# Index/Indeks


**Paper 1 / Vraestel 1 ..... 1**

Products & Factorisation / Produkte & Faktorisering ..... 2
Indices & surds / Eksponente & wortelvorme ..... 4
Quadratic equations / Kwadratiese vergelykings ..... 6
Simultaneous equations / Gelyktydige vergelykings ..... 8
Inequalities / Ongelykhede ..... 9
Patterns & sequences Pattern / Getalpatrone & rye ..... 11
Annuities / Annuïteite ..... 22
Straight line & parabola / Reguitlyn & parabool ..... 24
Exponential graph / Eksponensiële grafiek ..... 26
Properties of graphs / Eienskappe van grafieke ..... 27
Differential Calculus / Differensiaalrekeninge ..... 31
Linear Programming / Lineêre Programmering ..... 34

**Paper 2/ Vraestel 2 ..... 36**

Volume and surface area / Volume en buiteoppervlakte ..... 37
Analytical geometry / Analitiese meetkunde ..... 39
Trigonometry / Trigonometrie ..... 42
Compound & double angles / Saamgestelde- en dubbelhoeke ..... 43
Solving triangles / Oplos van driehoeke ..... 46
Transformations / Transformasies ..... 48
Data handling / Data Hantering ..... 52



## Compiled by / Saamgestel deur: Dr Gerrit Stols

Telephone & Fax (012) 653 5758 • Cell 082 415 7583 • gerrit.stols@up.ac.za

# Paper 1 / Vraestel 1

Patterns and sequences / Getalpatrone

Annuities and finance / Annuïteite en finansies

Functions and graphs / Funksies en grafieke

Algebra and equations / Algebra en vergelykings

Calculus / Differensiaalrekene

Linear programming / Lineêre programmering

# Products & Factorisation / Produkte & Faktorisering

## Products / Produkte

**General cases / Spesiale gevalle**

- $a(b + c) = ab + ac$
- $(a + b)(c + d) = ac + ad + bc + bd$
- $(a + b)(d + e + f) = ad + ae + af + bd + be + bf$

**Special cases / Spesiale gevalle**

- Difference of squares / Verskil van vierkante: $(2a^2 + 3b)(2a^2 - 3b) = 4a^4 - 9b^2$
- Square / Vierkant: $(a + 3b)^2 = a^2 + 6ab + 9b^2$

## Factorisation / Faktorisering

**Step 1: Find the common factor / Haal gemeenskaplike faktor uit:**

**Step 2: Count the number of terms / Tel die aantal terme**

**2 Terms / Terme**

- Different of squares: $x^2 - y^2 = (x - y)(x + y)$
- Sum of cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- Difference of cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

**3 Terms / Terme:** $6x^2 - 19x + 10 = (2x - 5)(3x - 2)$

**4 Terms / Terme:** Grouping / Groepeer 2-2 terms: $x^2 - xp - xq + pq = x(x - p) - q(x - p) = (x - p)(x - q)$

Use factor theorem / Gebruik die faktorstelling

## Worksheet 1

1. Simplify the expressions/Vereenvoudig die uitdrukkings:

   a) $-2x-(x-2x^2)$

   b) $(x-2x^2)-2x$

   c) $(x-2x^2)(-2x)$

   d) $-2x(x-2x^2)$

   e) $-2x+(x-2x^2)$

   f) $(-2x)(x-2x^2)$

   g) $(2x-y)^3$

   h) $(x+2)(x^2-2x+4)$

   i) $(4x^2+6x+9)(2x-3)$

2. Factorise/Faktoriseer:

   a) $x^2+x-12$

   b) $x^2-x-12$

   c) $x^2+x+12$

   d) $x^2-x+12$

   e) $x^2-7x+12$

   f) $x^2+7x+12$

   g) $(x-3)(x-2)-12$

   h) $7x^3-56$

   i) $18-8(x-y)^2$

   j) $(2x-y)^2+4(y-2x)$

   k) $2x^2+4y-8y^2-2x$

   l) $x^3-3x^2-16x+48$

   m) $x^3-4x^2-11x+30$

   n) $x^3-3x^2+4$

3. Simplify the expressions/Vereenvoudig die uitdrukkings:

   a) $\dfrac{2x}{5x^2-25x}+\dfrac{x+2}{25-x^2}$

   b) $\dfrac{a-b}{a+b}\div\dfrac{a^{2^2}-2ab+b^2}{a^3+b^3}$

   c) $\dfrac{x}{x^2-4}-\dfrac{2x+1}{x^2-2x}$

   d) $\dfrac{2x^2}{x^2-1}-\dfrac{x^3-1}{x-1}$

# Indices & surds / Eksponente & wortelvorme

**Definition:** $\sqrt{a} = b \Leftrightarrow b^2 = a$ and $a \geq 0$ & $b \geq 0$

$\therefore \sqrt{a} = a^{\frac{1}{2}}$ ; $a^{\frac{1}{n}} = \sqrt[n]{a}$ ; $x^{\frac{a}{even\ number}} < 0$

| Exponential laws<br>Eksponentwette | Definitions<br>Definisies | Surds<br>Wortelvorme |
|---|---|---|
| $a; b \in R$ and $n; m \in No$ | $a^0 = 1$ | $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$ |
| $a^k.a^s = a^{k+s}$ | $1^n = 1$ | $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ |
| $\frac{a^k}{a^s} = a^{k-s}$ | $a^{-n} = \frac{1}{a^n}$ | $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ |
| $(a^m)^n = a^{mn}$ | | |
| $a^k.b^k = (a.b)^k$ | | |

## Worksheet 2

1. Are the following true or false? / Is die volgende waar of vals?

   a) $2^4.2^5 = 4^{4+5} = 4^9$

   b) $2^2 + 2^3 = 2^5$

   c) $(-2)^2 = -2^2$

2. Simplify without using a calculator / Vereenvoudig sonder om 'n sakrekenaar te gebruik:

   a) $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

   b) $\frac{2^{x-3} - 2^x}{2^{x+1}}$

   c) $\sqrt{9^3} + 8^{-\frac{2}{3}}$

   d) $\frac{2.5^{x+1} + 3.5^{x+2}}{4.5^{x+1} - 3.5^x}$

   e) $\frac{8^{n-3}.32^{-n+1}.6^{2n}}{9^n}$

   f) $\frac{2^{x+1}.8^{x-1}}{4^{2(x-2)}}$

3. Simplify without using a calculator / Vereenvoudig sonder om 'n sakrekenaar te gebruik:

   a) $\frac{\sqrt{8} + \sqrt{8}}{\sqrt{2}.\sqrt{8}}$

   b) $\sqrt{32} - \sqrt{288} + \sqrt{800}$

   c) $\sqrt{2}.\sqrt{8}$

   d) $\frac{\sqrt{18} - \sqrt{32}}{\sqrt{8}}$

   e) $\sqrt[3]{4}.\sqrt[3]{16}$

   f) $\frac{\sqrt{27}.\sqrt{18}.\sqrt{32}}{\sqrt{12}.\sqrt{8}}$

   g) $\sqrt{3}\left(\sqrt{48} - 3\sqrt{75} + 2\sqrt{108}\right)$

4. Rationalise the denominator / Rasionaliseer die noemer:

   a) $\frac{2}{\sqrt{2}}$

   b) $\frac{5}{-2\sqrt{3}}$

# Quadratic equations / Kwadratiese vergelykings

**Methods to find the roots / Metodes om die wortels te bepaal:**

WRITE IN STANDARD FORM & Get rid of fractions: × LCM (denominators ≠ 0)

- **Factorising / Faktorisering**
  i. One side = 0 / Een kant = 0
  ii. Factorise / Faktoriseer
  iii. Let (…) = 0 or (…) = 0 / Stel (…) = 0 of (…) = 0
- **Formula / Formule:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **Completing the square / Vierkantsvoltooiing**
  i. Divide by the coefficient of $x^2$ / Deel deur koëffisiënt van $x^2$
  ii. Take constant term to right-hand side / Konstante term na regterkant
  iii. Add $\left(\dfrac{\text{Coef. of } x}{1} \times \dfrac{1}{2}\right)^2$ to both sides / Tel albei kante by $\left(\dfrac{\text{Koëf. van } x}{1} \times \dfrac{1}{2}\right)^2$
  iv. LHS: Factorise & RHS: LCM / LK: Faktoriseer & RK: KGV
  v. Extract $\pm\sqrt{\ }$ / Trek $\pm\sqrt{\ }$
  vi. Get $x$ alone on a side and LCM / LK: Kry $x$ alleen & RK: KGV

## Worksheet 3

1. Solve for $x$ / Los op vir $x$:

   a) $2x^2 + x - 6 = 0$

   b) $(2x - 3)(x - 1) = 15$

   c) $\frac{4}{x+3} = \frac{x-3}{4}$

   d) $(x + 3)(x - 5) = 9$

   e) $5x^2 - 80 = 0$

   f) $x - 18 - 6x(x - 5) = 0$

   g) $1 + \frac{2}{x+4} = 3 - \frac{x-2}{x+4}$

   h) $\frac{3x-2}{2x-3} + 2 = \frac{5x}{x+4}$

   i) $\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = 1$

2. Solve for $x$ by completing the square / Los op vir $x$ d.m.v. vierkantsvoltooiing:

   a) $2x^2 + 14x + 24 = 0$

   b) $2x^2 + x - 6 = 0$

   c) $ax^2 + bx + c = 0$

   d) $mx^2 + x^2 - x + m = -1$

3. Solve for $x$, using the formula / Los op vir $x$ deur die formule te gebruik.

   a) $3 - x^2 = 2x^2 + 1$

   b) $x - \frac{4}{x} + 2 = 0$

4. Factorise / Faktoriseer:

   a) $x^3 - 3x^2 + 4x - 12$

   b) $2x^3 + x^2 - 13x + 6$

   c) $x^3 + 6x^2 - 9x - 14$

   d) $x^3 + 3x^2 - 3x - 9$

5. Solve for $x$ / Los op vir $x$:

   a) $4x^5 - 3x^4 - 4x + 3 = 0$

   b) $-x^3 + 3x = 2$

   c) $-x^3 + 5x^2 + 8x = 12$

6. Solve for $x$ if/ Los op vir $x$ as: $(2x + 3)(x^2 - 5) = 0$ and/en

   a) $x \in Z$

   b) $x \in Q$

7. Given/Gegee: $\sqrt{-1} = i$

   Write the roots of $x^2 + 2x + 5 = 0$ in terms of $i$.

   Skryf die wortels van $x^2 + 2x + 5 = 0$ in terme van $i$.

# Simultaneous equations / Gelyktydige vergelykings

***Methods / Metodes:***

- **Elimination / Eliminasie**
  Use this method if there is no term with xy. / Gebruik hierdie metode indien daar geen term met xy is nie.
  i. Write similar terms underneath each other / Skryf gelyksoortige terme onder mekaar neer
  ii. Get coefficients of x or y to be the same by × / Kry koëffisiënte van x of y dieselfde
  iii. Eliminate one variable / Elimineer een onbekende
  iv. Solve equation / Los op
  v. Substitute back and determine other variable / Vervang terug en bepaal ander onbekende

- **Equate equations / Stel vergelykings gelyk**
  Use this method if you can get y alone on a side of both equations.
  Gebruik hierdie metode indien y alleen aan 'n kant van albei vergelykings gekry kan word.

- **Substitution / Substitusie**
  If you can't use method 1 or 2, this method always works. / Hierdie metode sal altyd werk, indien metode 1 of 2 nie gebruik kan word nie.
  i. Look for simplest equation / Soek eenvoudigste vergelyking
  ii. Get x or y alone (preferably not fraction) / Kry $x$ of $y$ alleen (verkieslik nie breuk nie)
  iii. Substitute in other equation and determine $x$ or $y$ / Vervang in ander vgl. en bepaal $x$ of $y$.
  iv. Substitute answer into any of the original equations to determine the other variable / Vervang antwoord om ander onbekende te bepaal.

# Inequalities / Ongelykhede

Step 1: One side = 0 / Een kant = 0

Step 2: Factorise / Faktoriseer

Step 3: Calculate roots / Bereken wortels: Let (…) = 0 or (…) = 0 / Stel (…) = 0 of (…) = 0

Step 4: Make a rough sketch of parabola / Maak rowwe skets van die parabool

Step 5: If (***)(***) < 0 read off the $x$ values of the graph under the $x$-axis
As (***)(***) < 0 lees dan die waardes van die grafiek af onder die $x$-as.

If (***)(***) > 0 read off the $x$ values of the graph above the $x$-axis
As (***)(***) > 0 lees dan die waardes van die grafiek af bo die $x$-as.

**Example: Solve for $x$ / Los op vir $x$: $x^2 - x \geq 6$**

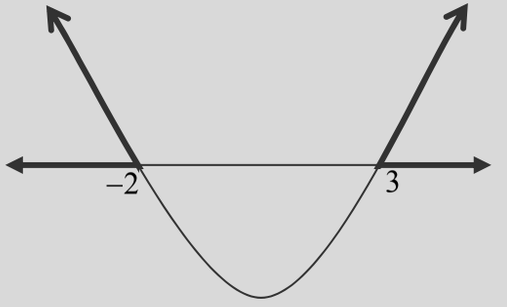
Step 1: $x^2 - x - 6 \geq 0$

Step 2: $(x - 3)(x + 2) \geq 0$

Step 3: $x$-interscepts / $x$-afsnitte: 3 and $-2$

Step 4: See sketch / Sien skets

Step 5: Solution / Oplossing: $x \leq -2$ or / of $x \geq 3$



## Worksheet 4

1. Solve for $x$ / Los op vir $x$:
   a) $2^x = 32$
   b $2^x = 5$
   c) $5^x - 100 = 0$
   d) $3.2^{x+1} - 18 = 0$
   e) $3.4^x - 8\sqrt{18} = 0$
   f) $3^{2x+1} = 12$
   g) $x^{\frac{1}{2}} = 4$
   h) $5.2^{x+1} - 8 = 32$
   i) $2.2^x - 64 = 0$
2. If $A = P(1+i)^n$, make $n$ the subject of the formula / Maak $n$ die onderwerp van die formule.
3. If $F = \dfrac{x[(1+i)^n - 1]}{i}$, make $n$ the subject of the formula / Maak $n$ die onderwerp van die formule.
4. Determine the points of intersection of the curves of: /
   Bepaal die snypunte van die krommes van:
   $y + 2 = 2x$ and/en $y = x^2 - 4x + 3$
5. Solve simultaneously for $x$ and y / Los gelyktydig vir $x$ en y op:
   a) $2x + 3y = 2$ and/en $x - 4y = 12$
   b) $3y - x - 4 = 0$ and/en $x^2 - xy + y^2 = 3$
   c) $y + 2x = 2$ and/en $y^2 + 2x^2 = 3xy$
   d) $y^2 + 2x^2 - 3xy = 0$ and/en $y + 2x = 2$
6. Solve for $x$ / Los op vir $x$:
   a) $1 - 2x > 0$
   b) $(x + 1)(x - 2) > 0$
   c) $(2x - 3)^2 < 4$
   d) $x^2 - x - 6 \geq 0$
   e) $x^2 < x + 12$
   f) $x^2 - 2x \leq 8$
   g) $x^2 < 16$
   h) $x^2 \geq 9$

# Patterns & sequences Pattern / Getalpatrone & rye

**Abbreviations / Afkortings**

a first term of the sequence / eerste term van die ry ($a = T_1$)

d constant difference / konstante verskil (RR): $d = T_2 - T_1$ or $d = T_{30} - T_{29}$

r constant ratio (GS) / konstante verhouding (MR): $r = \frac{T_2}{T_1}$ or $r = \frac{T_{45}}{T_{44}}$

n the number of terms / die aantal terme

$T_n$ the value of the nth term of the sequence / die waarde van die n de term van die ry

$S_n$ the sum of the first n terms / die som van die eerste n terme

## Patterns

- **Exponential:** 1; 2; 4; 8; … = $2^0$; $2^1$; $2^2$; $2^3$; … or $3^0$; $3^1$; $3^2$; $3^3$; …
- **Arithmetic:** If 1st difference is constant $\Rightarrow T_2 - T_1 = T_3 - T_2$
- **Geometric:** If the ratio is constant = $\frac{T_2}{T_1} = \frac{T_3}{T_2}$
- **Recursive sequences**
  - 1; 1; 2; 3; 5; 8; . $\Rightarrow$ Fibonacci sequence: $T_{n+1} = T_n + T_{n-1}$
  - 1; 9; 33; 105; 321; … $\Rightarrow T_n = 3(T_{n-1} + 2)$
- **Quadratic:** If 2nd difference is constant: $T_n = p.n^2 + q.n + r$
- **Combination**: $\frac{3}{1}$; $\frac{5}{2}$; $\frac{7}{3}$; $\frac{9}{4}$; … $\Rightarrow T_n = \frac{2n+1}{n}$

**Arithmetic sequences / Rekenkundige rye:**

2; 5; 8; … of –3; –7; –11; …

2) TEST / TOETS: $T_2 - T_1 = T_3 - T_2$

3) $T_n = a + (n-1)d$

4) $S_n = \frac{n}{2}[2a + (n-1)d]$

5) $S_n = \frac{n}{2}[a + l]$

**Geometric sequences / Meetkundige rye:**

2; 1; $\frac{1}{2}$; … of 2; –4; 8; …

1) TEST / TOETS: $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

1) $T_n = ar^{n-1}$

2) $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$

3) $S_\infty = \frac{a}{1-r}$ as $|r| < 1$

## Worksheet 5

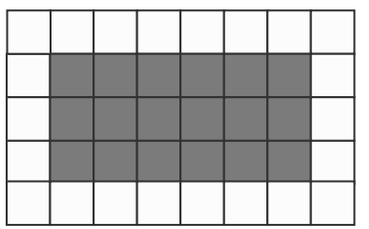
1. Determine the unit's digit (i.e. last digit of the number) of the product of the first 200 prime numbers.

