



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2009

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, an information sheet and 3 diagram sheets.

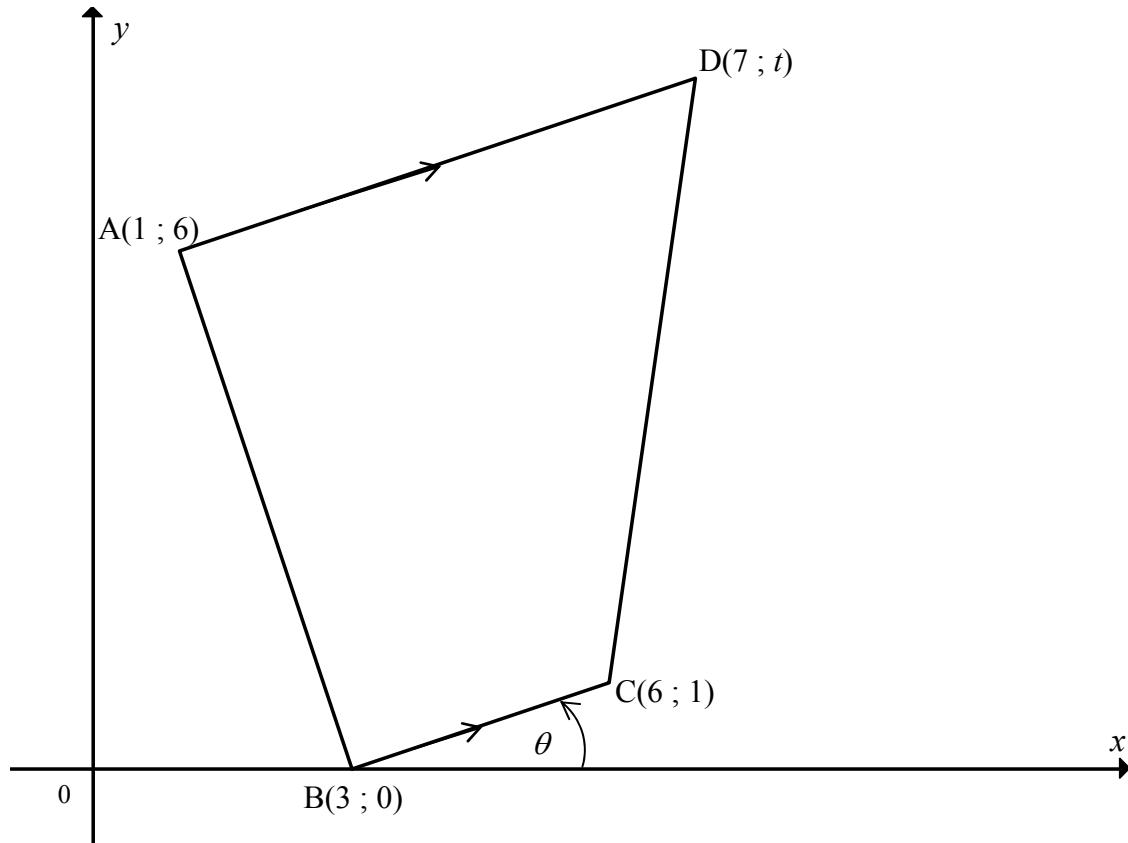
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Diagrams are NOT necessarily drawn to scale.
6. THREE diagram sheets for answering QUESTION 3.2.2, QUESTION 8.1, QUESTION 11.1, QUESTION 11.2 and QUESTION 12.1 are included at the end of this question paper. Write your examination number on these sheets in the spaces provided and hand them in together with your ANSWER BOOK.
7. Number the answers correctly according to the numbering system used in this question paper.
8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

ABCD is a quadrilateral with vertices $A(1 ; 6)$, $B(3 ; 0)$, $C(6 ; 1)$ and $D(7 ; t)$ in a Cartesian plane. $AD \parallel BC$.

