

# CHAPTER 10

## Data handling

In this chapter you will:

- Distinguish between discrete and continuous data
- Organise data into lists and into frequency tables
- Calculate the range, mode, median and mean of ungrouped data
- Group data in a frequency table
- Calculate the mode, median and mean of grouped data
- Calculate the lower quartile ( $Q_1$ ), upper quartile ( $Q_3$ ) and the Interquartile Range (IQR)
- Draw vertical bar graphs, horizontal bar graphs and compound bar graphs
- Draw pie charts
- Draw line graphs and broken line graphs
- Draw histograms of grouped data and frequency polygons

*This chapter covers material from Topic 4: Statistical and Probability Models*

### **SUBJECT OUTCOME 4.1:**

**Collect, organise and interpret univariate numerical data**

**Learning Outcome 1:** Calculate measures of central tendency, namely mean, median and mode, of grouped and ungrouped data

**Learning Outcome 2:** Calculate measures of dispersion including range, percentiles, quartiles, interquartile and semi-interquartile range

### **SUBJECT OUTCOME 4.2**

**Represent data effectively and appropriately**

**Learning Outcome 1:** Create and interpret bar and compound bar graphs

**Learning Outcome 2:** Create and interpret histograms of grouped data

**Learning Outcome 3:** Create and interpret frequency polygons

**Learning Outcome 4:** Create and interpret pie charts

**Learning Outcome 5:** Create and interpret line and broken line graphs

## § 4.1 TYPES OF DATA

- ✦ Data handling deals with:
  - a) the collection,
  - b) the organisation,
  - c) the presentation,
  - d) the analysis and
  - e) the interpretation of data.
- ✦ Data can be collected from questionnaires, interviews and experiments. Books, magazines, and the internet are also sources of data.
- ✦ Collected data, called **raw data**, has to be **organised** and **presented** in such a way that it makes sense and so that some deduction may be drawn from it.
- ✦ The data may be **discrete** or **continuous**.
  - With **discrete data**, each item of data is exact and distinct. This data is often a total of counted objects.
 

**Examples of discrete data** are:

    - a) Number of children (no fractions)
    - b) Shoe size (half sizes included, but no smaller fractions)
    - c) Number of objects
  - With **continuous data**, the data is measured to some degree of accuracy, e.g. to 1 decimal place. It is an approximation and not necessarily exact. This data is usually a measurement.
 

**Examples of continuous data** are

    - a) Age (measured to the nearest year or month)
    - b) Time (measured to the nearest hour, minute, second)
    - c) Length (measured to the nearest cm)

### Exercise 4.1

Classify each of the following examples of data as a discrete or continuous:

- 1) The number of goals scored in successive football matches .....
- 2) Shirt sizes of the men in an office .....
- 3) The height of each child in a class .....
- 4) The temperature of an oven in cake baking .....
- 5) The volume of water in each tank at a recycling centre .....
- 6) The number of children in each classroom in a school .....
- 7) The set of mathematics test marks obtained by a class .....
- 8) The heights of the mealies in a field .....
- 9) The number of sweets in various boxes .....
- 10) The mass of each of the players in a rugby team .....

## § 4.2 ORGANISING DATA

- ✦ If data is not organised, no pattern or trend can be seen. One way to organise data is to arrange it in descending or ascending order.

### EXAMPLE

Listed below are the results (raw data) of an experiment in which a dice is thrown 40 times and the number on the upper face is recorded after each throw.

2 3 1 5 4 2 3 5 2 1 6 2 4 5 2 3 5 2 2 5  
5 6 5 4 2 2 5 2 6 3 4 5 5 2 1 3 2 6 5 4

Arrange this data in descending order.

### SOLUTION

6 6 6 6 5 5 5 5 5 5 5 5 5 5 5 4 4 4 4 4  
3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1

- ✦ Organising the data in an array still does not give a clear picture. A **frequency table** shows better organisation.

### EXAMPLE

- 1) Organise the above data (the dice being thrown 40 times) using a frequency table.
- 2) What conclusion can be drawn from the data?
- 3) Which of the two forms of organisation presents the data more clearly?

### SOLUTION

1)

| Number | Tally        | Frequency |
|--------|--------------|-----------|
| 6      |              | 4         |
| 5      | <br>         | 11        |
| 4      |              | 5         |
| 3      |              | 5         |
| 2      | <br>    <br> | 12        |
| 1      |              | 3         |
| TOTAL  |              | 40        |

- 2) Conclusion: There are more 2's and 5's than of any other number. A fair dice would show about the same frequency for each number, especially with a greater number of throws. The dice is possibly biased (weighted).
- 3) The frequency table presents the data more clearly than a list.