2. A rubber ball is bounced from a height of 90 m and after each successive bounce loosed 30% of the previous height.
   a) What maximum height will the ball reach during the 5th bounce?
   b) After how many rebounces will the ball reach a height of less than 3 metres?
   c) Calculate the total distance travelled by the ball after it came to rest.

3. The following number pattern was given 1, 5, 11, 19, …
   a) Determine the 5th number in the pattern.
   b) Derive a formula for the nth number in the pattern.
   c) What is the 50th number in the pattern?

4. A swimming pool company builds rectangular pools and uses square paving blocks around it.
   a) How many square paving blocks are needed for an 8 × 5 pool?
   b) How many square paving blocks are needed for a $m \times n$ pool?
   c) How many square paving blocks are needed for a $m \times m$ pool?
   d) What is the largest pool that can be built with 200 square paving blocks?

6 by 3 pool



1. Bepaal die ene-syfer van die produk van die eerste 200 priemgetalle.

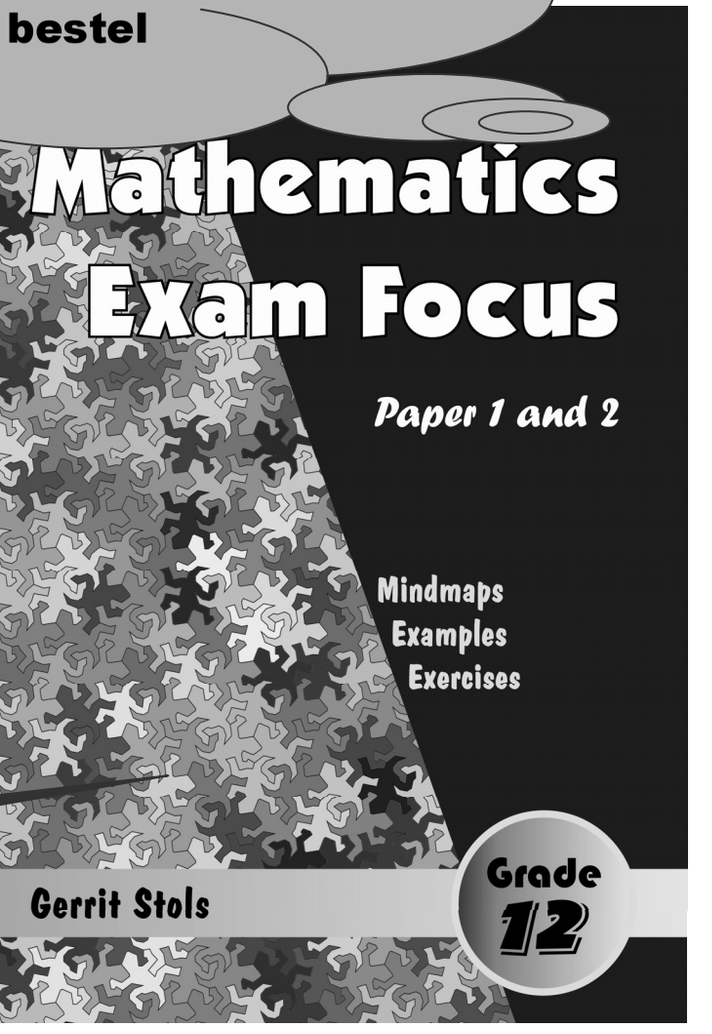
2. 'n Rubberbal wat van 'n hoogte van 90 m laat val word, verloor met elke terugsprong 30% van sy vorige hoogte.
   a) Wat is die maksimum hoogte wat die bal tydens die 5de terugsprong bereik?
   b) Na hoeveel spronge bereik die bal 'n hoogte van minder as 3 meter?
   c) Bereken die totale afstand wat die bal aflê het nadat dit tot rus gekom het.

3. Die volgende getal patroon word gegee: 1, 5, 11, 19, …
   a) Bepaal die 5de getal in die patroon.
   b) Lei 'n formule af vir die n-de getal in die patroon.
   c) Wat is die 50ste getal in die patroon?

4. 'n Swembad maatskappy bou reghoekige swembaddens en gebruik vierkantige blokke om die swembad as plaveisel.
   a) Hoeveel vierkantige plaveisel blokke word benodig vir 'n 8 × 5 swembad?
   b) Hoeveel vierkantige plaveisel blokke word benodig vir 'n $m \times n$ swembad?
   c) Hoeveel vierkantige plaveisel blokke word benodig vir 'n $m \times m$ swembad?
   d) Wat is die grootste swembad wat met 200 vierkantige plaveisel blokke gebou kan word?

**For more detailed summaries and examples order Mathematics Exam Focus!**

**Vir beter opsommings en voorbeelde bestel Wiskunde Eksamen Fokus!**




**This book will provide you with excellent summaries (mindmaps), examples and exercises. Available in English and Afrikaans.**

- Mathematics Exam Focus: Grade 12 mndmaps, examples and exercises. Paper 1 and 2 (ISBN: 978-0-620-41111-0). 156 pages.
- Wiskunde Eksamenfokus Graad 12 kopkaarte, voorbeelde en oefeninge. Vraestel 1 en 2. (ISBN 978-1-77021-729-4). 156 bladsye.

**For further queries please contact the Clever Books office at**

- **Phone: (012) 342 3263**
- **Fax: (012) 430 2376**
- **e-mail: heloise@cleverbooks.co.za**
- **webpage: www.cleverbooks.co.za**

Clever Books

The next 6 pages were copied from this book.
Die volgende 6 bladsye kom uit hierdie boek.

# Patterns & Sequences

**Abbreviations**
$a = T_1$: First term of the sequence
$n$: the number of terms
$T_n$: the value of the $n^{th}$ term

## Pattern recognition guidelines

1) Look at particular cases (draw picture if possible)

2) Draw and complete a table, e.g.

| Number of squares | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Number of matches | 4 | 7 | 10 | 13 |

3) Identify a pattern in the table (what must you add), e.g. matches: 4; 4 + 1(3); 4 + 2(3); ….
4) Try to find the 100th term
   E.g. $T_{100}$ = 4 + 99(3)
5) Us the same method to write the $n^{th}$ term (generalise), e.g. $T_n$ = 4 + (n-1)(3)
6) That is your conjecture (or statement about your finding)
7) Test your conjecture with values in table
8) Justify (or prove) your conjecture (when asked) - focus on the structure of the sketches.

## Different patterns

- **Exponential**
  - 1; 2; 4; 8; … = $2^0; 2^1; 2^2; 2^3; \ldots \Rightarrow T_n = 2^{n-1}$
  - $3^0; 3^1; 3^2; 3^3; \ldots \Rightarrow T_n = 3^{n-1}$
- **Arithmetic:** If 1st difference is constant $\Rightarrow T_2 - T_1 = \ldots$
- **Geometric:** If the ratio is constant = $\frac{T_2}{T_1} = \frac{T_3}{T_2}$
- **Quadratic:** If 2nd difference is constant: $T_n = a.n^2 + b.n + c$
- **Combination**: $\frac{3}{1}; \frac{5}{2}; \frac{7}{3}; \frac{9}{4}; \ldots \Rightarrow T_n = \frac{2n+1}{n}$
- **Alternate signs** $\Rightarrow$ add $(-1)^n$ or $(-1)^{n-1}$
  e.g. –1; 2; –4; 8; –16; … $\Rightarrow T_n = (-1)^n.2^{n-1}$

## Copied from Mathematics Exam Focus

**Arithmetic:** 2; 5; 8; … or –3; –7; –11; …

$d \Rightarrow$ constant difference: $d = T_2 - T_1$

$T_n = a + (n-1)d$

$\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}[2a + (n-1)d]$

**Geometric:** 2; 1; $\frac{1}{2}$; … or 2; –4; 8; …

$r \Rightarrow$ constant ratio: $r = \frac{T_2}{T_1} = \frac{T_{45}}{T_{44}}$

$T_n = ar^{n-1}$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r - 1}; r \neq 1$

$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}; -1 < r < 1$

- a = ½ (constant difference)
- Substitute 1st & 2nd term and solve simultaneously for q and r

**Σ - notation:** Indicates the sum of the terms

Number of terms = (Above Σ sign) – (Below Σ sign) + 1
= (40 – 5) + 1 = 36 terms

e.g. $\sum_{5}^{40}(7-3r) = (-8) + (-11) + (-14) + \ldots + (-113)$

(r = 5; r = 6; r = 7; r = 40)

## Formulae

$\sum_{i=1}^{n} 1 = n$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

# Getalpatrone en rye

**Afkortings**
$a = T_1$: Eerste term van die ry
$n$: Aantal terme
$T_n$ die waarde van die n$^{de}$ term

**Copied from Mathematics Exam Focus**

## Patroonherkenningsriglyne

1) Kyk na spesifieke gevalle (teken prent indien moontlik)

2) Teken en voltooi 'n tabel, byvoorbeeld:

| Getal vierkante | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Getal vuurhoutjies | 4 | 7 | 10 | 13 |

3) Identifiseer die patroon in die tabel :
Bv. 4; 4 + 1(3); 4 + 2(3); ….
4) Bepaal die 100$^{ste}$ term:
Bv. $T_{100}$ = 4 + 99(3)
5) Veralgemeen: Gebruik dieselfde metode om die n$^{de}$ term te bepaal: $T_n$ = 4 + (n-1)(3)
6) Maak 'n afleiding of bewering oor jou bevinding
7) Toets jou bewering met waardes in tabel
8) Regverdig of bewys jou bewering (indien gevra) – fokus op die struktuur van die sketse.

## Verskillende patrone

- **Eksponensieel**
  - $1; 2; 4; 8; \ldots = 2^0; 2^1; 2^2; 2^3; \ldots \Rightarrow T_n = 2^{n-1}$
  - $3^0; 3^1; 3^2; 3^3; \ldots \Rightarrow T_n = 3^{n-1}$
- **Rekenkundig:** As 1$^{ste}$ verskil konstant is $\Rightarrow T_2 - T_1 = T_3 - T_2$
- **Meetkundig:** As die verhouding konstant is $\frac{T_2}{T_1} = \frac{T_3}{T_2}$
- **Kwadraties:** As 2$^{de}$ verskil konstant is: $T_n = a.n^2 + b.n + c$
- **Kombinasie**: $\frac{3}{1}; \frac{5}{2}; \frac{7}{3}; \frac{9}{4}; \ldots \Rightarrow T_n = \frac{2n+1}{n}$
- **Tekens wat afwissel** $\Rightarrow$ voeg by $(-1)^n$ of $(-1)^{n-1}$
  Bv. $-1; 2; -4; 8; -16; \ldots \Rightarrow T_n = (-1)^n.2^{n-1}$

**Rekenkundig:** 2; 5; 8; . of –3; –7; –11; .
$d \Rightarrow$ Konstante verskil: $d = T_2 - T_1$

$$T_n = a + (n-1)d$$

$$\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}\left[2a + (n-1)d\right]$$

**Meetkundig:** 2; 1; $\frac{1}{2}$; . of 2; –4; 8; ...

$r \Rightarrow$ konstante verhouding: $r = \frac{T_2}{T_1} = \frac{T_{45}}{T_{44}}$

$$T_n = ar^{n-1}$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r}; -1 < r < 1$$

- a = ½ (konstante verskil)
- Vervang 1$^{ste}$; 2$^{de}$ terme in en los gelyktydig op.

**Σ-notasie:**

**Dui die som van die terme aan.**

Aantal terme = (Boonste – Onderste) + 1 = (40 – 5) + 1 = 36

Vb. $\sum_{5}^{40}(7 - 3r) = (-8) + (-11) + (-14) + \ldots + (-113)$

r = 5, r = 6, r = 7, r = 40

## Formules

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

## Example 1

**Find the next three terms and general expression of each sequence:**
**a) 1; 2; 4; 8; ...**
**b) –3; –7; –11; …**
**c) 2; –4; 8; …**
**d) 1; 4; 9; 16; …**
**e) 0; 3; 8; 15; …**

**Finding patterns**

a) …; 16; 32; 64 $T_n = 2^{n-1}$ ❶ Geometric: multiply by 2

b) …; –15; –19; –23 $T_n = 1 - 4n$ ❶ Arithmetic: add –4

c) …; –16; 32; –64 $T_n = -(-2)^n$ ❶ Geometric: multiply by –2

d) …; 25; 36; 49 $T_n = n^2$ ❶ $1^2; 2^2; 3^2; 4^2; 5^2;$

e) …; 24; 35; 48; 63; $T_n = n^2 - 1$ ❶ Quadratic: $T_n = a.n^2 + b.n + c$

9 11 13

2 2

2nd difference is constant:

$a = ½ \times (2) = 1$

## Example 2

**Calculate** $\sum_{k=3}^{30}(3k+4)$.

**Number of terms = (Above ∑ sign) – (Below ∑ sign) + 1 = 28**

$\sum_{k=3}^{30}(3k+4) = 13 + 16 + 19 + \ldots$ ❶ Substitute k = 3; 4; 5; … into 3k + 4

Arithmetic series ❷ $T_2 - T_1 = T_3 - T_2$ (= d)

$a = 13$ & $d = 3$ ❸ 1st term = a; constant difference = d

$n = 30 - 3 + 1 = 28$ ❹ Terms = (Above – Below) + 1

$\sum_{i=1}^{28} a + (i-1)d = \frac{28}{2}[2(13) + (27)(3)]$ ❺ Use $\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}[2a + (n-1)d]$

$= 1498$

## Example 3

**Thembani salary for 2008 is R 100 000 and his expenses are R 80 000. His salary increases by R 10 000 per annum while his expenses increase by R 12 500 per annum. Each year he manages to save the excess of income over expenditure.**
**a) Represent his accumulated savings in an arithmetic series.**
**b) After how many years will his savings be totally depleted?**
**c) To what amount must Thembani reduce the rate at which his annual expenses increase so as to let his savings last for 21 years?**

**Arithmetic sequence: *a* = 1st term; d = constant difference**

a) 1st year saving = R 20 000 ❶ Savings = Salary – Expenditure

Growth = – R 2 500 ❷ R 10 000 – R 12 500

20 000 + 17500 + 15 000 + … ❸ Arithmetic series

b) 20 000+17 500+ 15 000+… = 0 ❶ Savings totally depleted if sum = 0

$a = 20\,000$ & $d = -2\,500$ ❷ 1st term = a; constant difference = d

$0 = \frac{n}{2}[2(20\,000 + (n-1)(-2\,500)]$ ❸ Use $\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}[2a + (n-1)d]$

$0 = n(40\,000 - 2\,500n + 2\,500)$ ❹ Simplify

$2\,500n^2 - 42\,500n = 0$

$2\,500n(n - 17) = 0$ ❺ Factorise

$n = 0$ (n/a) or $n = 17$

Savings depleted in 17 years

c) Make expenses increase: $x$

$a = 20\,000$ & $d = 10\,000 - x$ ❶ Find a & d: n = 17 and Sum = 0

$0 = \frac{21}{2}[2(20\,000 + 20(10\,000 - x)]$ ❷ Use $\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}[2a + (n-1)d]$

$0 = 10{,}5(40\,000 + 200\,000 - 20x)$ ❸ Simplify

$x$ = R 12 000 per annum increase ❹ Answer question

## Example 4

**A rubber ball is dropped vertically from a building 20 m high. The ball bounces to a height 4/5 of the height of the previous bounce.**

**a) Find the maximum height of the ball once it has hit the ground three times.**

**b) Determine an algebraic expression for the maximum height reached once it has hit the ground *n* times**

**c) Use the expression derived in b) to find the maximum height reached once it has hit the ground 10 times**

**d) What distance did it travel the moment the ball hit the ground the $10^{th}$ time.**

**e) What is the total distance travelled by the ball?**

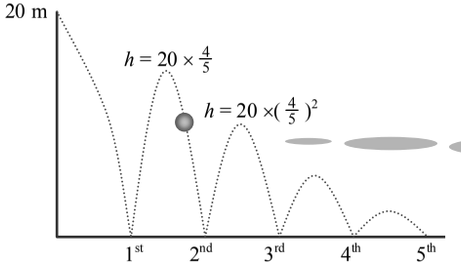
**Copied from Mathematics Exam Focus**

**Geometric sequence: *a* = 1st term; r = constant ratio**

❶ Look at particular cases (draw picture if possible)

❷ Draw and complete a table

| Number of times the ball hit the ground | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Maximum height (after hitting the ground) | $20(\frac{4}{5})^1$ | $20(\frac{4}{5})^2$ | $20(\frac{4}{5})^3$ | $20(\frac{4}{5})^4$ |




Although the ball moves only in a vertical direction the sketch adds a horizontal direction for illustration purposes!