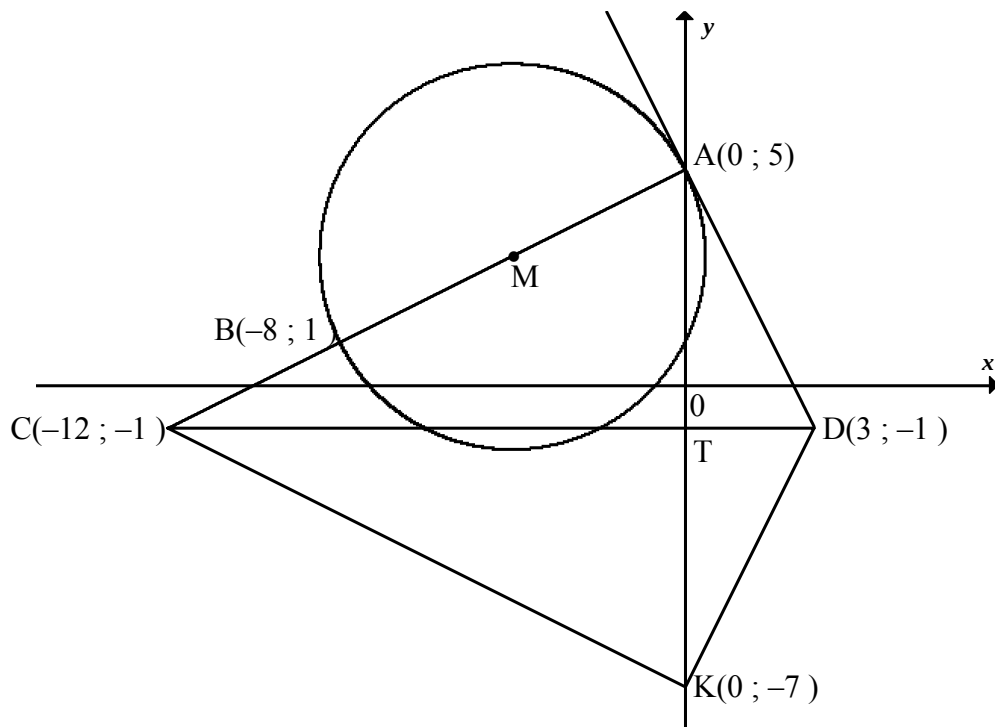


- 1.1 Calculate the gradient of BC. (2)
- 1.2 Determine the equation of AD in the form $y = \dots$ (3)
- 1.3 Show that $t = 8$. (2)
- 1.4 Calculate the lengths of AD, BC and AB. (4)
- 1.5 Show that AB is perpendicular to BC. (3)
- 1.6 Calculate the area of the quadrilateral ABCD. (Simplify your answer.) (4)
- 1.7 Determine θ , the angle of inclination of BC. (3)

[21]

QUESTION 2

$A(0 ; 5)$ and $B(-8 ; 1)$ are two points on the circumference of the circle centre M , in a Cartesian plane. M lies on AB . DA is a tangent to the circle at A . The coordinates of D are $(3 ; -1)$ and the coordinates of C are $(-12 ; -1)$. Points C and D are joined. K is the point $(0 ; -7)$. CTD is a straight line.



- 2.1 Show that the coordinates of M , the midpoint of AB , are $(-4 ; 3)$. (1)
- 2.2 Determine the equation of the tangent AD . (4)
- 2.3 Determine the length of AM . (3)
- 2.4 Determine the equation of circle centre M in the form $ax^2 + by^2 + cx + dy + e = 0$ (4)
- 2.5 Quadrilateral $ACKD$ is one of the following:
parallelogram; kite; rhombus; rectangle
Which one is it? Justify your answer. (4)

[16]

