= 30 \text{ square units}$	✓ formula for area of $\triangle$ ✓ ✓ $\frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ ✓ answer (4) ✓ formula for area of trapezium ✓ ✓ $\frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ ✓ Answer (4)
1.7	From 1.1 $m_{BC} = \frac{1}{3}$ $\tan \theta = \frac{1}{3}$ $\therefore \theta = 18,43^\circ$	✓ ✓ $\tan \theta = \frac{1}{3}$ ✓ $\theta = 18,43^\circ$ (3) [21]

QUESTION 2

2.1	Midpoint $AB \left(\frac{-8+0}{2}; \frac{1+5}{2} \right)$ $= (-4; 3)$	✓ substitution into midpoint formula (1)
2.2	$M_{AD} = \frac{5+1}{0-3} = \frac{-2}{1}$ $y - y_1 = m(x - x_1)$ $y - 5 = -2(x - 0)$ $y = -2x + 5$	✓ substitution into gradient formula ✓ $M_{AD} = \frac{-2}{1}$ ✓ substitution of (0 ; 5) in a straight line equation ✓ Answer (4)
2.3	$AM^2 = (5-3)^2 + (0+4)^2$ $AM^2 = 2^2 + 4^2$ $AM = \sqrt{20}$	✓ substitution into distance formula ✓ simplification ✓ answer (3)
2.4	$(x+4)^2 + (y-3)^2 = (\sqrt{20})^2$ $(x+4)^2 + (y-3)^2 = 20$ $x^2 + y^2 + 8x - 6y + 5 = 0$	✓ $(x+4)^2$ ✓ $(y-3)^2$ ✓ 20 ✓ answer (4)
2.5	$AT = TK = 6$ $CD \perp AK$ Therefore, ACKD is a kite since diagonal CD bisects diagonal AK at right angles. OR $\hat{CAD} = 90^\circ$ $M_{KC} \cdot M_{KD} = \frac{6}{-12} \cdot \frac{6}{3} = -1$ $\therefore \hat{CKD} = 90^\circ$ ΔCAD & ΔCKD are right angles & congruent ACKD is a kite	✓ $AT = TK$ ✓ CD perpendicular to AK ✓ Kite ✓ reason (4) ✓ ✓ $M_{KC} \cdot M_{KD} = \frac{6}{-12} \cdot \frac{6}{3} = -1$ ✓ ΔCAD & ΔCKD are right angles & congruent ✓ ACKD is a kite (4) [16]

QUESTION 3

3.1.1	$P'(-\sqrt{3}; -2)$	✓ ✓ coordinates P' (2)
3.1.2	$P'(-\sqrt{3}; 2)$	✓ ✓ coordinates P' (2)
3.2.1	$Q'(2; 2)$	✓ ✓ coordinates Q' (2)
3.2.2		✓ coordinates P' ✓ coordinates Q' ✓ coordinates R' ✓ coordinates S' (4)
3.2.3	$P''(4; 6)$	✓ ✓ answer (2)
3.2.4	Not rigid. The shape remains the same, whilst the size changes.	✓ Not rigid ✓ explanation (2)
3.2.5	$(x; y) \rightarrow (y; -x)$ $(y; -x) \rightarrow (2y; -2x)$ $\therefore (x; y) \rightarrow (2y; -2x)$	✓ $(y; -x)$ ✓ ✓ $(2y; -2x)$ (3)
3.2.6	Area of PQRS : area $P''Q''R''S''$ $= 1^2 : 2^2$ $= 1 : 4$	✓ squaring ✓ answer (2) [19]

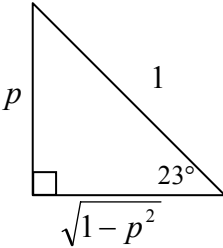
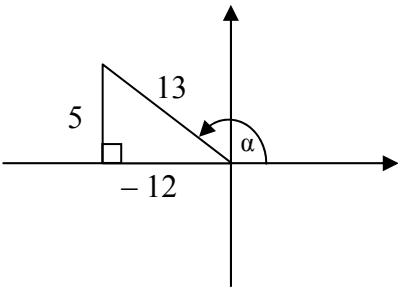
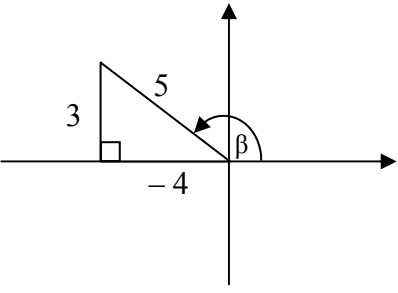
QUESTION 4