❸ Pattern emerge: $\times \frac{4}{5}$ every time: Geometric sequence

❹ Find general term by using the pattern

❺ Substitute n = 10 into general term

❻ Multiplied distance by 2: Up & down

❼ Geometric sequence (ignore the 20 from the series)

❽ Calculate $\sum_{i=1}^{9} ar^{i-1}$ : Sum of the last 9 terms & add 20 m

❾ Infinite convergent geometric sequence $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$

a) Maximum height (after hitting the ground 3 times) $= 20(\frac{4}{5})^3 = 10{,}24$ m

b) Pattern emerge: $T_n = 20(\frac{4}{5})^n$ or $H(n) = 20(\frac{4}{5})^n$

c) $T_{10} = 20(\frac{4}{5})^{10} = 2{,}15$ m

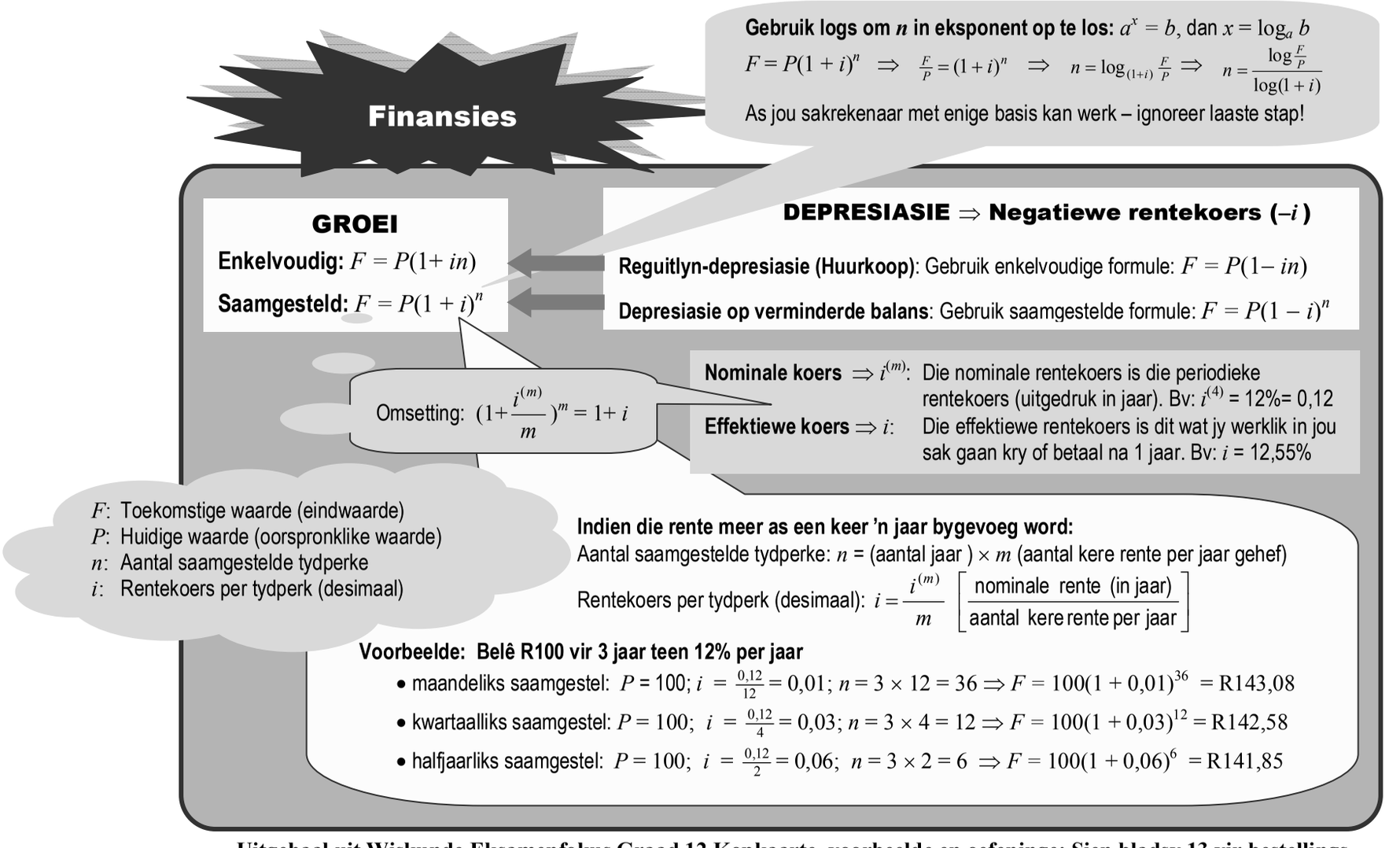
d) Distance travelled after $n^{th}$ hit: $20 + 2[20(\frac{4}{5})] + 2[20(\frac{4}{5})^2] + 2[20(\frac{4}{5})^3] + \ldots$

Bounce: 1st 2nd 3rd 4th

$a = 2[20(\frac{4}{5})] = 32$ & constant ratio: $r = \frac{4}{5}$

$\sum_{i=1}^{9} ar^{i-1} = \frac{32[(\frac{4}{5})^9 - 1]}{\frac{4}{5} - 1} = 138{,}53$ m $\therefore$ Total distance travelled = 138,53 + 20 = 158,53 m

e) $\sum_{i=1}^{\infty} ar^{i-1} = \frac{32}{1-\frac{4}{5}} = 160$ m $\therefore$ Total distance travelled = 160 + 20 = 180 m




**Uitgehaal uit Wiskunde Eksamenfokus Graad 12 Kopkaarte, voorbeelde en oefeninge: Sien bladsy 13 vir bestellings**

## Example 1

**A new car costs R 150 000. If inflation is calculated at 16% per annum and compounded monthly, what will be the cost of the car in 4 years time?**

**Compound growth: Use $F = P(1 + i)^n$ & Compounded MONTHLY**

$F = ?$ ❶ Future or final value of car
$P =$ R 150 000 Principal or current price of car
$n = 4 \times 12 = 48$ months Number of compounding MONTHS
$i = \frac{0{,}16}{12} = 0{,}133\ldots$monthly Interest rate per month (decimal form)
$F = P(1 + i)^n$ ❷ Use compounded growth formula
$= 150\,000(1 + \frac{0{,}16}{12})^{48}$ ❸ Substitute values
$=$ R 283 271,61 ❹ Use calculator

## Example 2

**Mokgadi buys a new fridge at a cash price of R 1 500 on a hire purchase agreement. She pays a deposit of 20% and pays the remaining balance off over 4 years with an interest rate of 18% per annum compounded monthly. Calculate the monthly payments.**

**Hire purchase ⇒ Simple growth formula: $F = P(1 + in)$**

$F = ?$ ❶ Future value of loan
$P = 1500 - (1500 \times 20\%) = 1200$ ❷ Loan (P) = total price – deposit
$n = 4$ years (and 48 payments) Number of years
$i = 0{,}18$ p.a. Interest rate per annum
$F = P(1 + in)$ ❸ Hire purchase ⇒ Simple formula
$= 1200[1 + (0{,}18)(4)] = 2064$ ❹ Substitute values
Instalments = R 2 064 ÷ 48 = R43 ❺ Instalments = $F \div$ Number payments

## Example 3

**The value of a computer decreases from R 5 000 to R 2 800 in three years. Calculate the rate of depreciation per annum if the calculation is done annually.**

**Compound depreciation: Use $F = P(1 + i)^n$ BUT rate will be negative**

**Another way to ask the question is to give the fraction of depreciation.**
**For example if the car loses $\frac{2}{3}$ of its original value then $\frac{F}{P} = \frac{1}{3}$ = $(1 + i)^n$**

$F =$ R 2 800 ❶ Future value of computer
$P =$ R 5 000 Principal or current price of computer
$n = 3$ Number of compounding years
$i = ?$ Interest rate per year
$F = P(1 + i)^n$ ❷ Use compounded formula
$2\,800 = 5\,000\,(1 + i)^3$ ❸ Substitute values
$\frac{2800}{5000} = (1 + i)^3$ ❹ Divide by 5 000 both sides
$\sqrt[3]{\frac{2800}{5000}} = 1 + i$ ❺ Cube root both sides
$\sqrt[3]{\frac{2800}{5000}} - 1 = i$
$i = -0{,}1757$ ❻ Use calculator
The rate of depreciation is 17,57%

## Voorbeeld 6

**David belê R1 000 vir 8 jaar. Hy ontvang rente van 12% per jaar, maandeliks saamgestel, vir die eerste 2 jaar. Die rentekoers verander na 24% per jaar, kwartaalliks saamgestel, vir die volgende 3 jaar, en verander weer na 16% per jaar, halfjaarliks saamgestel, vir die oorblywende tydperk. Bereken hoeveel geld David na 8 jaar sal kry.**

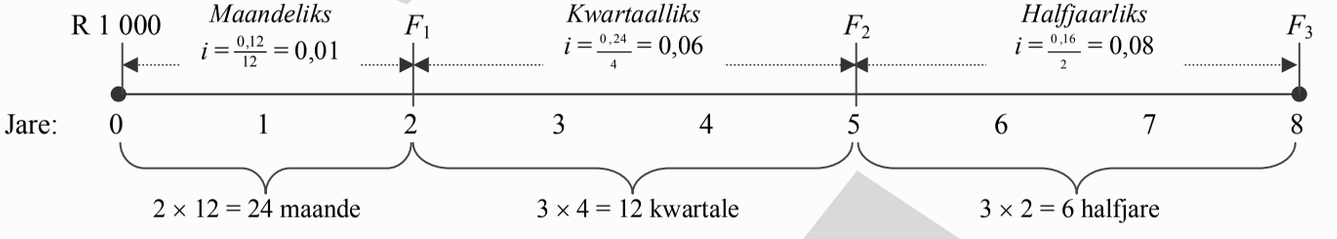
**Gebruik 'n tydlyn om meer ingewikkelde finansiële probleme op te los.**

**Om $F_1$ te bereken: Die toekomstige waarde van R1 000 na 2 jaar**

$F_1 = ?$ ❶ Toekomstige waarde
$P = 1\ 000$ Huidige waarde
$n = 24$ maande Aantal tydperke (maande)
$i = \frac{0,12}{12} = 0,01$ per maand Maandelikse rentekoers
$F_1 = P(1 + i)^n$ ❷ Saamgestelde groeiformule
$= 1\ 000(1+ 0,01)^{24}$ ❸ Vervang waardes
$= \text{R}1\ 269,73$ ❹ Gebruik sakrekenaar

**Om $F_3$ te bereken: Gebruik $F_2$ = R 2 554,95 as huidige waarde**

$F_3 = ?$ ❶ Toekomstige waarde
$P = F_2 = \text{R}2\ 554,95$ Huidige waarde
$n = 6$ halfjare Aantal tydperke
$i = \frac{0,16}{2} = 0,08$ Rentekoers per tydperk
$F_3 = P(1 + i)^n$ ❷ Saamgestelde groeiformule
$= 2\ 554,95(1+ 0,08)^6$ ❸ Vervang waardes
$= \text{R}4\ 054,38$ ❹ Gebruik sakrekenaar




**Om $F_2$ te bereken: Gebruik $F_1$ = R 1 269,73 as huidige waarde**

$F_2 = ?$ ❶ Toekomstige waarde
$P = F_1 = \text{R}1\ 269,73$ Huidige waarde
$n = 12$ kwartale Aantal tydperke (kwartale)
$i = \frac{0,24}{4} = 0,06$ per kwartaal Rentekoers per kwartaal
$F_2 = P(1 + i)^n$ ❷ Saamgestelde groeiformule
$= 1\ 269,73(1 + 0,06)^{12}$ ❸ Vervang waardes
$= \text{R}2\ 554,95$ ❹ Gebruik sakrekenaar

**FINALE ANTWOORD:**

David sal R4 054,38 na 8 jaar ontvang

## Worksheet 6

1. A new house costs R 240 000. If inflation is calculated at 18% per annum and compounded monthly, what will be the cost of the house in 12 years time?
2. Calculate the time in which an investment of R 450 will increase to R 1 200 if it is invested at a compound increase of 17% per annum if the interest is added monthly.
3. The value of a vehicle decreases from R 18 700 to R 12 475 in four years. Calculate the rate of depreciation per annum if the calculation is done annually.
4. Make $P$, $i$ and then $n$ the subject of the following formula: $F = P(1 + i)^n$
5. Ester invests a certain sum of money for 6 years. She receives interest of 12% per annum compounded monthly for the first two years. The interest rate changes to 16% per annum compounded semi-annually for the remaining term. The money appreciates to R80 000 at the end of the 6-year period.
   a) Calculate the effective interest rate per annum during the first year.
   b) Calculate how much money Ester invested initially.
6. After 4 years of reducing balance depreciation, a car has a ¼ of its original value. The original value was R120 000. Calculate the depreciation interest rate, as a percentage. (Correct your answer to 1 decimal place.)

1. 'n Nuwe huis kos R 240 000. As inflasie bereken word teen 18% per jaar en maandeliks saam word, wat sal die huis oor 12 jaar kos?
2. Bereken die tydperk waarin 'n belegging van R 450 sal oploop tot R 1 200 as dit belê word teen 'n saamgestelde koers van 17% per jaar as die rente maandeliks bygevoeg word.
3. Die waarde van 'n voertuig verminder van R 18 700 na R 12 475 in vier jaar. Bereken die waardeverminderingskoers per jaar as die berekening jaarliks gedoen word.
4. Maak P, i en dan n die onderwerp van die volgende formule: $F = P(1 + i)^n$
5. Ester belê 'n sekere bedrag geld vir 6 jaar. Vir die eerste twee jaar ontvang sy rente van 12% per jaar, wat maandeliks saamgestel word. Die rentekoers verander na 16% per jaar, wat halfjaarliks vir die oorblywende termyn vasgestel word. Die bedrag het tot R80 000 aan die einde van die 6 jaar-periode gegroei.
   a) Bereken die effektiewe rentekoers per jaar gedurende die eerste jaar.
   b) Bereken hoeveel geld Ester aanvanklik belê het.
6. Na 4 jaar van afnemende balansvermindering, is 'n motor 'n ¼ van die oorspronklike waarde werd. Die oorspronklike waarde was R120 000. Bereken die waardeverminderingsrentekoers, as 'n persentasie. (Rond jou antwoord korrek af tot een desimale plek.)

# Annuities / Annuïteite

## Home loans (bond repayment) / Huislenings (Verbandpaaiemente)

Interest on a home loan is calculated at the beginning of each month, and payments are calculated on a monthly basis. / Rente op ‘n huislening word bereken aan die begin van elke maand, en betalings word op ‘n maandelikse basis bereken.

**Use Present value formula to calculate *x* / Gebruik die Huidige waarde formule om *x* te bereken:** $P = \frac{x[1-(1+i)^{-n}]}{i}$

$P$ = Loan (Bond amount) / Lening (Verband bedrag)
$n$ = number payments / aantal terugbetalings (aantal paaiemente)
$i$ = interest rate per period / rentekoers per saamgestelde periode
$x$ = regular payment / gereelde betaling

**Total amount to be repaid / Totale bedrag wat terug betaal moet word:**
Regular payment × Number of repayments / Paaiement × Aantal terugbetalings

**Outstanding balance / Uitstaande balans:**

Working from the present time to the point in time when the balance must be calculated / Werk vanaf begintyd tot die tydstip waarop die balans bereken moet word: Balance = (loan + interest) – (sum of payments + interest) Balans = (lening + rente) – (som van paaiemente + rente)

## Sinking funds / Sinkende fondse

**Definition:** Start saving a fixed monthly instalment to provide for the purchase of new equipment in future. Use present equipment as a deposit.

**Definisie:** Begin vroegtydig ‘n gereelde paaiement te spaar om voorsiening te maak vir die aankoop van nuwe toerusting in die toekoms. Verkoop huidige toerusting vir ‘n deposito

Monthly instalment: Calculate *x* in Future Value Formula / Maandelikse paaiemente: Bereken *x* in die Toekomstige waarde formule $F = \frac{x[(1+i)^{n}-1]}{i}$

## Worksheet 7

1. Tokyo borrowed R50 000 and agrees to repay the loan by means of 50 equal monthly instalments. Interest is calculated at 16% per annum compounded monthly. Calculate the monthly instalment.
2. An amount of $x$ rand is invested $k$ times per annum in an annuity. The interest rate is $r$% per annum and is compounded $k$ times per annum. Derive a formula from first principles that you can use to calculate the amount available in the annuity after $n$ years.
3. Tabu decides to invest R5 000 quarterly in an annuity. His first payment is only at the end of the first quater. If the compound interest he earns is calculated quarterly at 16% per annum, calculate the value of this annuity in five years time from now.
4. The cost of a bus is R2 400 000. The cost is expected to rise at a rate of 16% per annum compound interest, while the value of the bus depreciates at a rate of 22% compounded depreciation per annum. The life expectancy of the bus is 5 years.
   a) Find the scrap value of the bus.
   b) Find the cost of a new bus in five years' time.
   c) A sinking fund is established to pay for the new bus. For the sinking fund, payments are made into an account paying 16% per annum, compounded monthly. Find the value of the sinking fund and the size of the monthly payments, if payments start in a year's time and stop the month before the purchase of the new bus.

1. Tokyo leen R50 000 en kom ooreen om die lening in 50 gelyke maandelikse paaiemente terug te betaal. Rente word bereken teen 16% per jaar maandeliks bereken. Bereken die maandelikse paaiement.
2. 'n Bedrag van $x$ rand word k keer per jaar in 'n annuïteit belê Die rentekoers is $r$% per jaar en word $k$ keer per jaar saamgestel. Lei 'n formule vanuit eerste beginsels af waarmee u die bedrag beskikbaar in die annuïteit na $n$ jaar sal kan bereken.
3. Tabu besluit om kwartaalliks R5 000 in 'n annuïteit te belê. Sy eerste betaling is eers aan die einde van die eerste kwartaal. As die saamgestelde rente wat hy verdien teen 16% per jaar kwartaalliks gekapitaliseer word, bereken die waarde van hierdie annuïteit oor vyf jaar van nou af.
4. 'n Bus kos R2 400 000. Die verwagte styging in koste is 16% saamgestelde rente per jaar. Die waarde van die bus verminder teen 'n saamgestelde koers van 22% per jaar. Die verwagte gebruiksleeftyd van die bus is 5 jaar.
   a) Bepaal die skrootwaarde van die bus.
   b) Vind die koste van die nuwe bus oor vyf jaar.
   c) 'n Delgingsfonds word gestig om vir die nuwe bus te betaal. Vir hierdie fonds geld die volgende: Die paaiemente word gemaak in 'n rekening wat 16% per jaar betaal, maandeliks saamgestel. Bepaal die waarde van hierdie delgingsfonds en die grootte van die maandelikse paaiement, indien paaiemente oor een jaar begin en eindig 'n maand voordat die bus aangekoop word.