QUESTION 3

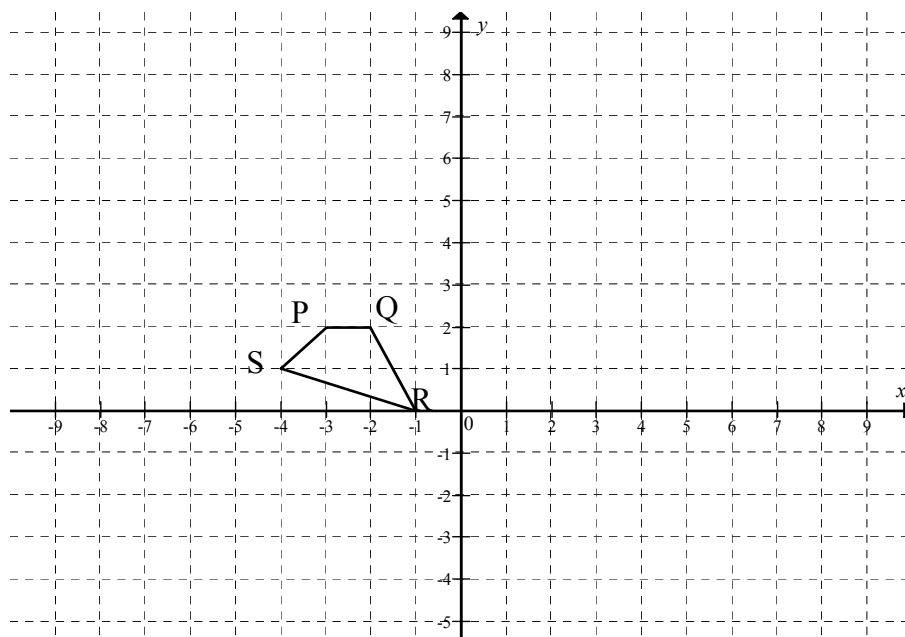
The point $P(\sqrt{3} ; -2)$ lies in a Cartesian plane.

3.1 Determine the coordinates of the image of P if:

3.1.1 P is reflected about the y -axis (2)

3.1.2 P is rotated about the origin through 180° in an anticlockwise direction (2)

3.2 The vertices of a polygon PQRS are shown in the grid below. The coordinates are $P(-3 ; 2)$, $S(-4 ; 1)$, $R(-1 ; 0)$ and $Q(-2 ; 2)$. Each of the points of PQRS in the grid below is rotated about the origin in a clockwise direction through an angle 90° .



3.2.1 Write down the coordinates of Q' , the image of Q . (2)

3.2.2 Sketch and label the vertices of the image $P'Q'R'S'$ of PQRS on the grid provided on DIAGRAM SHEET 1. (4)

3.2.3 The polygon $P'Q'R'S'$ is then enlarged through the origin by a scale factor of 2 to give the polygon $P''Q''R''S''$. Write down the coordinates of P'' the image of P' . (2)

3.2.4 State whether the transformation from PQRS to $P''Q''R''S''$ is rigid or not. Give a reason for your answer. (2)

3.2.5 Write down the general transformation of a point $(x ; y)$ in PQRS to $(x'' ; y'')$ after PQRS has undergone the above two transformations, namely rotation through 90° clockwise followed by an enlargement through the origin by a factor of 2. (3)

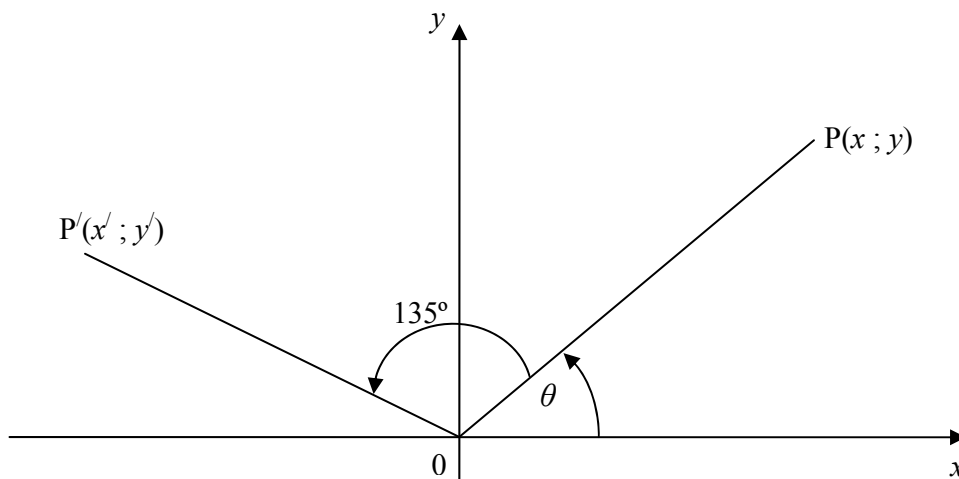
3.2.6 Calculate the ratio of area PQRS : Area $P''Q''R''S''$. (2)