4. 1	$x' = x \cos(135^\circ) - y \sin(135^\circ)$ $x' = -x \cos 45^\circ - y \sin 45^\circ$ $x' = x \left(\frac{-\sqrt{2}}{2} \right) - y \left(\frac{\sqrt{2}}{2} \right)$ $x' = -\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y$ and $y' = y \cos(135^\circ) + x \sin(135^\circ)$ $y' = -y \cos 45^\circ + x \sin 45^\circ$ $y' = y \left(-\frac{\sqrt{2}}{2} \right) + x \left(\frac{\sqrt{2}}{2} \right)$ $y' = -\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}x$	✓ 135° ✓ substitution ✓ answer for x ✓ answer for y (4)
4.2	$x' = -\frac{\sqrt{2}}{2}(2) - \frac{\sqrt{2}}{2}(4)$ $x' = -\sqrt{2} - 2\sqrt{2}$ $x' = -3\sqrt{2}$ $y' = -\frac{\sqrt{2}}{2}(4) + \frac{\sqrt{2}}{2}(2)$ $y' = -\sqrt{2}$ $\therefore M(-3\sqrt{2}; -\sqrt{2})$	✓ x coordinates ✓ y coordinates (2) [6]

QUESTION 5

5.1	$\frac{\tan(180^\circ + x) \cos(360^\circ - x)}{\sin(180^\circ - x) \cos(90^\circ + x) + \cos(540^\circ + x) \cos(-x)}$ $= \frac{\tan x \cdot (\cos x)}{(\sin x) \cdot (-\sin x) - \cos x \cdot \cos x}$ $= \frac{\frac{\sin x}{\cos x} \cos x}{-\sin^2 x - \cos^2 x}$ $= \frac{\sin x}{-(\sin^2 x + \cos^2 x)}$ $= -\sin x$	✓ $\tan x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ $\cos x$ ✓ simplification ✓ answer (8)
5.2	$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2\sin^2 x) - \sin x}{2\sin x \cdot \cos x - \cos x}$ $= \frac{2\sin^2 x - \sin x}{2\sin x \cdot \cos x - \cos x}$ $= \frac{\sin x(2\sin x - 1)}{\cos x(2\sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	✓ $1 - 2\sin^2 x$ ✓ $2\sin x \cdot \cos x$ ✓✓ factorisation ✓ answer (5) [13]

QUESTION 6

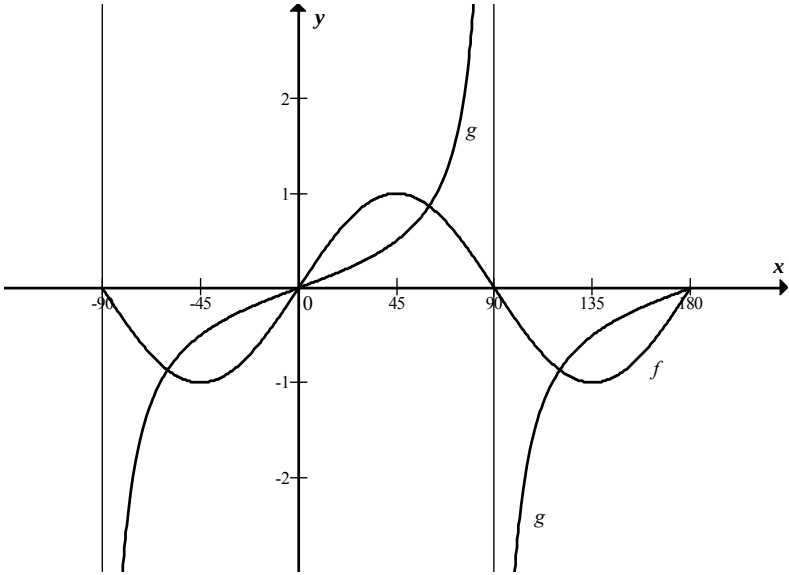
6.1.1	$\cos 113^\circ$ $= \cos (90^\circ + 23^\circ)$ $= -\sin 23^\circ$ $= -p$	✓ reduction ✓ answer (2)
6.1.2	$\cos 23^\circ$ $= \sqrt{1 - p^2}$ OR $\cos^2 23^\circ + \sin^2 23^\circ = 1$ $\cos^2 23^\circ = 1 - p^2$ $\cos 23^\circ = \sqrt{1 - p^2}$	 OR ✓ identity ✓ answer (2)
6.1.3	$\sin 46^\circ$ $= 2\sin 23^\circ \cdot \cos 23^\circ$ $= 2p\sqrt{1 - p^2}$	✓ expansion ✓ answer (2)
6.2.1	$\sin \alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos \alpha = -\frac{12}{13}$	 ✓ simplification ✓ diagram ✓ answer (3)
6.2.2	$\tan \beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$ $\cos(\alpha + \beta)$ $= \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$ $= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $= \frac{48 - 15}{65}$ $= \frac{33}{65}$	 ✓ diagram ✓ expansion ✓ $-\frac{4}{5}$ ✓ $\frac{3}{5}$ ✓ answer (5)