# Straight line & parabola / Reguitlyn & parabool

**Standard form / Standaardvorm**: $y = mx + c$

- m (slope / gradiënt) = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
  - || lines / lyne: $m_1 = m_2$
  - $\perp$ lines / lyne: $m_1 m_2 = -1$
- c is the *y*-intercept / $c$ is die $y$-afsnit

**Sketch (3 methods) / Skets (3 metodes)**

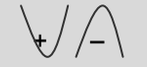
- Table: Choose *x* and determine *y* / Tabel: Kies $x$ en bepaal $y$
- Double intercept / Dubbel-afsnit metode:
  - *y*-intercept: let $x = 0$ / *y*-afsnit: stel $x$ = 0
  - *x*-intercept: let $y = 0$ / *x*-afsnit: stel $y$ = 0
- Gradient intercept / Gradiënt afsnit:
  - Get *y*-intercept / Kry $y$-afsnit
  - Use *m*: Work in *y* then in *x* direction / Gebruik $m$: Werk eers in $y$- en dan in $x$-rigting

**Determining the equation / Bepaling van die vergelyking**

- Determine gradient of line / Gebruik die gradiënt van die lyn
- Find the coordinates of a point on the line / Bepaal koördinate van 'n punt op die lyn
- Substitute $m$ and the point in / Vervang $m$ en die punt in: $(y - y_A) = m(x - x_A)$

**General form / Standaardvorm:** $y = a(x + p)^2 + q$

**Sketch / Skets**

- Turning point: / Draaipunt: (*p*; *q*)
- Form / Vorm: *a* value / a waarde 

- *x*-intercepts / *x*-afsnitte: Let / Stel *y* = 0
- *y*-intercept / *y*-afsnit: Let / Stel *x* = 0

**Sketch using transformations / Skets mbv transformasies:** $y = a(x - p)^2 + q$

- Draw the graph / Skets die grafiek van: $y = ax^2$
- Shift graph q units in the vertical direction / Skuif grafiek *q* eenhede vertikaal
- Shift graph (– *p*) units in the horizontal direction / Skuif grafiek (–*p*) eenhede horisontaal

**Determining the equation / Bepaling van die vergelyking**

- Turning point + any other point / Draaipunt + enige ander punt: $y = a(x - p)^2 + q$
- Two roots + any point / Twee wortels + enige punt: $y = a(x - x_1)(x - x_2)$

# Hyperbola / Hiperbool

**General form / Algemene vorm**: $y = \frac{a}{x}$ ; if a > 0 graph is in the 1st and 3rd quadrants / as $a$ > 0 dan is grafiek in 1st en 3rd kwadrante

**If graph is not in standard position / Indien grafiek nie in standaard posisie is nie:** $y = \frac{a}{(x+p)} + q$; with / met

- $a$ (vertical stretch / vertikale uitrekking)
- $p$ (horizontal shift / horisontale skuif)
- $q$ (vertical shift / vertikale skuif)

**Asymptotes of the graph / Asimptote van die grafiek:** $y = \frac{a}{(x+p)} + q$

- Horizontal asymptote / Horisontale asimptoot: $y = q$
- Vertical asymptote / Vertikaal asimptoot: $x = -p$

**Sketch using a table / Skets grafiek mbv tabel**

- Get *y* alone. / Kry $y$ alleen aan 'n kant.
- Table: Choose *x* and calculate *y*. / Tabel: Kies *x* en bereken $y$.

**Sketch using transformations / Skets mbv transformasies:** $y = \frac{a}{(x+p)} + q$

- Draw the graph / Skets grafiek van: $y = \frac{a}{x}$
- Shift graph q units in the vertical direction / Skuif grafiek q eenhede in die vertikale rigting
- Shift graph $(-p)$ units in the horizontal direction / Skuif grafiek $(-p)$ eenhede in die horisontale rigting

# Exponential graph / Eksponensiële grafiek

**General form / Algemene vorm**: $y = b^x;\ b > 0$

**If graph is not in standard position / Indien grafiek nie in standaard posisie is nie:** $y = a.b^{(x+p)} + q$, with / met

- $a$ (vertical stretch / vertikale uitrekking)
- $p$ (horizontal shift / horisontale skuif)
- $q$ (vertical shift / vertikale skuif)

**Asymptotes of the graph / Asimptote van die grafie:** $y = a.b^{(x+p)} + q$

- Horizontal asymptote / Horisontale asimptoot: $y = q$

**Sketch using a table / Skets grafiek mbv tabel**

- Get y alone. / Kry $y$ alleen aan 'n kant.
- Table: Choose $x$ and calculate $y$. / Tabel: Kies $x$ en bereken $y$.

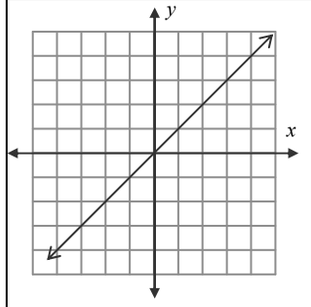
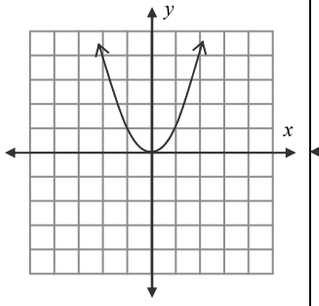
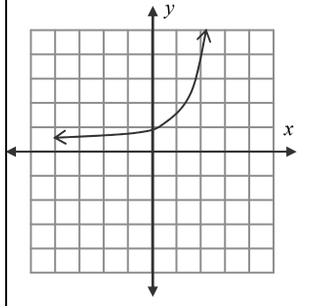
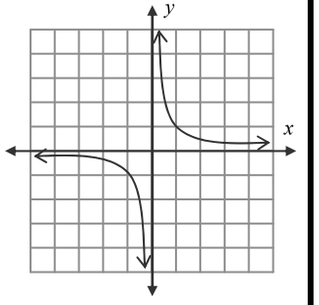
**Sketch using transformations / Skets mbv transformasies:** $y = a.b^{(x+p)} + q$

- Draw the graph / Skets grafiek van: $y = a.b^x$
- Shift graph q units in the vertical direction / Skuif grafiek $q$ eenhede in die vertikale rigting
- Shift graph $(-p)$ units in the horizontal direction / Skuif grafiek $(-p)$ eenhede in die horisontale rigting

**Determining the equation / Bepaling van die vergelyking:** $y = a.b^{(x+p)} + q$

- Find horizontal asymptote: If equation is $y$ = 3 then $q$ = 3 / Bepaal die horisontale asimptoot: As $y$ = 3 dan is $q$ = 3
- Use y-intercept to find a: If $p = 0$: $a$ = $y$-intercept / Gebruik $y$-afsnit om $a$ te bepaal: As $p$ = 0 dan is $a$ = $y$-afsnit
- Substitute any point on the exponential graph temporarily in $y = b^x$ and calculate $b$. / Vervang enige punt tydelik in die vergelyking om $b$ te bepaal.

# Properties of graphs / Eienskappe van grafieke

| | **Linear function** | **Quadratic** | **Exponential** | **Hyperbola** |
|---|---|---|---|---|
| | $y = x$ | $y = x^2$ | $y = b^x$ | $y = \frac{1}{x}$ |
| |  |  |  |  |
| domain | R | R | R | $(-\infty; 0) \cup (0; +\infty)$ |
| range | R | $[0; +\infty)$ | R | $(-\infty; 0) \cup (0; +\infty)$ |
| *x*- intercepts | 0 | 0 | - | - |
| *y*- intercepts | 0 | 0 | 1 | - |
| turning points | - | (0; 0) is a minimum | - | - |
| asymptotes | - | - | y = 0 | $x = 0$; $y = 0$ |
| symmetry | no symmetry | $x = 0$ | no symmetry | $y = -x$ |
| intervals on which the function increases | R | $(0; +\infty)$ | R | - |
| intervals on which the function decreases | - | $(-\infty; 0)$ | - | $(-\infty; 0) \cup (0; +\infty)$ |

## Worksheet 8

1. Given/Gegee: $f(x) = 2\cos(x + 30°)$
   Find the following properties of $f$.
   a) maximum value / maksimum waarde
   b) period / periode
   c) range / waardeversameling
   d) amplitude

2. Given/Gegee: $f(x) = 3\tan x - 2$.
   Find the following properties of f / Bepaal die volgende eienskappe van f:
   a) $y$-intercept / $y$-afsnitte
   b) general equation of the asymptotes
      algemene vergelyking van die asimptote
   c) period / periode
   d) range / waardeversameling

3. Sketch the graph of/Skets die grafiek van $f(x) = (x - 3)^2 + 2$.
   Use your graph to find the following properties about $f$
   Gebruik jou grafiek om die volgende eienskappe van $f$ te bepaal:
   a) $y$-intercept / $y$-afsnitte
   b) $x$-intercepts / $x$-afsnitte
   c) minimum value / minimum waarde
   d) domain / definisie versameling
   e) range / waardeversameling
   f) line(s) of symmetry / lyne van simmetrie

4. Draw the graph of / Skets die grafiek van: $f(x) = 3^x$
   a) Use a transformation to draw the graph of $y = 3^{-x}$ on the same set of axes.
      Gebruik 'n transformasie om die grafiek van $y = 3^{-x}$ op dieselfde assestelsel te teken.
   b) Give the equations of the line of symmetry between the two graphs.
      Gee die vergelyking van die simmetrie-as van die twee grafieke.

5. Given/Gegee: $g(x) = x^2$ and/en $f(x) = 3(x - 2)^2 - 1$
   a) Graph the function of g($x$) / Skets die grafiek van g($x$).
   b) Describe a series of transformations needed to obtain the graph of $f(x) = 3(x - 2)^2 - 1$ from the graph of $g(x) = x^2$
      Beskryf 'n reeks transformasies wat gebruik kan word om die grafiek van $f(x) = 3 - (x + 4)^2$ vanaf die grafiek $g(x) = x^2$ te verkry.
   c) Graph the function $f(x)$ by starting with the graph of g($x$) by using transformations.
      Skets die funksie $f(x)$ deur met die grafiek van $g(x)$ te begin en deur transformasies te gebruik.
   d) Give the domain of $f(x)$
      Gee die definisie versameling van $f(x)$.
   e) Give the range of $f(x)$
      Gee die waardeversameling van $f(x)$.

## Worksheet 9

1. The monthly electricity cost depends on the number of units used during the month. Ann found that in April her electricity bill was R460 for 420 units and in May the bill was R620 for 720 units.
   a) Express the monthly cost C in terms of the number of units used $x$, assuming that a linear relationship gives a suitable model.
   b) Use a) to predict the cost of 2000 units per month.
   c) Draw the graph of the linear equation.
   d) What does the slope of the line represent?
   e) What does the y-intercept of the graph represent?

2. Draw the graphs of $y = x^2 + x - 6$ and $y = 2x + 1$ and use the graphs to answer the following questions:
   If $f(x) = \dfrac{x^2 + x - 6}{2x + 1}$, then complete:
   a) For which value(s) of $x$ is $f(x) = 0$
   b) For which value(s) of $x$ is $f(x) \leq 0$
   c) For which value(s) of $x$ is $f(x) \geq 0$
   d) For which value(s) of $x$ does $f(x)$ not exist?

3. Given: $f(x) = x^2 - x - 6$ and $g(x) = x + 2$
   a) Sketch the graphs of $g(x)$ and $f(x)$ on the same set of axes.
   b) Use your graphs to solve the equation $x^2 - 2x - 8 = 0$

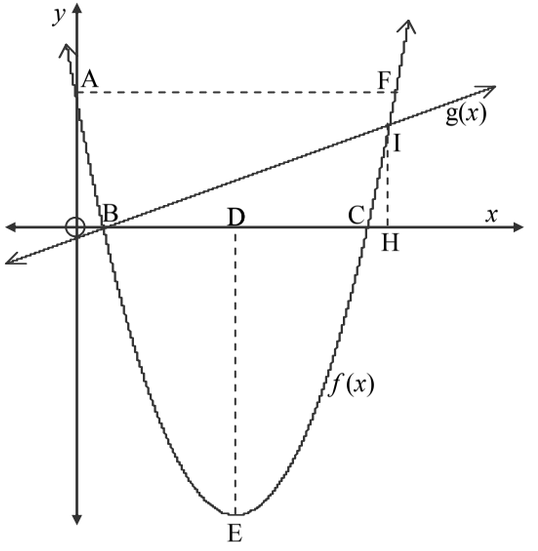
1. Die maandelikse koste van elektrisiteit hang af van die aantal eenhede wat gebruik word gedurende die maand. Ann se koste in April maand was R460 vir 420 eenhede wat sy gebruik het en in Mei was dit R620 vir 720 eenhede wat sy gebruik het.
   a) Druk die maandelikse koste C uit in terme van die aantal eenhede $x$ wat gebruik is, as aanvaar word dat die model deur 'n lineêre verwantskap voorgestel kan word.
   b) Gebruik die antwoord in a) en voorspel die koste as 2000 eenhede per maand gebruik word.
   c) Stel die lineêre vergelyking grafies voor.
   d) Wat stel die gradiënt van die lyn voor?
   e) Wat stel die y-afsnit van die grafiek voor?

2. Teken die grafieke van $y = x^2 + x - 6$ en $y = 2x + 1$ en gebruik hulle om die volgende vrae te beantwoord:
   As $f(x) = \dfrac{x^2 + x - 6}{2x + 1}$ voltooi:
   a) Vir watter waardes van $x$ is $f(x) = 0$
   b) Vir watter waardes van $x$ is $f(x) \leq 0$
   c) Vir watter waardes van $x$ is $f(x) \geq 0$
   d) Vir watter waardes van $x$ sal $f(x)$ ongedefinieer wees?

3. Gegee: $g(x) = x + 2$ & $f(x) = x^2 - x - 6$
   a) Skets die grafieke van $g(x)$ en $f(x)$ op dieselfde assestelsel.
   b) Gebruik jou grafieke om die vergelyking $x^2 - 2x - 8 = 0$ op te los.

## Worksheet 10

1. In the figure the graphs of the following functions are shown:
   In die figuur word die grafieke van die volgende funksies aangetoon:



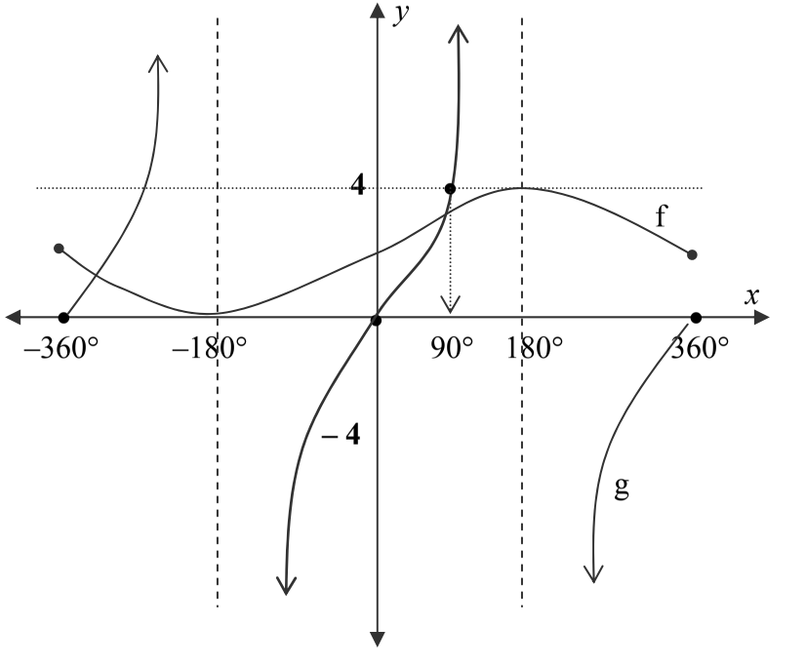

E is the turning point of $f$ / E is die draaipunt van $f$
$f(x) = x^2 - 10x + 9$ and/en $g(x) = 2x - 2$

a) Determine the coordinates of the following points:
   i) A
   ii) B and/en C

b) Determine the lengths of / Bepaal die lengtes van:
   i) OD
   ii) DE
   iii) AF
   iv) HI

2. The figure shows the graphs of two trigonometric functions, $f(x)$ and $g(x)$. Determine the equation of $f(x)$ and $g(x)$.

   Die figuur toon grafieke van twee trigonometriese funksies, $f(x)$ en $g(x)$. Bepaal die vergelykings van $f(x)$ en $g(x)$.



# Differential Calculus / Differensiaalrekeninge

**Average decline / Gemiddelde helling:** $\frac{f(x+h)-f(x)}{h}$ (Average speed / Gemiddelde spoed)

**The incline in a point / Die helling in 'n punt:** $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

**Notation / Notasie:** $f'(x)$ or $D_x(**)$ or $\frac{dy}{dx}$ or $\frac{d}{dx}(**)$

**Rate of change / Veranderingstempo**: f '(x)

Remember $s(t) \Rightarrow$ distance & $s'(t) \Rightarrow$ speed & $s''(t) \Rightarrow$ speed

**Wordproblems / Woordsomme:**

- Draw a sketch / Teken 'n skets
- Draw up an equation, based on the problem, eg. f(x). / Stel 'n vergelyking op, gebaseer op die probleem, bv. f(x).
- Determine / Bepaal $f'(x)$
- Put $f'(x) = 0$, to deliver a maximum or minimum value for x / Stel $f'(x) = 0$ om 'n maksimum of minimum waarde vir x te lewer.
- Substitute x in f(x) to get the maximum or minimum value. / Vervang x in f(x) om die maksimum of minimum waarde te kry

**Sketches of polynomials of the third degree / Sketse van derdegraadse polinome:**

General equation / Algemene vergelyking: $y = ax^3 + bx^2 + cx + d$

- X-intercepts / X-afsnitte: f(x) = 0
- Factorise through grouping / factor theorem
- Faktoriseer deur groepering / faktorstelling
- Y-intercept / Y-afsnit: Let / Stel x = 0
- Stationary points / Stationêre punte: Put / Stel $f'(x) = 0$
    - Solve for x / Los op vir x.
    - Find f(x) for abovementioned x-values.
    - Vind f(x) vir bogenoemde x-waardes.