[19]

QUESTION 4

- 4.1 Show that the coordinates of P' , the image of $P(x ; y)$ rotated about the origin through an angle of 135° , in the anti-clockwise direction, is given by

$$\left(-\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y, -\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}x \right). \quad (4)$$



- 4.2 M' is the image of $M(2 ; 4)$ under a rotation about the origin through 135° , in the anti-clockwise direction.

Determine the coordinates of M' , using the results in QUESTION 4.1.

(2)
[6]

QUESTION 5

Simplify each of the following to a single trigonometric ratio: (Show ALL the calculations.)

5.1
$$\frac{\tan(180^\circ + x) \cos(360^\circ - x)}{\sin(180^\circ - x) \cos(90^\circ + x) + \cos(540^\circ + x) \cos(-x)} \quad (8)$$

5.2
$$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} \quad (5)$$

[13]

QUESTION 6

6.1 If $\sin 23^\circ = p$, write down the following in terms of p . Do NOT use a calculator.

6.1.1 $\cos 113^\circ$ (2)

6.1.2 $\cos 23^\circ$ (2)

6.1.3 $\sin 46^\circ$ (2)

6.2 It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90^\circ; 270^\circ]$ and $\beta \in [90^\circ; 270^\circ]$. Determine, without using a calculator, the values of the following:

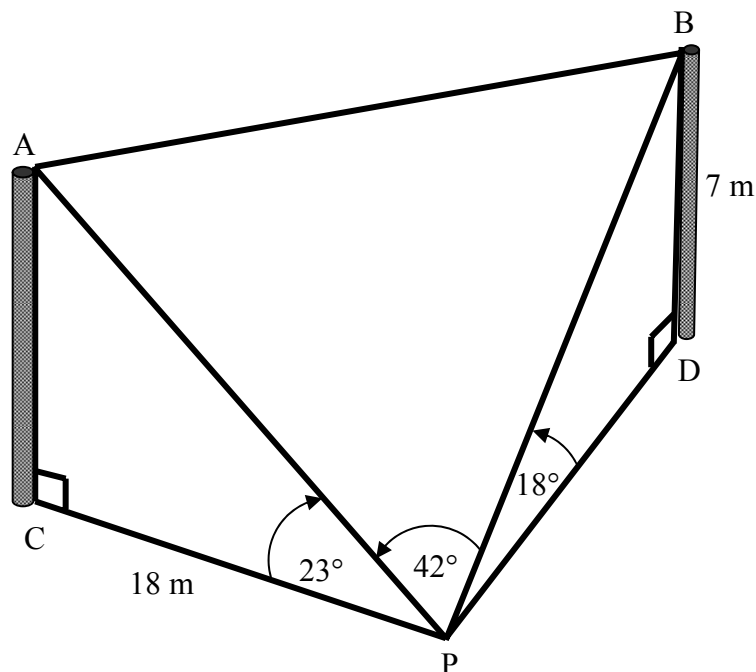
6.2.1 $\cos\alpha$ (3)

6.2.2 $\cos(\alpha + \beta)$ (5)

6.3 Solve for $x \in [0^\circ; 360^\circ]$ if $\frac{1}{2}\cos x = 0,435$. (3)
[17]

QUESTION 7

Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.



Calculate, correct to TWO decimal places:

- 7.1 The distance from Thandi to the top of pole BD (2)
 - 7.2 The distance from Thandi to the top of pole AC (2)
 - 7.3 The distance between the tops of the poles, that is the length of AB, if $\hat{APB} = 42^\circ$ (4)
- [8]**

QUESTION 8

Consider the functions defined by $f(x) = \sin 2x$ and $g(x) = \frac{1}{2} \tan x$ for $x \in [-90^\circ; 180^\circ]$.