6.3	$\frac{1}{2} \cos x = 0,435$ $\cos x = 0,87$ $x = 29,54^\circ$ or $x = 330,46^\circ$	✓ simplification ✓✓ answers (3) [17]
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QUESTION 7

7.1	$\frac{7}{PB} = \sin 18^\circ$ $PB = \frac{7}{\sin 18^\circ}$ $PB = 22,65 \text{ m}$ (22,65247584...)	✓ ratio ✓ answer (2)
7.2	$\frac{18}{PA} = \cos 23^\circ$ $PA = \frac{18}{\cos 23^\circ}$ $PA = 19,55 \text{ m}$ (19,55448679..)	✓ ratio ✓ answer (2)
7.3	$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55) \cdot \cos 42^\circ$ $= 237,0847954...$ $AB = 15,40 \text{ m}$ (15,3975581...)	✓ use of cosine rule ✓ substitution ✓ 237,0847... ✓ answer (4) [8]

QUESTION 8

8.1		tan graph ✓ shape ✓ asymptotes ✓ intercepts Sine graph ✓ shape ✓ intercepts ✓ period (6)
8.2	$\sin 2x = \frac{1}{2} \tan x$ $2 \sin x \cdot \cos x = \frac{\sin x}{2 \cos x}$ $4 \sin x \cdot \cos^2 x - \sin x = 0$ $\sin x (4 \cos^2 x - 1) = 0$ $\sin x = 0 \qquad \cos^2 x = \frac{1}{4}$ $x = 0^\circ \text{ or } 180^\circ \text{ or } \cos x = \pm \frac{1}{2}$ $x = 60^\circ ; -60^\circ \text{ or } 120^\circ$	✓ equating ✓ $2 \cdot \sin x \cdot \cos x$ ✓ $\frac{\sin x}{2 \cos x}$ ✓ simplification ✓ factorisation ✓ $\sin x = 0$ ✓ $x = 0^\circ \text{ or } 180^\circ$ ✓ $\cos^2 x = \frac{1}{4}$ ✓ $\cos x = \pm \frac{1}{2}$ ✓ answers (10)
8.3	$\{x \mid -60^\circ < x < 0^\circ\} \cup \{x \mid 60^\circ < x < 90^\circ\} \cup \{x \mid 120^\circ < x < 180^\circ\}$ OR $x \in (-60^\circ ; 0^\circ) \cup (60^\circ ; 90^\circ) \cup (120^\circ ; 180^\circ)$ OR $-60^\circ < x < 0^\circ \text{ or } 60^\circ < x < 90^\circ \text{ or } 120^\circ < x < 180^\circ$	✓✓✓ answers (3) [19]