## Worksheet 11

1. Given / Gegee: $f(x) = 3x - 2x^2$
   Determine the average rate of change of the function between $x = -2$ and $x = 3$.
   Bepaal die gemiddelde veranderingstempo van die funksie tussen $x = -2$ en $x = 3$.

2. Given / Gegee: $f(x) = \dfrac{2x}{x-1}$
   a) Determine the average rate of change of the function between $x = -1$ and $x = 2$.
      Bepaal die gemiddelde veranderingstempo van die funksie tussen $x = -1$ en $x = 2$.
   b) Determine the average rate of change of the function between $x = a$ and $x = a + h$.
      Bepaal die gemiddelde veranderingstempo van die funksie tussen $x = a$ en $x = a + h$.

3. If $f(x) = -4x^2$, determine $f'(x)$ from first principles.
   As $f(x) = -4x^2$, bepaal $f'(x)$ vanaf eerste beginsels.

4. Determine $f'(x)$ / Bepaal $f'(x)$:
   a) $f(x) = (x - 1)(x^2 + 2)$
   b) $f(x) = 2x - \dfrac{4}{x^2} + 3$
   c) $f(x) = 3x^{-4} + \sqrt{x}$
   d) $f(x) = 3x^3 + 2x^2 - 4x + 6$
   e) $f(x) = \sqrt{x} + (x + 3)^2$

5. a) Find the equations of the tangents to the parabola $y = x^2 - 2x - 3$ at the points where $x = 0$ and $x = 2$ respectively.
      Bepaal die vergelykings van die raaklyne aan die parabool $y = x^2 - 2x - 3$ by die punte waar $x = 0$ en $x = 2$ respektief is.

   b) Find the point of intersection of these two tangents.
      Bepaal die snypunt van hierdie twee raaklyne.

6. Sketch the following graphs / Skets die volgende grafieke:
   a) $f(x) = x^3 - 4x^2 - 11x + 30$
   b) $f(x) = x^3 - 7x^2 + 15x - 9$
   c) $f(x) = x^3 - x^2 - 8x + 12$

7. A cold drink company wants to optimise its manufacturing cost. They want to use a closed cylindrical can that holds 350 ml of cold drink to sell their products. Calculate the height and radius that will minimise the amount of material needed to manufacture a can.

   'n Koeldrank maatskappy wil die vervaardigingskoste optimeer. Hulle wil 'n geslote silindriese houer gebruik wat 350 ml bevat om die produk te verkoop. Wat moet die radius en die hoogte van die blikkie wees om die hoeveelheid materiaal wat benodig word om die blikkie te vervaardig te optimeer?

# Worksheet 12

1. A biologist states that when a specific kind of anti-bacterium is added to a culture of bacteria, the number of bacteria present will be given by the formula $B(t) = -5t^2 + 60\,t + 1200$ where $B(t)$ represents in millions the number of bacteria on moment t.

   'n Bioloog beweer dat wanneer 'n sekere soort antibakterie tot 'n kultuur van bakterieë gevoeg word, die getal bakterieë teenwoordig gegee sal word deur die formule $B(t) = -5t^2 + 60\,t + 1200$ waar $B(t)$ in miljoene die getal bakterieë op tydstip $t$, gemeet in uur, voorstel.

a) How many bacteria were present at the beginning? / Hoeveel bakterieë was aan die begin teenwoordig?

b) Calculate the tempo of change in relation to the time on moment $t = 10$ hours. / Bereken die tempo van verandering met betrekking tot die tyd op tydstip $t = 10$ uur.

c) Was the bacteria population shrinking or growing on moment $t = 10$ hours? / Was die bakterieë bevolking aan die afneem of toeneem op tydstip $t = 10$ uur?

d) At which moment was the maximum number of bacteria present? / Op watter tydstip was die maksimum getal bakterieë teenwoordig?

e) After how many hours did the number of bacteria start decreasing? / Na hoeveel uur het die getal bakterieë begin afneem?

2. The humidity (H) of air in relation to the temperature (t in ºC) is given by the formula $H = -t^3 + 24t^2 - 84t + 80$

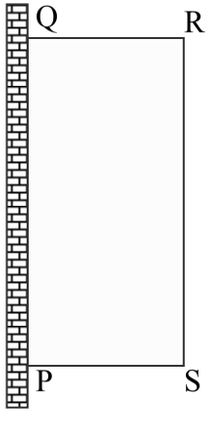
   Die humiditeit (H) van lug met betrekking tot die temperatuur (t in ºC) word gegee deur die formule $H = -t^3 + 24t^2 - 84t + 80$

a) Determine the humidity at temperature 0 ºC / Bepaal die humiditeit by temperatuur 0 ºC.

b) At which temperature is the humidity 0? / By watter temperatuur is die humiditeit 0?

c) At which temperature does the humidity reach a maximum in the interval [0º ; 20º]? / By watter temperatuur bereik die humiditeit 'n maksimum in die interval [0º ; 20º]?

d) Sketch the graph of H with relation to t in the interval [0º; 20º]. / Skets die grafiek van H met betrekking tot t in die interval [0º; 20º].




3. A rectangular camp PQRS that must be fenced in by 72 cm of wire netting. PQ is an existing wall. PS, SR and QR must be fenced in. Calculate the maximum area of the camp.

   Die meegaande figuur toon 'n reghoekige kamp PQRS wat omhein moet word deur 72 cm ogiesdraad. PQ is 'n bestaande muur. PS, SR en QR moet toegekamp word. Bereken die maksimum oppervlakte van die kamp.

# Linear Programming / Lineêre Programmering

**Step 1 / Stap 1:** Determine what you are going to name x and y / Bepaal wat jy x en y gaan noem

**Step 2 / Stap 2**
Identify the constraints and highlight them. (Search for words like "at most", "at least", "not more than" or "not less than").
Identifiseer die beperkings en "highlight" dit. (Soek vir woorde soos "hoogstens", "minstens", "nie meer as" of "nie minder as").

| | |
|---|---|
| $y$ is not more than 20 / $y$ in nie meer as 20 nie | $y \leq 20$ |
| $x$ is at most 12 / $x$ is hoogstens 12 | $x \leq 12$ |
| $y$ is at least 200 / $y$ is minstens 200 | $y \geq 200$ |
| $x$ is not less than 12 / $x$ is nie minder as 12 | $x \geq 12$ |
| twice $x$ is at most three times $y$ / twee keer $x$ is hoogstens drie keer $y$ | $2x \leq 3y$ |
| $x$ is at least twice y / $x$ is ten minste 2 keer $y$ | $x \geq 2y$ |
| $y$ is at most 3 times $x$ / $y$ is hoogstens 3 keer $x$ | $y \leq 3x$ |

**Step 3 / Stap 3**
Compile the constraints (all the inequalities together) algebraically. Remember the implicit constraints (conditions that go without saying).
Stel die beperkings (al die ongelykhede saam) algebraïes voor. Onthou die implisiete beperkings (voorwaardes wat vanselfsprekend is).

**Step 4 / Stap 4**
Represent information graphically and determine the feasible region (the region that satisfies all the inequalities
Stel inligting grafies voor en bepaal die gangbare gebied (die gebied wat al die ongelykhede bevredig).

**Step 5 / Stap 5:**
Determine the objective function / Bepaal die doelfunksie.

**Step 6 / Stap 6**
Search line method: Gradient of objective function
Soeklyn-metode: Helling van doelfunksie
Name the vertices of the feasible region and the coordinates.
Determine the value of the objective function at each vertex.
Benoem die hoekpunte van toelaatbare gebied en die koördinate.
Bepaal die waarde van die doelfunksie by elke hoekpunt.

$x, y \in R$ - shade region / kleur gebied in
$x, y \in Z$ - use dots / maak kolle

$y >$ …. Shade above the line
arseer bo die lyn
$y <$ ….. Shade below the line
arseer onder die lyn

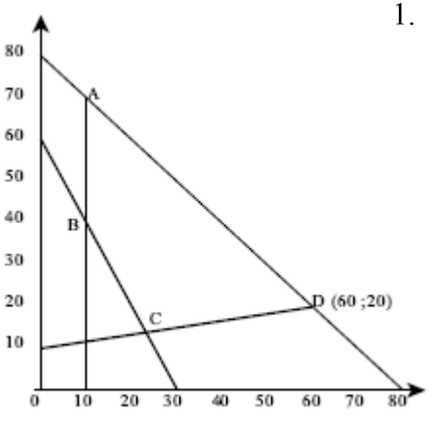
$\leq$ or $\geq$: solid line/soliede lyn
$\leq$ or $>$: dotted line/stippellyn

**Step 7 / Step 7:** Answer the question / Beantwoord die vraag.

## Worksheet 13

1. In the accompanying sketch there is a set of inequalities that leads to the feasible region *ABCD* as shown by the shaded area.
   Use the graph to answer the following questions:

   a) Write down the set of inequalities that describe the feasible region
   b) Maximise $2x + 3y$ for the given feasible region.
   c) The coordinates of point *C* minimise the function value of c where $y = mx + c$. Write down the possible values of *m* and *c*.




1. In die meegaande skets word die gangbare gebied *ABCD* deur 'n aantal ongelykhede voorgestel. Gebruik die grafiek om die volgende vrae te beantwoord:

   a) Skryf die ongelykhede neer wat die gangbare gebied beskryf.
   b) Maksimeer $2x + 3y$ vir die gegewe gangbare gebied.
   c) Bepaal die moontlike waardes van m en c waarvoor $y = mx + c$ 'n minimumwaarde by *C* sal hê.

2. A farmer plans to plant maize and sunflowers on his farm. The land available for the maize is *x* hectares and for the sunflower is *y* hectares. The planting is subject to the following restrictions:
   $2 \le x \le 8$ $0 \le y \le 7$ $5x + 7y \le 70$ $8x + 5y \le 80$

   a) Use graph paper and draw on one set of axes the graphs of all the restrictions. Shade the feasible region clearly, indicating the coordinates of all the vertices on the graph.

   b) If the profit P is given by $P = 6x + 5y$, use the graph to determine how man hectares of each crop must be planted for a maximum profit. Indicate on your graph where this value may be.

2. 'n Boer beplan om mielies en sonneblomme te plant op sy plaas. Daar is *x* hektaar lande beskikbaar vir mielies en *y* hektaar vir sonneblomme. Die aanplant van die gewasse is onderhewig aan die volgende beperkings:
   $2 \le x \le 8$ $0 \le y \le 7$ $5x + 7y \le 70$ $8x + 5y \le 80$

   a) Gebruik grafiekpapier en maak op een assestelsel 'n skets van al hierdie beperkings. Arseer die gangbare gebied duidelik en dui al die hoekpunte se koördinate aan op die grafiek.
   b) As die profyt P gegee word deur $P = 6x + 5y$, gebruik die grafiek om vas te stel hoeveel hektaar van elke gewas geplant moet word vir 'n maksimum wins. Dui aan op die grafiek waar die afleespunt is.

# Paper 2/ Vraestel 2

Coordinate geometry / Analitiese meetkunde

Transformations / Transformasies

Trigonometry / Trigonometrie

Data handling / Data hantering

# Volume and surface area / Volume en buiteoppervlakte

### Cylinder

- Volume of a cylinder $= \pi r^2 h$
- Surface area (closed cylinder) $= 2\pi r^2 + 2\pi rh$

### Prism

- Volume of prism = (area of base) $\times$ height
- Surface area $= A + ph = ab + h(a + b)$

### Cone

- Volume of cone = $\frac{1}{3}\pi r^2 h$
- Surface area of closed cone $= \pi r^2 + \pi rH$

### Pyramid

- Volume of pyramid $= \frac{1}{3}Ah$
- Surface area of pyramid $= A + \frac{1}{2}pH$

### Sphere

- Volume of sphere = $\frac{4}{3}\pi r^3$
- Surface area of sphere = $4\pi r^2$

## Worksheet 14

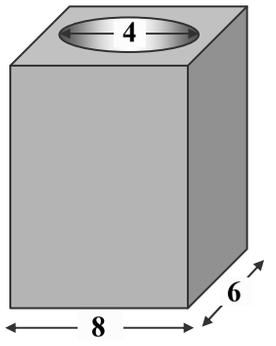
1. Calculate the number of diagonals in a polygon with 12 sides / Bepaal hoeveel diagonale 'n veelhoek met 12 sye het.
2. Complete the following table / Voltooi die volgende tabel:

| Number of sides of a polygon / Aantal sye van veelhoek | 3 | 4 | 5 | 100 | n |
|---|---|---|---|---|---|
| Sum of interior angles / Som van die binnehoeke | 180° | 360° | | | |
| In the case of a regular polygon find the measure of each angle.<br>In die geval van 'n reëlmatige veelhoek bepaal die grootte van elke hoek | 60° | 90° | | | |

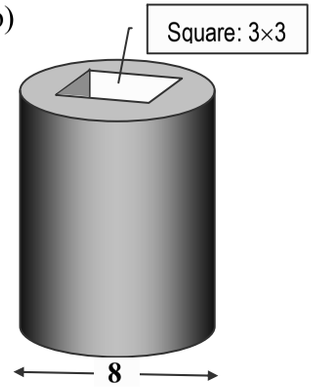
3. Determine the measure of each of the twelve angles of a regular dodecagon / Bepaal die mates van die binnehoeke van 'n reëlmatige twaalfhoek.

4. Calculate the surface area of each solid correct to two decimal places. Lengths are in cm and the height of each of the prisms and pyramids is 10 cm.
   Bereken die buite oppervlakte van elk van die volgende vaste liggame korrek tot twee desimale syfers. Lengtes is in cm en die hoogtes van alle prismas en piramiedes is 10cm.

a)

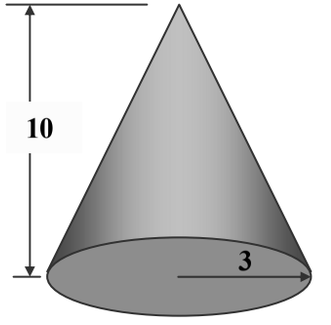



b)

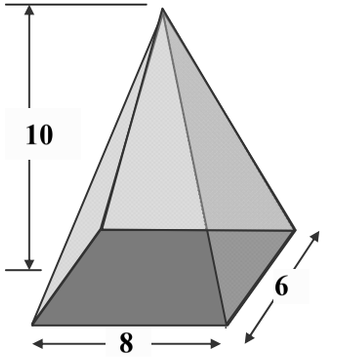



c)

d)




e)



# Analytical geometry / Analitiese meetkunde

**Formulae / Formules**

Distance between points A and B / Afstand tussen twee punte A en B: $AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$

Midpoint M($x_M$ ; $y_M$) of line segment AB / Middelpunt M($x_M$ ; $y_M$) of AB: $x_M = \frac{x_A + x_B}{2}$ and $y_M = \frac{y_A + y_B}{2}$

Gradient of a line / Helling (gradiënt): $m_{AB} = \frac{y_A - y_B}{x_A - x_B}$

Angle of inclination / Inklinasie hoek: $\tan\theta = m$

Equation of a straight line / Vergelyking van 'n reguitlyn: $y = mx + c$ of/or $y - y_A = m(x - x_A)$

Circle with centre $(a; b)$ and radius $r$ / Sirkel met middelpunt $(a; b)$ & radius $r$: $(x - a)^2 + (y - b)^2 = r^2$

## Worksheet 15

1. A (–5 ; 6), B ($t$ ; –2) and C (–7 ; 2) are three points in a Cartesian plane. Calculate
   a) the co-ordinates of the midpoint of AC
   b) the value(s) of $t$ if AB = BC
2. P (–6 ; 2) and Q ($q$ ; 6) are points on the circumference of a circle with the origin as centre. Determine
   a) the equation of the circle
   b) the value(s) of $q$
   c) the size of the angle between the line PQ and the positive $x$-axis if $q = 2$.
3. B(3; 2) and C(3; –6) are points on the circumference of a circle with centre Q and with equation $x^2 + y^2 - 2x + 4y - 15 = 0$.
   a) Determine the coordinates of the centre of the circle and the length of the radius.
   b) Determine the equation of the tangent to the circle at C.
4. A(–4 ; 5), B (–6; –6) and C (3; –2) are the vertices of $\Delta ABC$. Calculate the measure of angle $B\hat{C}A$
5. A circle with centre M(4 ; 2) passes through the point A(8 ; –2). Determine
   a) the equation of the circle in the form $x^2 + ax + y^2 + by + c = 0$
   b) the co-ordinates of the y-intercepts of the circle
6. What is the radius of the circle with equation $9x^2 = 4 - 9y^2$ ?

1. A (–5 ; 6), B ($t$ ; –2) en C (–7 ; 2) is drie punte in 'n Cartesiese vlak. Bereken
   a) die koördinate van die middelpunt van AC
   b) die waarde(s) van $t$ as AB = BC
2. P (–6 ; 2) en Q ($q$ ; 6) is punte op die omtrek van 'n sirkel met die oorsprong as middelpunt. Bepaal
   a) die vergelyking van die sirkel
   b) die waarde(s) van $q$
   c) die grootte van die hoek wat die lyn PQ met die positiewe x-as maak as $q = 2$.
3. B(3; 2) en C(3; –6) is punte op die omtrek van 'n sirkel met middelpunt Q en met vergelyking $x^2 + y^2 - 2x + 4y - 15 = 0$.
   a) Bepaal die koördinate van die middelpunt van die sirkel en die radius van die sirkel.
   b) Bepaal die vergelyking van die raaklyn aan die sirkel by C.
4. A(–4 ; 5), B (–6; –6) en C (3; –2) is die hoekpunte van $\Delta ABC$. Bereken die grootte van hoek $B\hat{C}A$ .
5. 'n Sirkel met middelpunt M(4 ; 2) gaan deur die punt A(8 ; –2). Bepaal
   a) die vergelyking van die sirkel in vorm $x^2 + ax + y^2 + by + c = 0$
   b) die koördinate van die y-afsnitte van die sirkel.
6. Wat is radius van die sirkel met vergelyking $9x^2 = 4 - 9y^2$ ?