- 8.1 Sketch the graphs of f and g on the same system of axes on DIAGRAM SHEET 1. (6)
 - 8.2 Calculate the x -coordinates of the points of intersection of f and g . (10)
 - 8.3 Determine the values of x for which $g(x) > f(x)$. (3)
- [19]**

QUESTION 9

Determine the minimum and maximum values of the following:

$$f(x) = \frac{1}{3 \sin^2 x + 4 \cos^2 x} \quad [4]$$

QUESTION 10

The data below shows the energy levels, in kilocalories per 100 g, of 10 different snack foods.

440 520 480 560 615 550 620 680 545 490

- 10.1 Calculate the mean energy level of these snack foods. (2)
- 10.2 Calculate the standard deviation. (4)
- 10.3 The energy levels, in kilocalories per 100 g, of 10 different breakfast cereals had a mean of 545,7 kilocalories and a standard deviation of 28 kilocalories. Which of the two types of food show greater variation in energy levels? What do you conclude? (2)
- [8]**

QUESTION 11

The heights, h , of the learners at Nkosi High School in Grades 10, 11 and 12 were recorded as follows:

HEIGHT (IN CM)	FREQUENCY
$118 \leq h < 127$	16
$127 \leq h < 136$	26
$136 \leq h < 145$	42
$145 \leq h < 154$	54
$154 \leq h < 163$	26
$163 \leq h < 172$	22
$172 \leq h < 181$	14

- 11.1 Set up a cumulative frequency table for the data on DIAGRAM SHEET 2. (2)
- 11.2 Draw an ogive for the data on the grid provided on DIAGRAM SHEET 2. (3)
- 11.3 Use the ogive, or otherwise, to determine the lower quartile, median and upper quartile. (3)
- 11.4 If the minimum height was 119 cm and the maximum height was 178 cm, draw a box and whisker diagram for the data. (3)
- 11.5 Comment on the distribution of the heights of the learners. (1)
- 11.6 Approximately how many learners are between 138 cm and 158 cm tall? (1)

[13]**QUESTION 12**

A motor company did research on how the speed of a car affects the fuel consumption of the vehicle. The following data was obtained:

Speed in km/h	60	75	115	85	110	95	120	100	70
Fuel consumption in $\ell/100$ km	11,5	10	8,4	9,2	7,8	8,9	8,8	8,6	10,2

- 12.1 Represent the data as a scatter plot on DIAGRAM SHEET 3. (3)
- 12.2 Suggest whether a linear, quadratic or exponential function would best fit the data. (1)
- 12.3 What advice can the company give about the driving speed in order to keep the cost of fuel to a minimum? (2)

[6]**TOTAL: 150**

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$$

$$\sum_{i=1}^n ar^{i-1} = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} ; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