	$= \frac{1}{3 \sin^2 x + 3 \cos^2 x + \cos^2 x}$ $= \frac{1}{3(\sin^2 x + \cos^2 x) + \cos^2 x}$ $= \frac{1}{3 + \cos^2 x}$	✓ expansion ✓ identity ✓ simplification ✓ answer(s) (4) [4]
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10.1	Mean = $\frac{5500}{10} = 550$ kilocalories	✓✓ mean (2)																																				
10.2	$\sigma = 69,03$ kilocalories (done by calculator) OR <table border="1"> <thead> <tr> <th>x</th><th>$(x - \bar{x})$</th><th>$(x_i - \bar{x})^2$</th></tr> </thead> <tbody> <tr><td>440</td><td>-110</td><td>12100</td></tr> <tr><td>520</td><td>-30</td><td>900</td></tr> <tr><td>480</td><td>-70</td><td>4900</td></tr> <tr><td>560</td><td>10</td><td>100</td></tr> <tr><td>615</td><td>65</td><td>4225</td></tr> <tr><td>550</td><td>0</td><td>0</td></tr> <tr><td>620</td><td>70</td><td>4900</td></tr> <tr><td>680</td><td>130</td><td>16900</td></tr> <tr><td>545</td><td>-5</td><td>25</td></tr> <tr><td>490</td><td>-60</td><td>3600</td></tr> <tr> <td colspan="2">Sum</td><td>47650</td></tr> </tbody> </table> $\sigma^2 = \frac{47650}{10}$ $= 4765$ $\sigma = 69,03$	x	$(x - \bar{x})$	$(x_i - \bar{x})^2$	440	-110	12100	520	-30	900	480	-70	4900	560	10	100	615	65	4225	550	0	0	620	70	4900	680	130	16900	545	-5	25	490	-60	3600	Sum		47650	✓✓✓✓ standard deviation (4) ✓✓ $(x_i - \bar{x})^2$ ✓ sum = 47650 ✓ answer
x	$(x - \bar{x})$	$(x_i - \bar{x})^2$																																				
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Sum		47650																																				
10.3	Snack foods have a greater variation. The standard deviation for snack foods is 69,03 kilocalories whilst the standard deviation for breakfast cereals is 28 kilocalories. i.e energy levels of breakfast cereals is spread closer to the mean than in those of the snack food.	✓snack foods ✓ explanation (2)																																				

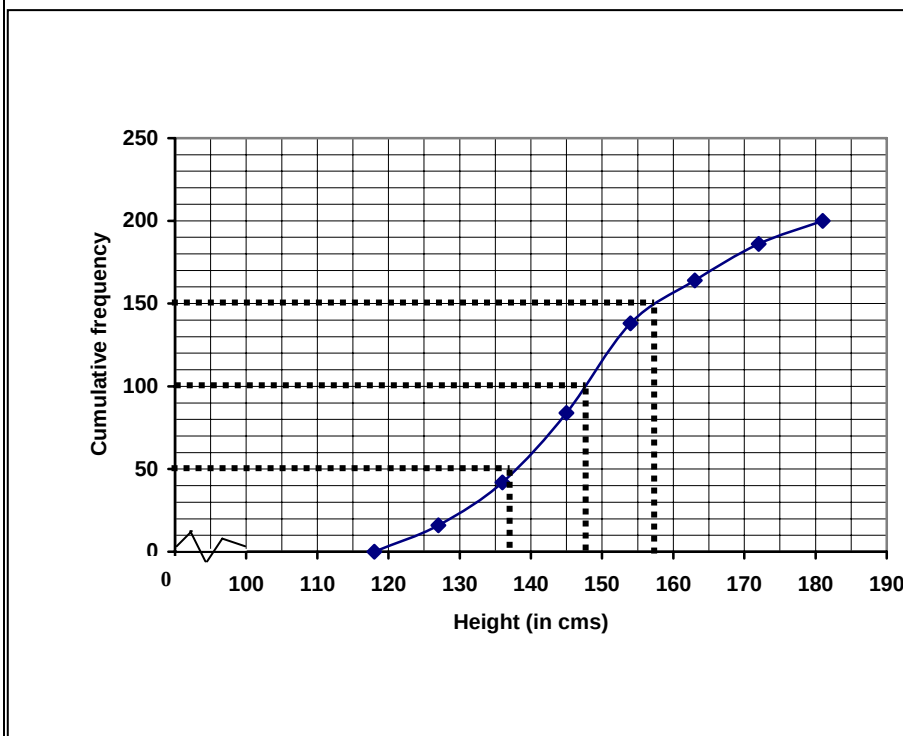
QUESTION 11

11.1

Height (in cms)	Frequency	Cumulative Frequency
$118 \leq h < 127$	16	16
$127 \leq h < 136$	26	42
$136 \leq h < 145$	42	84
$145 \leq h < 154$	54	138
$154 \leq h < 163$	26	164
$163 \leq h < 172$	22	186
$172 \leq h < 181$	14	200