## Worksheet 16

1. Determine the co-ordinates of the centre and the radius of the circle with equation $x^2 - 10x + y^2 + y = 0$

2. AB and DE are two parallel lines. The equation of AB is $3y - 2x = 15$ and the co-ordinates D and E are D(0 ; −1) and E($e$ ; −3) respectively. Determine

a) the gradient of the straight line AB

b) the value(s) of $e$

c) the equation of the straight line through the point C(−3 ; 8) which is perpendicular to AB, in the form $ax + by + c = 0$.

1. Bepaal die koördinate van die middelpunt en radius van die sirkel met vergelyking $x^2 - 10x + y^2 + y = 0$

2. AB en DE is twee ewewydige lyne. Die vergelyking van AB is $3y - 2x = 15$ en die koördinate van D en E is onderskeidelik D (0 ; −1) en E ($e$ ; −3). Bepaal

a) die gradiënt van die reguitlyn AB

b) die waarde(s) van $e$

c) die vergelyking van die reguitlyn deur die punt C (−3 ; 8) wat loodreg is op AB, in die vorm $ax + by + c = 0$.

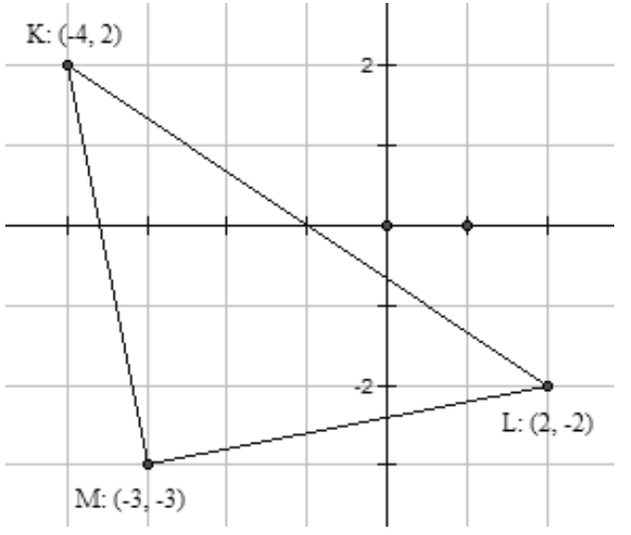
3. K(−4; 2), L(2; −2) and M(−3; −3) are vertices of ΔKLM in a Cartesian plane.

a) Calculate the perimeter of ΔKLM.

b) Calculate the inclination angle of LM.

c) Determine the co-ordinates the midpoint of segment KM.

d) Determine the equation of the perpendicular bisector of KM.




3. K(−4; 2), L(2; −2) en M(−3; −3) is hoekpunte van ΔKLM op ’n Kartesiese vlak.

a) Bereken die omtrek van ΔKLM

b) Bereken die inklinasiehoek van LM

c) Bepaal die koördinate van die middelpunt van lynstuk KM.

d) Bepaal die vergelyking van die loodregte verdeler van KM.

# Trigonometry / Trigonometrie

## Definitions

$\sin\theta = \frac{y}{r} = \frac{t}{s}$ $\quad \cos\theta = \frac{x}{r} = \frac{a}{s}$ $\quad \tan\theta = \frac{y}{x} = \frac{t}{a}$

## Simplifying with reducing formulae:

**Step 1** Make negative ∠s positive **then** cos *x* is positive, the rest is negative / Maak negatiewe ∠e positief dan cos x is positief, en die res negatief
e.g. sin (–300°) = – sin 300° & cos (–250°) = + cos 250°

**Step 2** Make angle greater than 360° / smaller than 360° by subtracting any multiple of 360°, then nothing changes. / Maak hoeke groter as 360° kleiner as 360° deur veelvoude van 360° af te trek, maar geen tekens verander nie.
e.g. cos 1000° = cos 280°

**Step 3** Make all ∠'s acute angles (< 90°). Discard multiples of 180°± or 360°±' and determine the sign by means of CAST table. Maak alle ∠e skerphoeke (< 90°), 'gooi 180°± of 360°± weg' en bepaal die teken mbv die CAST-tabel:
e.g. cos 150° = cos (180° – 30°) = –cos 30°

**Step 4** For further simplification or if there is (90°– β), then use cofunctions / Vir verdere vereenvoudig of as daar 'n (90°– β), gebruik ko-funksies

## Proof of identities (LHS & RHS individually) / Bewys van identiteite (LK & RK apart)

1) Choose most difficult side / Kies moeilikste kant
2) Look for square identities / Kyk vir vierkant identiteite (Kyk vir '1' en kwadrate)
3) Everything to sin *x* and cos *x* / Alles na sin x & cos x
4) Fractions: Get LCM / Breuke: Kry KGV
5) Factorise or simplify / Faktoriseer of vereenvoudig
6) Emergency plan: If cos$x$ – 1, then multiply above and below by (cos$x$ + 1) / Noodplan: Indien (cos$x$ – 1) maal dan bo en onder: (cos$x$ + 1)
7) See for which values of *x* the expression is not defined / Kyk vir watter waardes van x is die uitdrukking nie gedefinieer nie

## Identities / Identiteite

$\tan x = \frac{\sin x}{\cos x}$ $\quad \sin^2 x + \cos^2 x = 1$ $\quad$ **Cofunction** $\sin(90° \pm x) = \cos x$ $\quad \cos(90° + x) = -\sin x$

# Compound & double angles / Saamgestelde- en dubbelhoeke

## Compound angles / Saamgestelde hoeke

- $\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$
- $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$
- $\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$
- $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

## Double angles / Dubbelhoeke

- $\sin 2A = 2\sin A.\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$

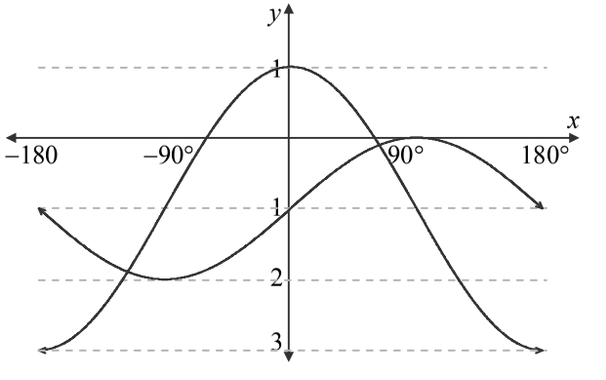
## Worksheet 17

1. Given/Gegee: $f(x) = \sin 2x$ and/en $g(x) = \sin(45° + x)$
   a) Sketch the graphs of f and g on the same set of axes for $x \in [-180°;180°]$. Clearly show the co-ordinates of the intercepts with the axes and of all turning points.
   Skets die grafieke van f en g op dieselfde assestelsel vir $x \in [-180°;180°]$. Dui die koördinate van die afsnitte op die asse en die draaipunte van die grafieke duidelik aan.
   b) Write down the period of $f$ and $g$
   Skryf die periode neer van $f$ en $g$

2. a) Sketch the graphs of f, g end h for the interval $[-180^o; 180^o]$, where: / Skets die grafieke van $f$, $g$ en $h$ vir die interval $[-180^o ; 180^o]$, waar: $f(x) = 2\cos x$ ; $g(x) = 2 + \cos x$ ; $h(x) = \cos 2x$
   b) For which value(s) of $x$ will $\cos 2x = 0$?
   Vir watter waarde(s) van $x$ sal $\cos 2x = 0$?
   c) Use your graphs to determine the greatest value of the expression $2 - \cos x$. Indicate clearly where you have read your answer, again using capital letters.
   Gebruik u grafiek om die grootste waarde van die uitdrukking $2 - \cos x$ te bepaal. Toon duidelik aan waar u die antwoord afgelees het. Gebruik weer eens hoofletters daarvoor.

3.a) Sketch, on the same set of axes, the graphs of / Skets op dieselfde assestelsel, die grafieke van:
   $f = \{(x; y) / y = \frac{1}{2}\tan x; -180^o \le x \le 180^o\}$ and/en
   $g = \{(x; y) / y = \tan \frac{1}{2} x ; -180^o \le x \le 180^o\}$
   b) What is the period of $f$? / Wat is die periode van $f$?
   c) For which value(s) of $x$ will $f(x) = g(x)$?
   Vir watter waarde(s) van $x$ sal $f(x) = g(x)$?
   d) For which value(s) of $x$ will $f(x) > g(x)$?
   Vir watter waarde(s) van $x$ sal $f(x) > g(x)$?

4. In the figure, the graphs of two functions / In die figuur word die grafieke van die twee funksies:
   $f = \{(x; y) / y = a\cos x + k; -180^o \le x \le 180^o\}$ and/en
   $g = \{(x; y) / y = \sin bx + m; -180^o \le x \le 180^o\}$




Determine the values of $a$, $b$, $k$, and $m$. / Bepaal die waardes van $a$, $b$, $k$ en $m$.

## Worksheet 18

1. Simplify, without using a calculator, the following expressions: (Show ALL the calculations.)
   Vereenvoudig, sonder die gebruik van 'n sakrekenaar, die volgende uitdrukkings (Toon AL die berekeninge.):
   a) $\dfrac{\tan(360°-x).\cos(180°+x)}{\cos(90°+x)}$
   b) $\dfrac{\cos 330°.\tan 135°}{\sin 570°.\cos(-60°)}$
   c) $\dfrac{\tan 240°.\cos 150° - \tan 225° + 4\sin 90°}{\cos(-300°)}$
   d) $\tan 30°.\sin 240° + \cos 180° - \cos^2 135°$
   e) $\dfrac{3\cos(90°-x).\sin(360°+x)}{\sin(-90°)} + \dfrac{\cos^2(360°+x).\tan 150°}{\tan(-120°)}$
   f) $\sin^2 x\left[\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}\right]$

2. Determine, without using a calculator, the value of the following in terms of *t,* if sin 17° = *t* / Bepaal, sonder die gebruik van 'n sakrekenaar, die waarde van die volgende in terme van *t* indien sin 17° = *t*:
   a) sin 523°
   b) cos 73°
   c) tan(–17°)

3. If $x \in [0°; 180°]$ solve the following equations / As $x \in [0°; 180°]$ los die volgende vergelykings op:
   a) $\cos A = -\dfrac{\sqrt{3}}{2}$
   b) $\sin^2 x = 1 - \cos^2 x$
   c) $\cos(2x - 72°) = \sin(x + 23°)$

4. Solve for *x* if / Los op vir *x,* indien:
   a) $5\cos 3x + 4 = 0$ and/en $x \in [0°; 360°]$.
   b) $2\sin(A - 20°) = -0{,}428$ ; $\hat{A} \in [-90°; 270°]$

5. If/As $4.\cos x = 2\sqrt{3}$ , $\sqrt{2}\sin y + 1 = 0$ and/en $x, y \in [180°; 360°]$ calculate without the use of a calculator the value of / bereken, sonder die gebruik van 'n sakrekenaar die waarde van
   a) $x + y$ b) $\tan x$

6. Determine the general solution of / Bepaal die algemene oplossing van:
   a) $2.\cos x.\sin x - \cos x = 0$
   b) $7\cos\theta - 2\sin^2\theta + 5 = 0$

7. Prove the identity / Bewys die identiteit:
   $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = \dfrac{4\tan x}{\cos x}$

8. If *x* and *y* are complementary angles and $13\cos x - 5 = 0$, calculate, without using a calculator, the value of $\tan x + \cos y$

   As *x* en *y* komplementêre hoeke is en $13\cos x - 5 = 0$, bereken, sonder om 'n sakrekenaar te gebruik die waarde van $\tan x + \cos y$

# Solving triangles / Oplos van driehoeke

### Sin rule / reël

If ∠ and side opposite each other are known + any other angle or side (i.e. ∠∠s or ss∠)
Indien ∠ en sy regoor mekaar bekend is + enige ander hoek of sy m.a.w. ∠∠s of ss∠)

If you want to calculate ∠ / Indien ∠ gevra word: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

If you want to calculate a side / Indien sy gevra word: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

### Cos rule / reël

If no ∠ and side opposite each other are known (i.e. sss or s∠s)
Indien geen ∠ en sy regoor mekaar nie bekend is nie (m.a.w. sss of s∠s)

If side / Indien sy gevra: $a^2 = b^2 + c^2 - 2bc.\cos A$

If ∠ / As∠ gevra word: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

### Area rule / Opp reël

If 90° ∠ / Indien 90° ∠ in driehoek: Area = ½ $b.h$

Determine s∠s: Area $\Delta ABC$ = ½ $b.c.\sin A$

# Worksheet 19

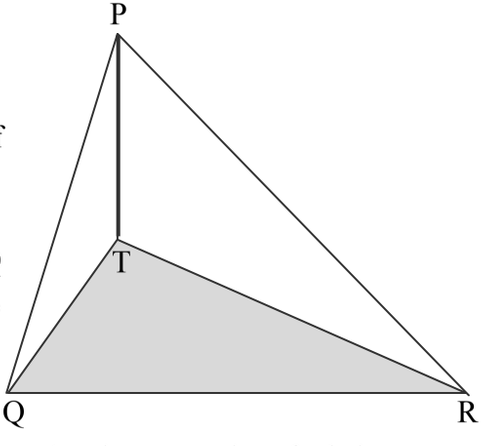
1. If in ΔABC, c = 8 units, b = 7 units and $\hat{B} = 60°$, calculate the following without using a calculator:

   As in ΔABC , c = 8 eenhede, b = 7 eenhede en $\hat{B} = 60°$, bereken die volgende sonder die gebruik van die sakrekenaar:

   a) the value of a / die waarde van a.

   b) the area of ΔABC for the greatest value of a.
   die oppervlakte van ΔABC vir die grootste waarde van a.

2. In the figure Q, T and R are points in the same horizontal plane such that TQ = TR = y and TP represents a vertical pole positioned at T.
   In die figuur is Q, T en R punte in dieselfde horisontale vlak sodat TQ = TR = y en TP stel 'n vertikale paal voor.




   a) Prove that / Bewys dat:
   PQ = PR.

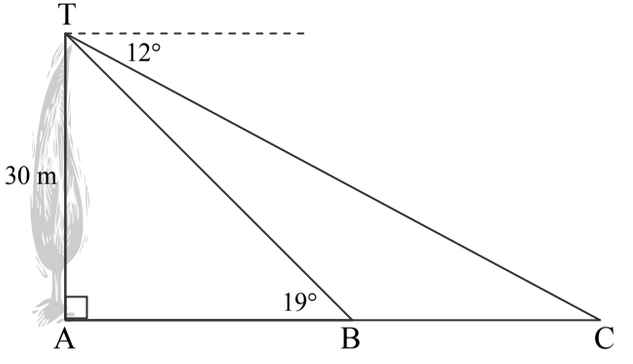
   b) If the angle of elevation of P from Q is α and $P\hat{R}Q = \beta$ / As die hoogtehoek van P vanaf Q gelyk is aan α en $P\hat{R}Q = \beta$

   i) express PQ in terms of y and a trigonometric value of α / druk PQ uit in terme van y en 'n trigonometriese funksie van α.

   ii) Prove that / Bewys dat: $QR = \dfrac{y \sin 2\beta}{\sin \beta . \cos \alpha}$

3. The diagram below is a representation of a tree (TA) with height of 30 m and two observers K and L on the ground. The angle of depression from T to person C is 12°. The angle of elevation from person B to the top of the tree is 19°.

   Die diagram hieronder is 'n voorstelling van 'n 30 m hoë boom (TA) en twee persone op die grond. Die dieptehoek van T na persoon C is 12°. Die hoogtehoek van B na die top van die boom 19°.




   a) Calculate the size of $\hat{C}$ / Bereken die grootte van $\hat{C}$.

   b) Calculate the length of TB / Bereken die lengte van TB.

   c) Hence calculate the distance between the two observers B and C / Bereken vervolgens die afstand tussen die twee persone B en C

# Transformations / Transformasies

**Notation:** $A(x; y) \rightarrow A'(x'; y')$ where A' is the image of A under the transformation
**Notasie:** $A(x; y) \rightarrow A'(x'; y')$ waar A' die beeld van A na die transformasie is.