#### Note:

- By adding the frequencies we can check that all the data is entered on the table

### Exercise 4.2

A dice is thrown 20 times and the number is recorded after each throw. Write the data in ascending order and also in a frequency table.

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 3 | 1 | 6 | 4 | 6 | 1 | 1 | 2 | 1 | 6 | 2 | 4 | 5 | 6 | 3 | 6 | 1 | 6 | 1 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

| Face      | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|-----------|---|---|---|---|---|---|-------|
| Frequency |   |   |   |   |   |   |       |

### § 4.3 THE RANGE AND THE MODE

- ✦ The **range** is a measure of spread. It is the difference in value between the highest and the lowest values. **Range = highest value – lowest value.**
- ✦ The **mode** is the value that occurs most often amongst all the data items. It is the value having the greatest frequency.

| EXAMPLES                                                                                                                                                                                                                                                            | SOLUTIONS                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |             |   |   |   |   |                  |   |   |   |   |   |                                                                                                                                                                                                                                                                        |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|---|---|---|---|------------------|---|---|---|---|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1) Find the <b>range</b> and the <b>mode</b> of the following raw data:<br>2, 3, 1, 2, 4, 4, 1, 3, 2, 4, 2, 2, 1                                                                                                                                                    | We can arrange the data in ascending order:<br>1, 1, 1, <u>2</u> , <u>2</u> , <u>2</u> , <u>2</u> , 3, 3, 4, 4, 4<br><b>OR</b> we can use a frequency table: <table><tr><td><b>data</b></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td><b>frequency</b></td><td>3</td><td>4</td><td>2</td><td>3</td></tr></table><br>Range = highest value – lowest value<br>= 4 – 1 = 3<br>The data item occurring most often (having the greatest frequency) is 2, so the <b>Mode = 2</b> . | <b>data</b> | 1 | 2 | 3 | 4 | <b>frequency</b> | 3 | 4 | 2 | 3 |   |                                                                                                                                                                                                                                                                        |
| <b>data</b>                                                                                                                                                                                                                                                         | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2           | 3 | 4 |   |   |                  |   |   |   |   |   |                                                                                                                                                                                                                                                                        |
| <b>frequency</b>                                                                                                                                                                                                                                                    | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 4           | 2 | 3 |   |   |                  |   |   |   |   |   |                                                                                                                                                                                                                                                                        |
| 2) Find the <b>range</b> and the <b>mode</b> of the following data:<br>3, 4, 6, 4, 1, 5, 6, 1, 3, 5                                                                                                                                                                 | Arrange the data in ascending order:<br>1, 1, 3, 3, 4, 4, 5, 5, 6, 6,<br>Range = highest value – lowest value<br>= 6 – 1 = 5<br>There is <b>no mode</b> , since no one data item occurred more frequently than any of the others.                                                                                                                                                                                                                                                       |             |   |   |   |   |                  |   |   |   |   |   |                                                                                                                                                                                                                                                                        |
| 3) Find the <b>range</b> and the <b>mode</b> of the data on the frequency table: <table><tr><td><b>data</b></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td><b>frequency</b></td><td>2</td><td>5</td><td>1</td><td>5</td><td>4</td></tr></table> | <b>data</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 1           | 2 | 3 | 4 | 5 | <b>frequency</b> | 2 | 5 | 1 | 5 | 4 | Range = highest data value – lowest data value<br>= 5 – 1 = 4<br>The outcomes 2 and 4 each have the greatest frequency of <b>5</b> , so there are <b>two</b> modes.<br>We say that the set of data is <b>bimodal</b> .<br>The <b>Modes</b> are <b>2</b> and <b>4</b> . |
| <b>data</b>                                                                                                                                                                                                                                                         | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2           | 3 | 4 | 5 |   |                  |   |   |   |   |   |                                                                                                                                                                                                                                                                        |
| <b>frequency</b>                                                                                                                                                                                                                                                    | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 5           | 1 | 5 | 4 |   |                  |   |   |   |   |   |                                                                                                                                                                                                                                                                        |

#### Note:

- It is not the frequency that is the mode, it is the data item

#### Exercise 4.3

1) Find the range and the mode of the following set of raw data: 4, 2, 6, 3, 3, 2, 4, 3, 7, 2, 7, 6, 6, 7, 6