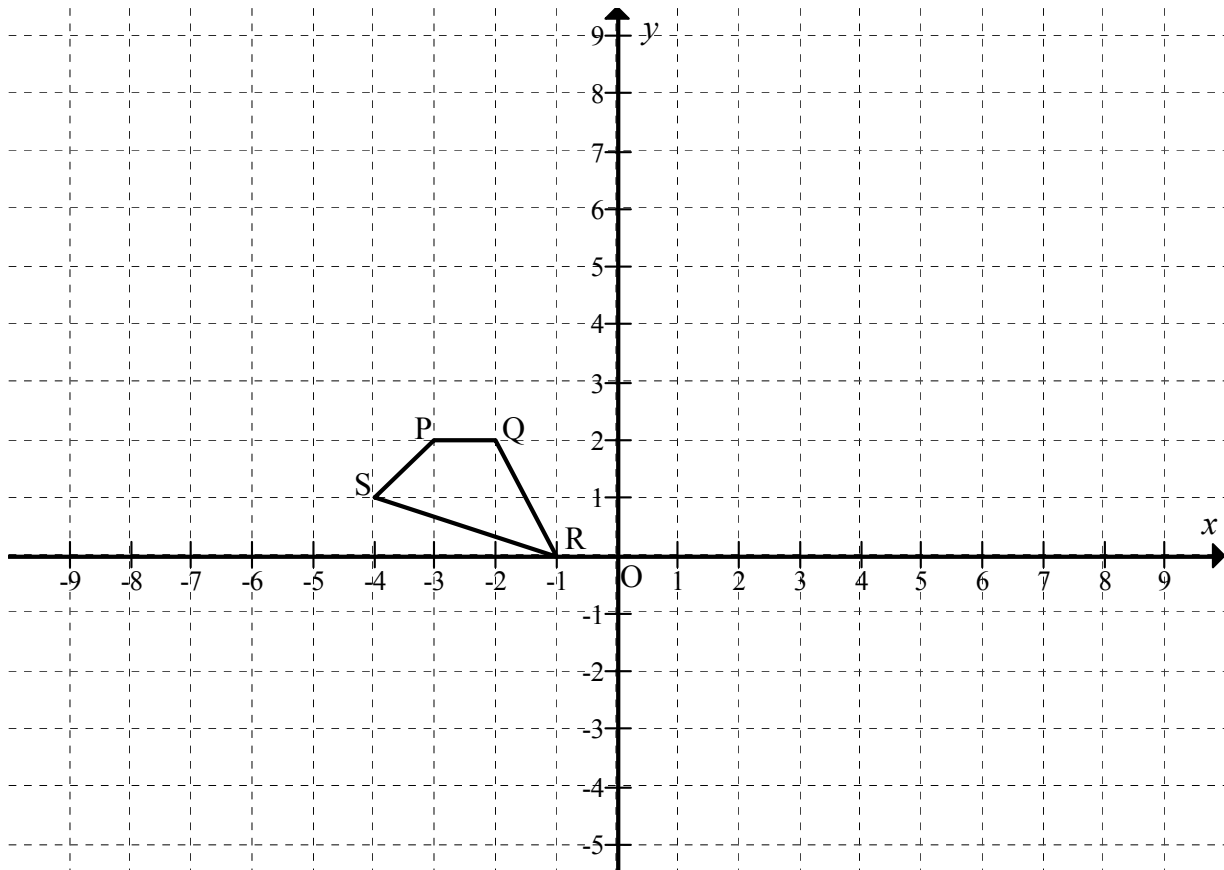
$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

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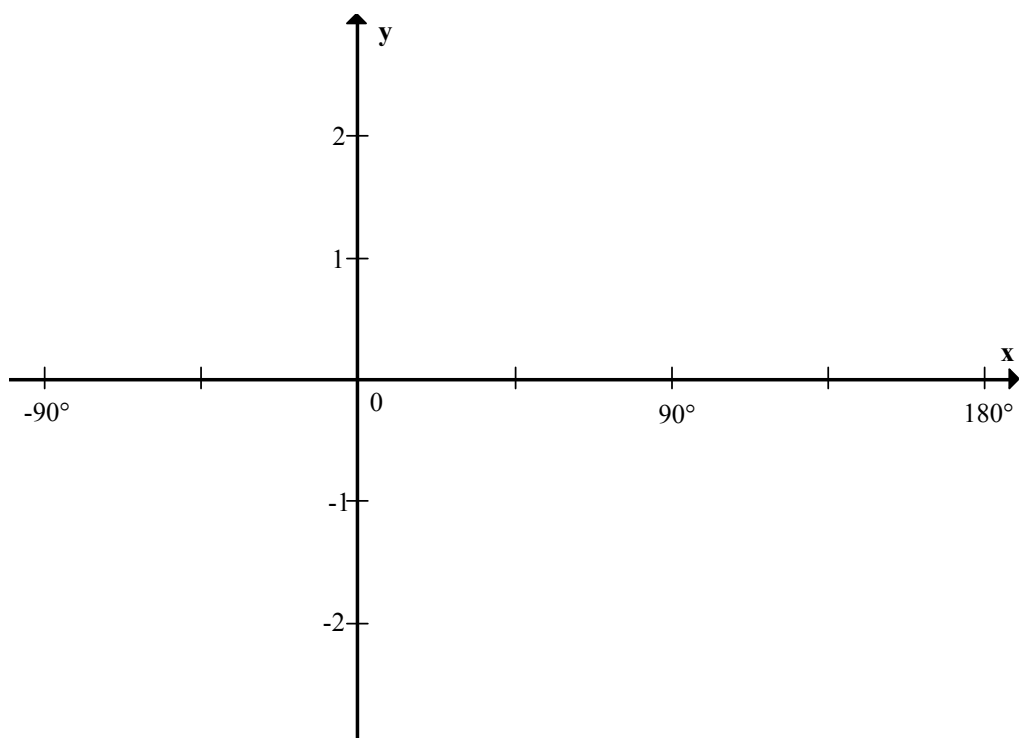
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DIAGRAM SHEET 1