## Translations (shift) / Translasie (skuif)

p units horizontally / p eenhede horisontaal:: $(x; y) \rightarrow (x+p; y)$

q units vertically / q eenhede vertikaal: $(x; y) \rightarrow (x; y+q)$

p units horizontally and q units vertically / p eenhede horisontaal & q eenhede vertikaal: $(x; y) \rightarrow (x+p; y+q)$

## Reflection / Refleksie

About the *x*-axis / Ten opsigte van *x*-as: $(x; y) \rightarrow (x; -y)$

About the *y*-axis / Ten opsigte van y-as: $(x; y) \rightarrow (-x; y)$

About the line $y = x$ /Ten opsigte van die lyn $y = x$: $(x; y) \rightarrow (y; x)$

## Rotation / Rotasie

Around the origin through an angle of 90° / 90° rotasie ten opsigte van die oorsprong: $(x; y) \rightarrow (-y; x)$

Around the origin through an angle of 180° / 180° rotasie ten opsigte van die oorsprong: $(x; y) \rightarrow (-x; -y)$

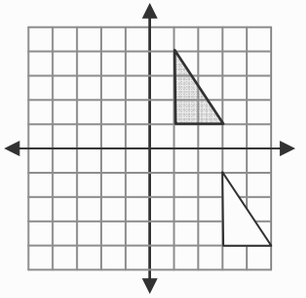
Around the origin through an angle $\beta$ / Rotasie van 'n hoek $\beta$ ten opsigte van die oorsprong:
$(x; y) \rightarrow (x\cos\beta - y\sin\beta; y\cos\beta + x\sin\beta)$

## Enlargement / Vergroting

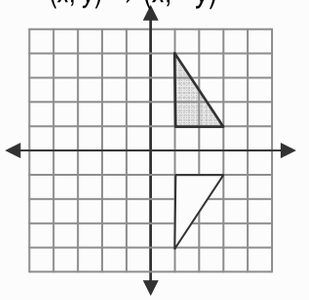
Through the origin, by a constant factor k / Vergroting van 'n konstante faktor k ten opsigte van die oorsprong: $(x; y) \rightarrow (kx; ky)$

# Examples of Transformations

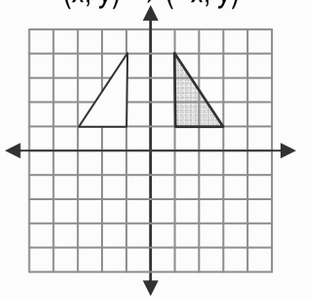
Translation
$(x; y) \rightarrow (x+2; y-5)$



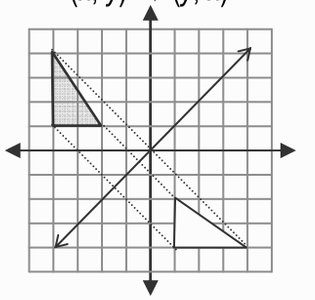
Reflection
$(x; y) \rightarrow (x; -y)$



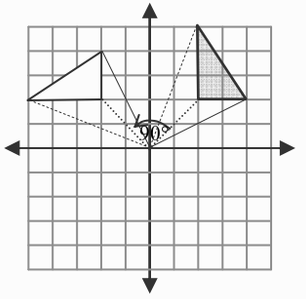
Reflection
$(x; y) \rightarrow (-x; y)$



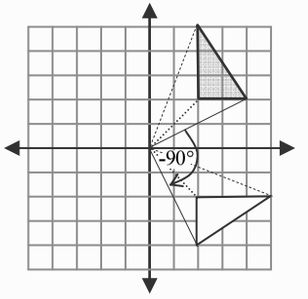
Reflection
$(x; y) \rightarrow (y; x)$



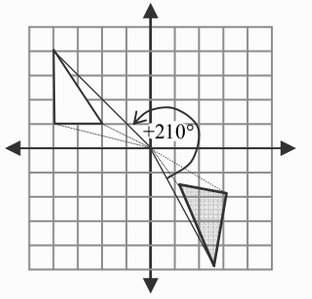
Rotation
$(x; y) \rightarrow (-y; x)$



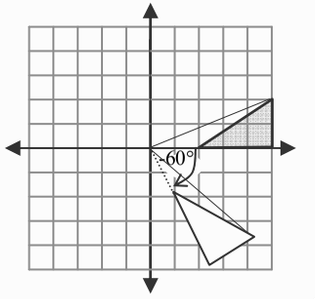

Rotation
$(x; y) \rightarrow (-x; y)$




Rotation
+210° about (0; 0)




Rotation
–60° about (0; 0)



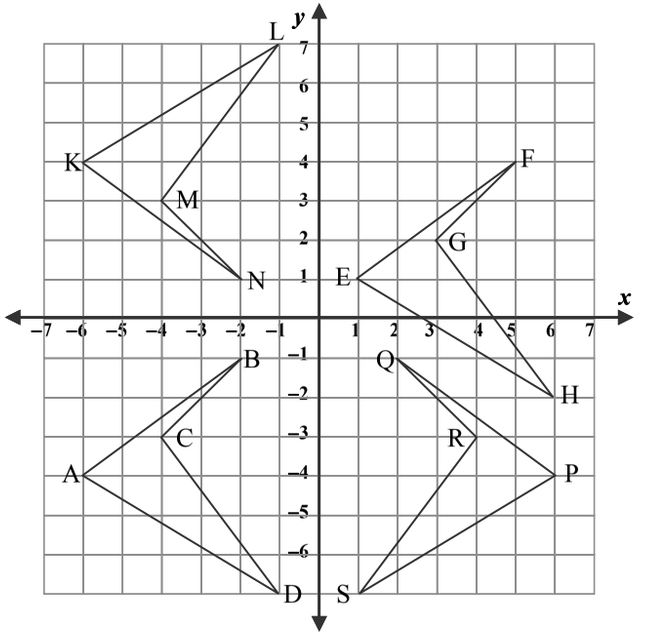
## Worksheet 20

1. B(3; 1), C(1; 3) and D(−1; 2) are the coordinates of the vertices of triangle BCD. BCD has to be enlarged with the origin as centre by a factor of 2.
   a) Give the coordinates of vertices of triangle B'C'D' of the enlargement.
   b) If the area of ΔBCD is $y$ square units, determine the area of the enlarge ΔB'C'D'.

   B(3; 2), C(2; –1) en D(1; 0) is die koördinate van die hoekpunte van ΔBCD. ΔBCD moet vergroot word met 'n faktor 2 met die oorsprong as middelpunt.
   a) Gee die koördinate van hoekpunte van ΔB'C'D' van die vergroting.
   b) Indien die area van ΔBCD $y$ vierkante eenhede is, bepaal die area van vergroting ΔB'C'D'.

2. The diagram shows quadrilateral ABCD and its transformations.
   a) State the general rule for the coordinates of any point representing the transformation of quadrilateral ABCD to quadrilateral KLMN.
   b) State the general rule for the coordinates of any point representing the transformation of quadrilateral ABCD to quadrilateral EFGH.
   c) Describe two possible transformations of quadrilateral ABCD to quadrilateral PQRS.
   d) Give the coordinates of the reflection of quadrilateral ABCD in the line $y = x$.

   Die diagram toon vierhoek ABCD asook die vierhoek se transformasies.
   a) Gee die algemene reël vir die koördinate van enige punt wat die transformasie van vierhoek ABCD na vierhoek KLMN voorstel.
   b) Gee die algemene reël vir die koördinate van enige punt wat die transformasie van vierhoek ABCD na vierhoek EFGH voorstel.
   c) Beskryf TWEE moontlike transformasies van vierhoek ABCD na vierhoek PQRS.
   d) Gee die koördinate van die refleksie van vierhoek ABCD in die lyn $y = x$.




3. A(−2; 6) is rotated 120° in a anti-clockwise direction through the origin as centre of the rotation. Calculate the coordinates of point A' after the rotation.

   A(−2; 6) word 120° in 'n antikloksgewys rigting met die oorsprong as middelpunt van die rotasie geroteer. Bereken die koördinate van A' na die rotasie.

## Worksheet 21

1. A translation maps the origin to the point (1; 6).
   a) What is the image of point (–5; –1) after the same translation?
   b) What is the pre-image of point (6; 2)?

   ‘n Translasie beeld die oorsprong op die punt (1; 6) af.
   a) Wat is die beeld van die punt (–5; –1) na die translasie T?
   b) Bepaal die voorafbeeld van die punt (6; 2)?

2. Give the coordinates of the vertices of quadrilateral with vertices P(1; 1), Q(–2; 0), R(3; –3) and S(6; –2) after a rotation with the origin as centre:
   a) 90° in a clockwise direction
   b) 180° in a clockwise direction
   c) 270° in a clockwise direction

   Gee die koördinate van die hoekpunte van ‘n vierhoek met hoekpunte P(1; 1), Q(–2; 0), R(3; –3) en S(6; –2) na ‘n rotasie, met die oorsprong as middelpunt:
   a) 90° in 'n kloksgewys rigting
   b) 180° in 'n kloksgewys rigting
   c) 270° in 'n kloksgewys rigting

3. State the general rule for the coordinates of a point after a rotation with the origin as centre of:
   a) 90° in a clockwise direction
   b) 180° in a clockwise direction
   c) 270° in a clockwise direction

   Gee die algemene reël wat die koördinate van 'n punt na ‘n rotasie, met die oorsprong as middelpunt:
   a) 90° in 'n kloksgewys rigting
   b) 180° in 'n kloksgewys rigting
   c) 270° in 'n kloksgewys rigting

# Data handling / Data Hantering

## Measures of Central Tendency / Bepalers van sentrale neiginge

- Mean (Average) / Gemiddeld
  - The sum of a set of data divided by the number of data: / Die som van die gegewe data gedeel deur die aantal data: $\bar{x} = \frac{\sum x}{n}$
- Median (middle value) / Mediaan (middelwaarde)
  - The value halfway through an ordered data set. / Die waarde halfpad in die geordende data versameling.
- Mode (most frequent value) / Modus
  - Value that appears the most. / Waarde wat die meeste voorkom.

## Measures of dispersion (variability ) / Maat van dataverspreiding

- Range (Largest value – smallest value) / Omvang (Grootste waarde – kleinste waarde)
- Percentiles (% of total frequency $\leq x$) / Persentiel (% van die totale frekwensie $\leq x$)
- Percentiles divide the data in 100 groups containing the same number of observations. / Persentiele deel die data in 100 groepe wat die selfde aantal observasies bevat.
- Variance / Variansie: $\frac{\sum (x-\bar{x})^2}{n}$
- Standard deviation / Standaard afwyking: $\sqrt{\frac{\sum (x-\bar{x})^2}{n}}$

# Worksheet 22

1. Given data / Gegewe data: 5, 9, 7, 8, 3, 10, 8, 10, 9, 17, 33
   Find the following for the data / Bepaal die volgende vir die data:
   a) mean / gemiddeld  b) median / mediaan
   c) mode / modus  d) range / omvang
   e) lower quartile / onderste kwartiel
   f) upper quartile / boonste kwartiel
   g) interquartile / inter-kwartiel
   h) standard deviation / standaard afwyking
   i) variance / variansie

2. Given data/Gegewe data: 5, 9, 7, 8, 3, 10, 8, 10, 9, 17, 33
   Represent the data using a box and whisker diagram / Stel die data voor deur van 'n mond-en-snordiagram gebruik te maak.

3. The frequency table represents the marks in terms of %, obtained by a group of Grade 12 learners in a Mathematics examination.
   Die frekwensietabel verteenwoordig die punte in terme van %, deur 'n groep Graad 12-leerders in 'n Wiskunde-eksamen behaal.

| Test scores/ Toetspunte | Frequency/ Frekwensie | Cumulative frequency/ Kumulatiewe frekwensie |
|---|---|---|
| 1-20 | 3 | |
| 21-40 | 5 | |
| 41-60 | 9 | |
| 61-80 | 6 | |
| 81-100 | 2 | |

   a) Find the following / Bepaal die volgende:
      i) lower quartile / onderste kwartiel
      ii) upper quartile / boonste kwartiel
      iii) interquartile / inter-kwartiel
      iv) semi-interquartile range / semi-inter-kwartiel
   b) Use the table on the diagram sheet to complete the cumulative frequency column / Gebruik die tabel op die diagramvel om die kumulatiewe frekwensie kolom te voltooi.
   c) Draw the ogive for the given data on a grid / Teken die ogief op ‘n rooster.
   d) Use the ogive to determine the median mark / Gebruik die ogief om die mediaanpunt te bepaal.

4. The table represents the number of cars sold by a car manufacturer from 2003 to 2008:
   Die onderstaande tabel stel die aantal motors wat verkoop is deur ‘n motorvervaardiger 2003 tot 2008, voor:

| Year / Jaar | Cars sold / Motors verkoop |
|---|---|
| 2002 | 1234 |
| 2003 | 1432 |
| 2004 | 1672 |
| 2005 | 1752 |
| 2006 | 2013 |
| 2007 | 2193 |
| 2008 | 2345 |

   a) Draw the scatter plot to represent the above data. / Teken 'n spreidiagram om bogenoemde inligting voor te stel.
   b) Explain whether a linear, quadratic or exponential curve would be a line of best fit for the above-mentioned data. / Verduidelik of 'n lineêre, kwadratiese of eksponensiële kurwe die beste paslyn vir die bogenoemde inligting sal wees.
   c) If the same trend continued, estimate, by using your graph, the number of cars that will be sold in 2010. / Indien dieselfde neiging/patroon voortgaan, skat, deur jou grafiek te gebruik, die aantal motors wat teen 2010 verkoop sal word.

# Solutions / Oplossings

# PAPER 1

**Worksheet 1**

1. a) $2x^2 - 3x$
   b) $-2x^2 - x$
   c) $-2x^2 + 4x^3$
   d) $-2x^2 + 4x^3$
   e) $-2x^2 - x$
   f) $-2x^2 + 4x^3$
   g) $8x^3 - 12x^2y + 6xy^2 - y^3$
   h) $x^3 + 8$
   i) $8x^3 - 27$
2. a) $(x + 4)(x - 3)$
   b) $(x - 4)(x + 3)$
   c) cannot factorise
   d) cannot factorise
   e) $(x - 4)(x - 3)$
   f) $(x + 4)(x + 3)$
   g) $(x - 6)(x + 1)$
   h) $7(x - 2)(x^2 + 2x + 4)$
   i) $2(3 - 2x + 2y)(3 + 2x - 2y)$
   j) $(2x - y)(2x - y - 4)$
   k) $(x - 2y)(2x + 4y - 1)$
   l) $(x - 3)(x - 4)(x + 4)$
   m) $(x - 2)(x - 5)(x + 3)$
   n) $(x + 1)(x - 2)^2$
3. a) $\dfrac{-3x^2}{5x(x-5)(x+5)}$
   b) $\dfrac{a^2 - ab + b^2}{a - b}$
   c) $\dfrac{-x-1}{x(x-2)}$
   d) $\dfrac{x^3 + 2x^2 + 1}{(x-1)(x+1)}$

**Worksheet 2**

1. a) False
   b) False
   c) False
2. a) $\dfrac{9}{4}$
   b) $-\dfrac{7}{16}$
   c) 27 ¼
   d) $\dfrac{85}{17}$
   e) $\dfrac{1}{16}$
   f) 64
3. a) $\sqrt{2}$
   b) $12\sqrt{2}$
   c) 4
   d) – ½
   e) 4
   f) $3\sqrt{2}$
   g) 3
4. a) $\dfrac{\sqrt{2}}{2}$
   b) $\dfrac{5\sqrt{3}}{-6}$

**Worksheet 3**

1. a) 1½ or –2
   b) –1½ or 4
   c) ±5
   d) 6 or –4
   e) ±4
   f) 4½ or $\frac{2}{3}$
   g) –8
   h) 3 or $10\frac{2}{3}$
   i) no solution
2. a) –3 or –4
   b) no real solution
   c) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   d) $x = \dfrac{1 \pm \sqrt{4m^2 + 8m + 5}}{2(m+1)}$
3. a) $x = \pm\sqrt{\frac{3}{2}}$
   b) 1,24 or –3,23
4. a) $(x - 3)(x^2 + 4)$
   b) $(x - 2)(2x - 1)(x + 3)$
   c) $(x - 2)(x + 7)(x + 1)$
   d) $(x + 3)(x - \sqrt{3})(x + \sqrt{3})$
5. a) $x = \frac{3}{4}$ or $x = 1$ or $x = -1$
   b) $x = 1$ or $x = -2$
   c) $x = 1$ or $x = 6$ or $x = -2$
6. a) no solution
   b) $-\frac{3}{2}$
7. $-1 \pm 2i$

**Worksheet 4**

1. a) 5
   b) 2,32
   c) 2,86
   d) 1,58
   e) 1,75
   f) 0,63
   g) 16
   h) 2
   i) 5
2. $n = \log_{(1+i)}\left(\frac{A}{P}\right)$
3. $n = \log_{(1+i)}\left(\frac{Fi}{x} + 1\right)$
4. (1; 0) or (5; 8)
5. a) (4; –2)
   b) (–1; 1) or $(\frac{11}{7}; \frac{13}{7})$
   c) $(\frac{2}{3}; \frac{3}{2})$ or $(\frac{1}{2}; 1)$
   d) (0,5; 1) or $(\frac{2}{3}; \frac{2}{3})$
6. a) $x < \frac{1}{2}$
   b) $x < -1$ or $x > 2$
   c) $\frac{1}{2} < x < \frac{5}{2}$
   d) $x \le -2$ or / of $x \ge 3$
   e) $-3 < x < 4$
   f) $-2 \le x \le 4$
   g) $-4 < x < 4$
   h) $x \le -3$ or / of $x \ge 3$

**Worksheet 5**

1. 2; 3; 5; 7; 11; 13; 17; …
   Product 1st; 2nd; 3rd; …
   = 2; 6; 30; 210; 2310; …
   ∴ Units digit is 0
2. a) $90(0{,}7)^5 = 15{,}126$ m
   b) 11 bounces
   c) 510 m
3. a) 29
   b) 1; 5; 11; 19; 29; …
      4 ; 6 ; 8 ; 10 ; …

2;  2 ; 2 ; …

$\therefore T_n = \left(\frac{T_1}{2}\right)n^2 + qn + r$

Substitute $T_1 = 1$ & $T_2 = 5$:

$\therefore T_n = 0{,}5n^2 + 2{,}5n - 2$

c) $T_{50} = 1373$

4. a) $6 \times 3$ pool: $2(6 + 2) + 2(3)$
$= 22$

$8 \times 5$ pool: $2(8 + 2) + 2(5) = 30$

b) $m \times n$ pool:

$2(m + 2) + 2(n) = 2(m + n) + 4$

c) $m \times n$ pool:

$2(m + m) + 4 = 4m + 4$

d) $49 \times 49$ pool

**Worksheet 6**

1. R27 228,80
2. 5,9 years
3. 9,62%
4. $P = F(1 + i)^{-n}$

$i = \sqrt[n]{\frac{F}{P}} - 1$

$n = \log_{(i+1)}\left(\frac{F}{P}\right)$

5. a) 12,68%
b) R34039,80
6. 29,3%

**Worksheet 7**

1. $50000 = \frac{x\left[1 - \left(1 + \frac{0{,}16}{12}\right)^{-50}\right]}{\frac{0{,}16}{12}}$

$x$ = R1376,51

2. $F = \frac{x\left[\left(1 + \frac{r}{k}\right)^{nk} - 1\right]}{\frac{r}{k}}$

3. $F = \frac{5000[(1{,}04)^{20} - 1]}{0{,}04}$

= R 148 890,39

4. a) $F = 2\,400\,000(1 + 0{,}16)^5$
= R 5 040 819,98

b) $F = 2\,400\,000(1 - 0{,}22)^5$
= R 6 486 499,59

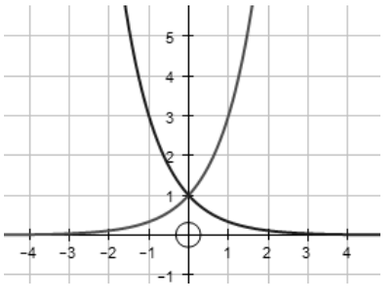
c)

$1445679{,}61 = \frac{x\left[\left(1 + \frac{0{,}16}{12}\right)^{60} - 1\right]}{\frac{0{,}16}{12}}$

$\therefore x$ = R15 880,39

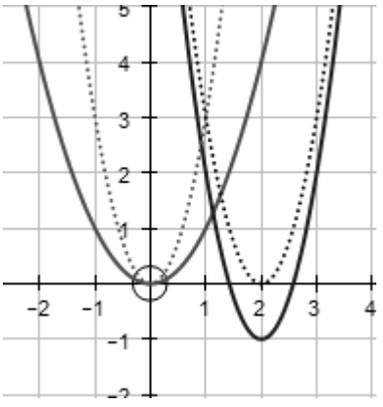
**Worksheet 8**

1. a) 2
b) 360°
c) $[-2; 2]$
d) 2
2. a) $-2$
b) $x = 90° + k.180°$
c) 180°
d) R
3. a) 11
b) no $x$-intercepts
c) 2
d) R
e) $[2; \infty)$
f) $x = 3$
4. a)



b) $x = 0$

5. a) & c)



b) 3 × vertical stretch

Translation 2 units to the right

Translation 1 unit downwards

d) R

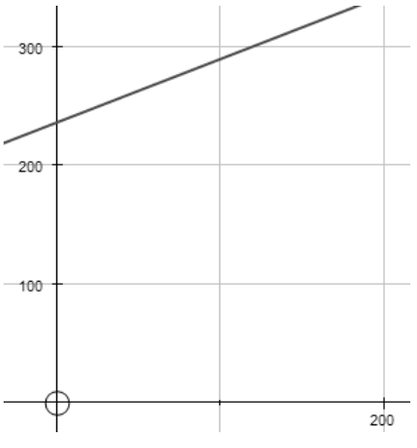
e) $[-1; \infty)$

**Worksheet 9**

1. a) $C(x) = \frac{8}{15}x + 236$

b) C(2000) = R1302,67

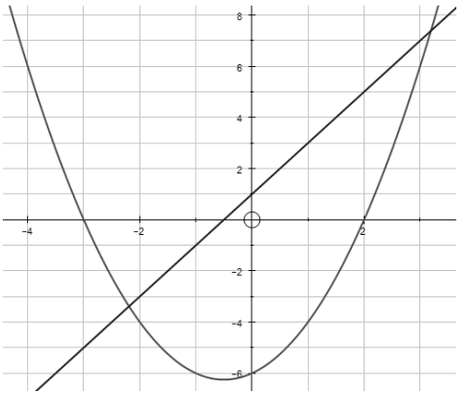
c)




d) The rate that the cost change with respect to the units used.

e) Basic cost

2.



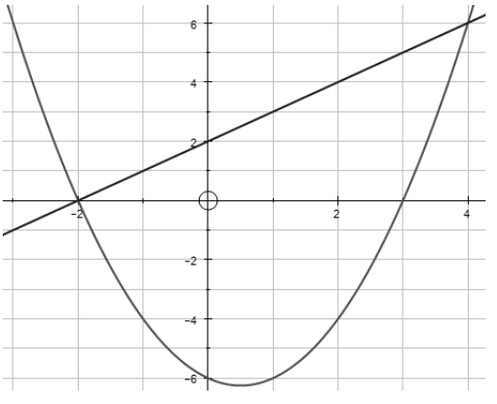
a) $x = -3$ or $x = 2$

b) $x \leq -3$ or $-½ < x \leq 2$

c) $-3 \leq x \leq -½$ or $x \geq 2$

d) $x = -½$

3. a)



b) $x^2 - 2x - 8 = 0$

$\Leftrightarrow x^2 - x - 6 = x + 2$

$\therefore (-2; 0)$ & $(4; 6)$

**Worksheet 10**

1. a) A(0; 9)

   B(1; 0)

   C(9; 0)

   b) OD = 5

   DE = 16

   AF = 10

   HI = 20

2. $f(x) = 2\sin(\frac{1}{2}x) + 2$

   $g(x) = 4\tan x$

**Worksheet 11**

1. m = 1
2. a) m = 1

   b) $\dfrac{-2}{(a+h-1)(a-1)}$
3. $-8x$
4. a) $f'(x) = 3x^2 - 2x + 2$

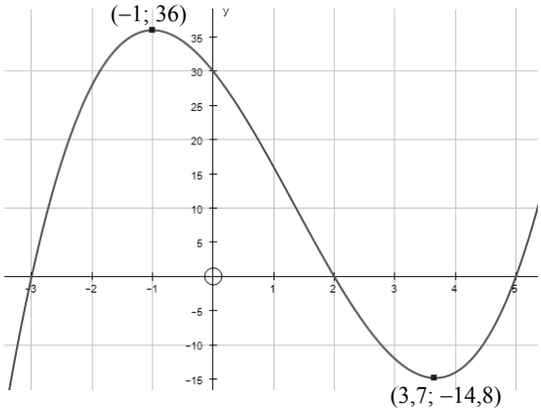
   b) $f'(x) = 2 + \dfrac{8}{x^3}$

   c) $f'(x) = \dfrac{-12}{x^4} + \dfrac{1}{2\sqrt{x}}$

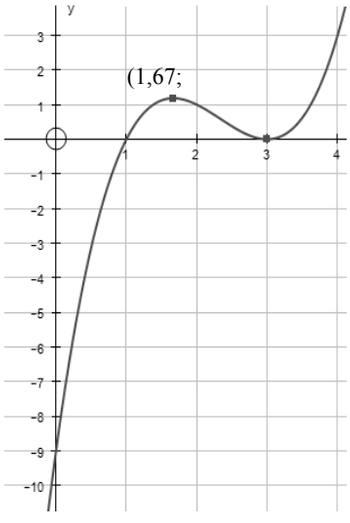
   d) $f'(x) = 9x^2 + 4x - 4$

   e) $f'(x) = \dfrac{1}{2\sqrt{x}} + 2x + 6$
5. a) $y = -2x - 3$ & $y = 2x - 7$

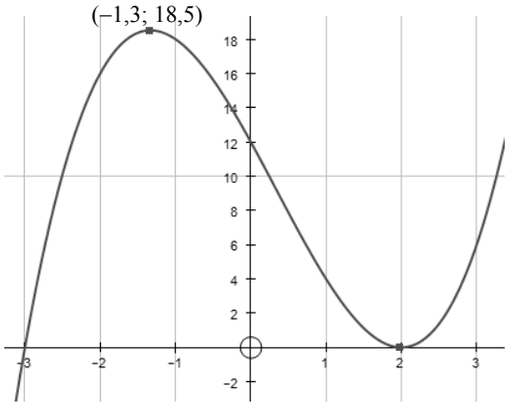
   b) $(1; -5)$
6. a)




b)




c)




7. Volume = $\pi r^2 h$ $\therefore h = \dfrac{350}{\pi r^2}$

   $A(r) = 2(\pi r^2) + 2\pi rh$

   $A'(r) = 4\pi r - \dfrac{700}{r}$

   $\therefore r = 3{,}81$ cm & $h = 7{,}67$ cm

**Worksheet 12**

1. a) B(0) = 1 200

   b) B'(t) = –40

   c) Decrease with 40 million per hour.

   d) Shrinking

   e) t = 6

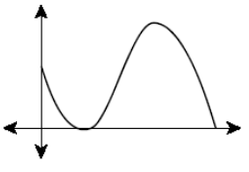
   f) 6 hours

2. a) $H(0) = 8$

   b) t = 2ºC or/of t = 20ºC

   c) t = 14ºC

   d)



3. a) $SR = (72 - 2x)$ m

   b) $A = 72x - 2x^2$

   c) $x = 18$ m

   d) $A = 648$ m²

**Worksheet 13**

1. a) $x \geq 10$

   $y \geq 0$

   $y \geq -2x + 60$

   $y \leq -x + 80$

   $y \geq \frac{1}{6}x + 10$

   b) $y = -\frac{2}{3}x + \frac{1}{3}P$

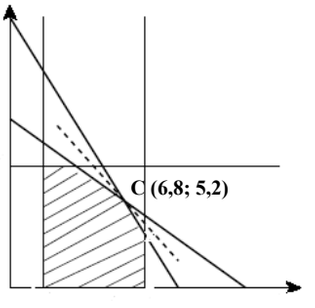
   $P_{max}$ at A(10; 70)

   P = 230 units

   c) $-2 < m < \frac{1}{6}$

   $10 < c < 60$

2. a)




   b) At the point C(6,8; 5,2)

   ∴ 6,8 ha maize & 5,2 ha sunflower

# PAPER 2

**Worksheet 14**

1. $\frac{1}{2}n^2 - \frac{3}{2}n$

2.

| 5 | 100 | n |
|---|---|---|
| 540° | 17640° | 180°(n – 2) |
| 108° | 176,4° | $\frac{180°(n-2)}{n}$ |

3. 150°

4 a) 441,1 cm$^2$

b) 412,6 cm$^2$

c) 414,7 cm$^2$

d) H = $\sqrt{91}$

$\pi r^2 + \pi rH$

$= \pi(3)^2 + \pi(3)\sqrt{91}$

$= 118{,}2$ cm$^2$

e) $(8)(6) + 2\left(\frac{1}{2}(6)(\sqrt{116})\right) + 2\left(\frac{1}{2}(8)(\sqrt{109})\right)$

$= 196{,}1$ cm$^2$

**Worksheet 15**

1. a) (–6; 4)

b) t = 6

2. a) $x^2 + y^2 = 40$

b) q = ± 2

c) θ = 26,6°

3. a) (1; –2) & r = $\sqrt{20}$

b) $m_{radius} = -2$

$m_{tangent} = ½$

$y + 6 = ½(x - 3)$

$y = \frac{1}{2}x - \frac{15}{2}$

4. $m_{AC} = -1$ & $m_{BC} = \frac{4}{9}$

Angle = 45° + 23,9° = 68,9°

5. a) $x^2 - 8x + y^2 - 4y - 12 = 0$

b) (0; 6); (0; –2)

6. $r = \frac{2}{3}$

**Worksheet 16**

1. $(x - 5)^2 + (y + \frac{1}{2})^2 = 25\frac{1}{4}$

M( 5; $-\frac{1}{2}$ ) & r = 5,025

2. a) $m_{AB} = \frac{2}{3}$

b) e = –3

c) $3x + 2y - 7 = 0$

3. a) $\sqrt{26} + \sqrt{26} + \sqrt{52} = 17{,}4$

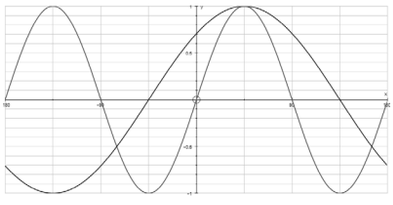
b) 11,3°

c) (–3,5; −0.5)

d) $y = 0{,}2x + 0{,}2$

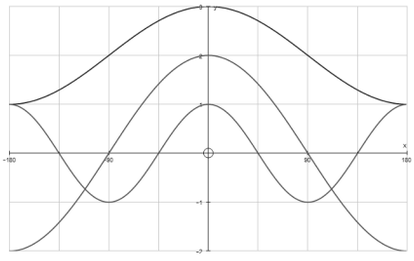
**Worksheet 17**

1. a)



b) $f(x)$: 180° & $g(x)$: 360°

2. a)



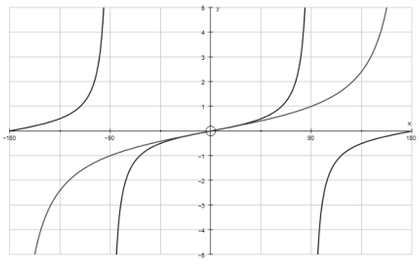
b) $x = -135^\circ ; -45^\circ ; 45^\circ ; 135^\circ$

c) $2 - \cos x = 2 + \cos x - 2\cos x$

$= (2 + \cos x) - (2\cos x)$

$x = -180^\circ; 180^\circ$

3. a)



b) $180^\circ$

c) $x = 0^\circ$

d) $-180^\circ < x < -90^\circ$ or $0^\circ < x < 90°$

4. a = 2 & k = –1

b = 1 & m = –1

**Worksheet 18**

1. a) 1

b) $-2\sqrt{3}$

c) 3

d) –2

e) –3

f) 2

2. a) t

b) t

c) $\frac{-t}{\sqrt{1-t^2}}$

3. a) 150°

b) $x \in R$

c) $x = 46{,}3°$

4. a) 12,3° ; 107,7°

b) 212,4° ; 7,6°

5. a) 555°

b) $\frac{-\sqrt{3}}{3}$

6. a) $x = k.180°$

$x = 30° + k.360°$

$x = 150° + k.360°$

$k \in Z$

b) $\theta = 60° + k.360°$

$\theta = 300° + k360°$

$k \in Z$

7. Prove LHS = RHS

8. $\frac{181}{65}$

**Worksheet 19**

1. a) a = 5 or a = 3

   b) Area = $10\sqrt{3}$

2. a) $\Delta PQT \equiv \Delta PRT$ (S∠S)

   $\therefore$ PQ = PR

   b) PQ = $\frac{y}{\cos\alpha}$; Use sine rule

3. a) 12°

   b) $\sin 19° = \frac{30}{TB}$

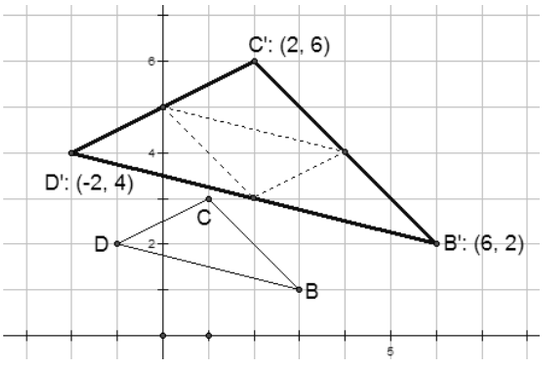
   TB = 9,77 m

   c) $\frac{BC}{\sin 7°} = \frac{9,77}{\sin 12°}$

   BC = 5,73 m

**Worksheet 20**

1.




   a) B'(6; 2); C'(2; 6); D'(−2; 4)

   b) 4 times $\Delta BCD = 4y$

2. a) $(x; y) \rightarrow (x; -y)$

   b) $(x; y) \rightarrow (x + 7; y + 5)$

   c) $(x; y) \rightarrow (-x; y)$

   d) A'(−4;−6); B'(−1; −2)

   C'(−3; −4); D'(−7; −1)

3. A'(6,2; −1,27)

**Worksheet 21**

1. a) (−4; 5)

   b) (5; −4)

2. a) P'(1; −1); Q'(0; 2)

   R'(−3; 3); S'(−2; −6)

   b) P'(−1; 1); Q'(2; 0)

   R'(−3; 3); S'(−6; 2)

   c) P'(−1; 1); Q'(0; −2)

   R'(3; 3); S'(2; 6)

3. a) $(x; y) \rightarrow (y; -x)$

   b) $(x; y) \rightarrow (-x; -y)$

   c) $(x; y) \rightarrow (-y; x)$

**Worksheet 22**

1. a) 10,82

   b) 9

   c) 9

   d) 30

   e) 7

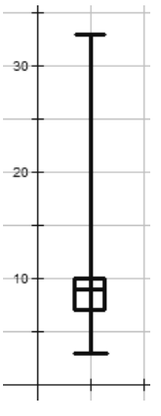
   f) 10

   g) 317

   h) 8,15

   i) 66,4

2.

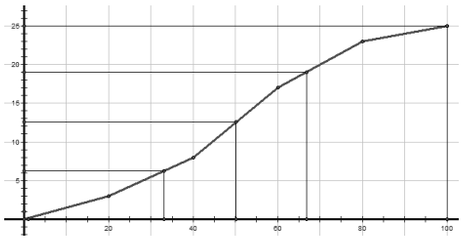


3. a) i) 33
ii) 67
iii) 34
iv) 17

b)

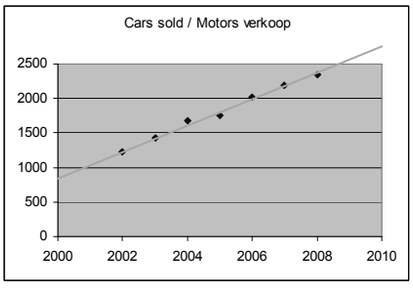
| | | |
|---|---|---|
| 1-20 | 3 | 3 |
| 21-40 | 5 | 8 |
| 41-60 | 9 | 17 |
| 61-80 | 6 | 23 |
| 81-100 | 2 | 25 |

c)



d) 50

4. a)




b) Linear

c) 3000