✓✓ answers in cumulative frequency column (2)

11.2



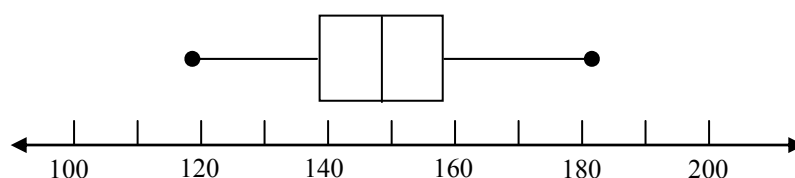
✓ cumulative totals
✓ points at upper limits of intervals
✓ curve (3)

11.3

Lower quartile ≈ 138 cms
Median ≈ 148 cms
Upper quartile ≈ 158 cms

✓✓✓ correctly read off ogive (3)

11.4



✓ minimum and maximum values
✓ quartiles and median
✓ whiskers (3)

11.5

The heights of players are spread fairly evenly.

✓ spread evenly (1)

11.6

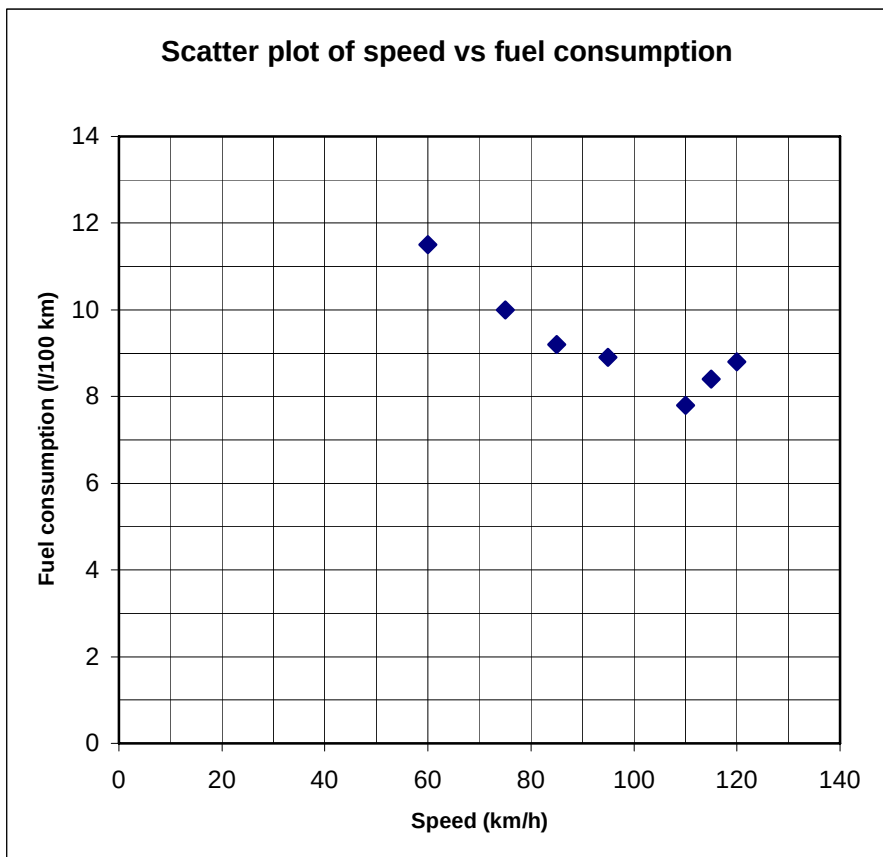
100 players fall in this height interval.

✓ 100 (1)

[13]

QUESTION 12

12.1



✓✓ plotting points
✓ labels

(3)

12.2

✓ quadratic

(1)

12.3

Quadratic

✓✓ answer

(2)

The quadratic pattern shows that the best fuel consumption occurs when the car is driven at 110 km/h. In order for the company to keep its fuel bill to a minimum, drivers should be urged to travel at this speed where possible.

[6]**TOTAL: 150**