QUESTION 3.2.2



QUESTION 8.1



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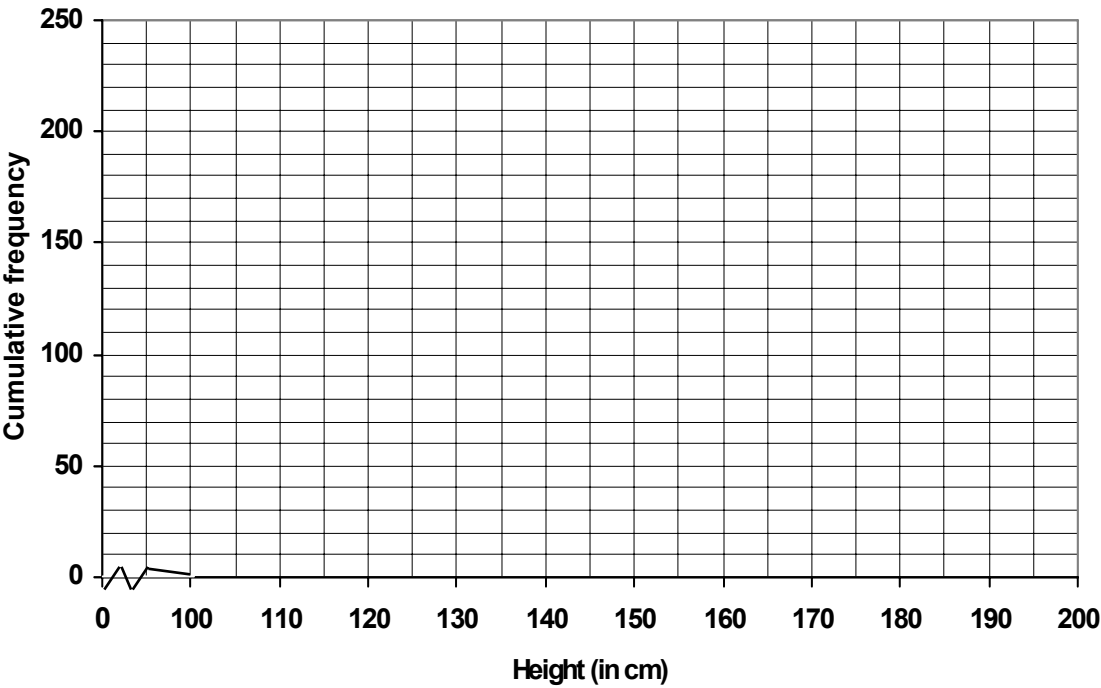
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DIAGRAM SHEET 2

QUESTION 11.1

HEIGHT (IN CM)	FREQUENCY	CUMULATIVE FREQUENCY
$118 \leq h < 127$		
$127 \leq h < 136$		
$136 \leq h < 145$		
$145 \leq h < 154$		
$154 \leq h < 163$		
$163 \leq h < 172$		
$172 \leq h < 181$		

QUESTION 11.2



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DIAGRAM SHEET 3

QUESTION 12.1