2) Use the frequency table to find the range and the mode of the data:

|                  |          |          |          |          |          |          |
|------------------|----------|----------|----------|----------|----------|----------|
| <b>data</b>      | <b>1</b> | <b>2</b> | <b>3</b> | <b>4</b> | <b>5</b> | <b>6</b> |
| <b>frequency</b> | <b>3</b> | <b>2</b> | <b>4</b> | <b>3</b> | <b>1</b> | <b>2</b> |

3) Organise the following raw data in a frequency table and then find the mode:

A, B, B, D, C, E, B, D, C, C, A, B, A, C, D, D, D, E, A, D, B, A, B

## § 4.4 THE MEDIAN

- ✦ The **median** is the value of the middle item in a distribution once all the data has been arranged in order of size.
- ✦ The median does not have to be one of the data items.

| EXAMPLES                                                                                                                                                                                                                                                                                                     | SOLUTIONS                                                                                                                                                                                                                                                                               |          |          |          |          |              |              |           |   |   |   |   |   |    |                                                                                                                                                                                                                                                                                                        |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|----------|----------|----------|--------------|--------------|-----------|---|---|---|---|---|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1) Find the median of the following set of data: 6, 3, 4, 7, 2, 5, 3, 4, 5                                                                                                                                                                                                                                   | Arrange the data in ascending order:<br>2, 3, 3, 4, <b>4</b> , 5, 5, 6, 7<br>There are 9 data items, an <b>odd</b> number.<br>The <b>5<sup>th</sup> data item</b> , the middle item, is the <b>median = 4</b>                                                                           |          |          |          |          |              |              |           |   |   |   |   |   |    |                                                                                                                                                                                                                                                                                                        |
| 2) Find the median of the following set of data: 5, 9, 1, 7, 2, 6, 9, 3.                                                                                                                                                                                                                                     | Arrange the data in ascending order:<br>1, 2, 3, <b>5</b> , <b>6</b> , 7, 9, 9<br>There are 8 data items, an <b>even</b> number.<br>The middle item is the <b>average of the 4<sup>th</sup> and 5<sup>th</sup> terms</b><br>So the <b>median</b> = $\frac{5+6}{2} = \frac{11}{2} = 5,5$ |          |          |          |          |              |              |           |   |   |   |   |   |    |                                                                                                                                                                                                                                                                                                        |
| 3) Find the median of the data on the frequency table:<br><table border="1"><tr><td>Data</td><td><b>1</b></td><td><b>2</b></td><td><b>3</b></td><td><b>4</b></td><td><b>5</b></td><td><b>Total</b></td></tr><tr><td>Frequency</td><td>2</td><td>4</td><td>1</td><td>3</td><td>2</td><td>12</td></tr></table> | Data                                                                                                                                                                                                                                                                                    | <b>1</b> | <b>2</b> | <b>3</b> | <b>4</b> | <b>5</b>     | <b>Total</b> | Frequency | 2 | 4 | 1 | 3 | 2 | 12 | We can write the data items in a list:<br>1, 1, 2, 2, 2, <b>2</b> , <b>3</b> , 4, 4, 4, 5, 5.<br>There are 12 data items, an <b>even</b> number.<br>The middle item is the <b>average of the 6<sup>th</sup> and 7<sup>th</sup> terms</b><br>So the <b>median</b> = $\frac{2+3}{2} = \frac{5}{2} = 2,5$ |
| Data                                                                                                                                                                                                                                                                                                         | <b>1</b>                                                                                                                                                                                                                                                                                | <b>2</b> | <b>3</b> | <b>4</b> | <b>5</b> | <b>Total</b> |              |           |   |   |   |   |   |    |                                                                                                                                                                                                                                                                                                        |
| Frequency                                                                                                                                                                                                                                                                                                    | 2                                                                                                                                                                                                                                                                                       | 4        | 1        | 3        | 2        | 12           |              |           |   |   |   |   |   |    |                                                                                                                                                                                                                                                                                                        |

### Exercise 4.4

- 1) Find the median of the raw data: 3, 6, 1, 0, 2, 5, 2, 3, 1

Write the data in numerical order: .....

Number of data items = .....

Median = .....

- 2) Find the median of the raw data: 9, 2, 1, 9, 7, 4, 2, 3

Write the data in numerical order: .....

Number of data items = .....

Median = .....

- 3) Find the median of the data in the frequency table :

| DATA         | FREQUENCY |
|--------------|-----------|
| 0            | 2         |
| 1            | 2         |
| 3            | 2         |
| 4            | 5         |
| 5            | 3         |
| <b>TOTAL</b> |           |

List of data items: .....

Number of data items = .....

Median = .....

## § 4.5 THE MEAN

- ✦ The **mean** is found by dividing the sum of all the data by the number of data items.
- ✦ It is called the **equal shares** average.
- ✦ When finding the mean, it is not necessary to first arrange the data items in order.

| EXAMPLES                                                                                     | SOLUTIONS                                                                                                                                                                                                             |
|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1) Find the mean of the data:<br>2, 3, 4, 5, 6, 7, 8                                         | Mean = $\bar{x} = \frac{2+3+4+5+6+7+8}{7} = \frac{35}{7} = 5$                                                                                                                                                         |
| 2) Find the value of $x$ so that the mean of the given data: 8, 3, 1, $x$ , 4, 6, 2, 6, is 4 | Mean = $\bar{x} = \frac{8+3+1+x+4+6+2+6}{8} = 4$<br>$\frac{30+x}{8} = 4$<br>$\frac{30+x}{8} \times 8 = 4 \times 8$<br>$30+x = 32$<br>$x = 2$                                                                          |
| 3) Find the mean, correct to 1 decimal place, of the data in the following frequency table:  | Raw data: 2, 2, 2, 2, 3, 4, 4, 5, 5<br>$\bar{x} = \frac{2+2+2+2+3+4+4+5+5}{9} = \frac{29}{9} = 3,2$<br><b>OR</b> $\bar{x} = \frac{(4 \times 2) + (1 \times 3) + (2 \times 4) + (2 \times 5)}{9} = \frac{29}{9} = 3,2$ |

### Exercise 4.5

- 1) Calculate the mean of the data: 7, 3, 5, 2, 9, 3, 1, 2

- 2) Find the mean, to 1 decimal place, of the data whose frequency table is given.

|           |   |   |   |   |   |       |
|-----------|---|---|---|---|---|-------|
| data      | 1 | 2 | 3 | 4 | 5 | Total |
| frequency | 3 | 1 | 4 | 2 | 4 |       |

List of raw data: .....

Mean =  $\bar{x} =$

- 3) The following data items are given: 7, 3, 1,  $x$ , 6. The mean of the data is 5. Find the value of  $x$ .

- 4) Find the range, mode, median and mean of the following set of data correct to 1 decimal place:  
32; 32; 33; 35; 37; 40; 40; 42; 46; 54; 57; 57; 57; 57; 61; 62; 62; 63; 65; 66; 70

## § 4.6 GROUPING DATA

✦ When data is being collected, it is sometimes convenient to record it in groups or **class intervals**.

### EXAMPLE

The test marks, out of 40, for of a class of 20 students are recorded below.

30 22 18 17 28 40 39 13 6 19  
39 12 16 37 29 9 17 20 1 28

Collect the data items on a grouped frequency table.

### SOLUTION

- ✦ Presenting these marks on a frequency distribution table would need 17 rows.
- ✦ We can group the marks in equal intervals for fewer rows and a more compact presentation.

| Interval     | Tally           | Frequency |
|--------------|-----------------|-----------|
| 1 – 10       |                 | 3         |
| 11 – 20      | <del>    </del> | 8         |
| 21 – 30      | <del>    </del> | 5         |
| 31 – 40      |                 | 4         |
| <b>TOTAL</b> |                 | 20        |

#### Note:

By adding the frequencies we can check that all 20 pieces of data are on the table.

### Exercise 4.6

The number of runs scored by the batsmen in the 18 cricket matches played by a team are:

35 0 12 34 21 6 15 39 44 7 19 33 34 40 29 14 22 39

- 1) What is the highest score? .....
- 2) What is the lowest score? .....
- 3) Use a suitable grouping of this data to complete the frequency distribution table below.

| Interval     | Tally | Frequency |
|--------------|-------|-----------|
| 0 – 9        |       |           |
|              |       |           |
|              |       |           |
|              |       |           |
|              |       |           |
| <b>TOTAL</b> |       |           |

## § 4.7 FINDING THE MODE AND MEDIAN OF GROUPED DATA

- ✦ The interval with the largest frequency is called the **modal class** or **modal group** or **modal interval**.
- ✦ With a grouped frequency table it is not possible to find a single value that is the median. We can, however, find the interval in which the median lies.

| EXAMPLE                                                                                                                                                                                                                                                                                                                                                                                                                         | SOLUTION      |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|-----------|--------|---|---------|---|---------|---|---------|---|--------------|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Find the mode and the median of the following set of data:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Mark interval</th><th>Frequency</th></tr> </thead> <tbody> <tr> <td>1 – 10</td><td>3</td></tr> <tr> <td>11 – 20</td><td>8</td></tr> <tr> <td>21 – 30</td><td>5</td></tr> <tr> <td>31 – 40</td><td>4</td></tr> <tr> <td><b>TOTAL</b></td><td>20</td></tr> </tbody> </table> | Mark interval | Frequency | 1 – 10 | 3 | 11 – 20 | 8 | 21 – 30 | 5 | 31 – 40 | 4 | <b>TOTAL</b> | 20 | <p>The <b>modal class</b> or <b>modal group</b> or <b>modal interval</b> is a score of 11 – 20 marks as this is the interval with the greatest frequency.</p> <p>There are 20 data values.<br/>Half of 20 is 10.<br/>The median lies between the 10<sup>th</sup> and the 11<sup>th</sup> values.<br/>Add up the frequencies either from the top or from the bottom. Both the 10<sup>th</sup> and the 11<sup>th</sup> values lie in the interval 11 – 20.<br/><b>The median lies in the interval 11 – 20.</b></p> |
| Mark interval                                                                                                                                                                                                                                                                                                                                                                                                                   | Frequency     |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 1 – 10                                                                                                                                                                                                                                                                                                                                                                                                                          | 3             |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 11 – 20                                                                                                                                                                                                                                                                                                                                                                                                                         | 8             |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 21 – 30                                                                                                                                                                                                                                                                                                                                                                                                                         | 5             |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 31 – 40                                                                                                                                                                                                                                                                                                                                                                                                                         | 4             |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <b>TOTAL</b>                                                                                                                                                                                                                                                                                                                                                                                                                    | 20            |           |        |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |

### Exercise 4.7

For each of the following, find

- a) the modal class
- b) the interval in which the median lies

1)

| Class interval | Frequency |
|----------------|-----------|
| 0 – 4          | 2         |
| 5 – 9          | 5         |
| 10 – 14        | 6         |
| <b>TOTAL</b>   |           |

- a) .....
- b) .....

2)

| Age group    | Number of people |
|--------------|------------------|
| 0 – 9        | 6                |
| 10 – 19      | 7                |
| 20 – 29      | 10               |
| 30 – 39      | 3                |
| 40 – 49      | 1                |
| <b>TOTAL</b> |                  |

- a) .....
- b) .....



## § 4.8 FINDING THE MEAN OF GROUPED DATA

✦ Since you do not know the exact data when data is grouped, you can only find an **estimate** of the mean

| EXAMPLE                                                                                                                                                                                                                                                                                                                                                               |                                       | SOLUTION       |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------|----------------|-----------|-------|---|-------|---|---------|---|---------|---|---------|---|--------------|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|----------------|-----------|-----------|-------|-----------------------------------|---|-------|------------------------------------|---|---------|---------------------------------------|---|---------|---------------------------------------|---|---------|---------------------------------------|---|--------------|--|----|
| <p>Find the mean (correct to 2 decimal places) of the following set of data</p> <table><tr><th>Class interval</th><th>Frequency</th></tr><tr><td>0 – 4</td><td>2</td></tr><tr><td>5 – 9</td><td>5</td></tr><tr><td>10 – 14</td><td>6</td></tr><tr><td>15 – 19</td><td>5</td></tr><tr><td>20 – 24</td><td>6</td></tr><tr><td><b>TOTAL</b></td><td>24</td></tr></table> |                                       | Class interval | Frequency | 0 – 4 | 2 | 5 – 9 | 5 | 10 – 14 | 6 | 15 – 19 | 5 | 20 – 24 | 6 | <b>TOTAL</b> | 24 | <p><u>STEP 1:</u> Find the midpoint of the class interval:<br/><i>Mid-point of the class interval</i> = <math>\frac{1}{2}</math> (smallest value + greatest value)</p> <table><tr><th>Class interval</th><th>mid-point</th><th>Frequency</th></tr><tr><td>0 – 4</td><td><math>\frac{0+4}{2} = \frac{4}{2} = 2</math></td><td>2</td></tr><tr><td>5 – 9</td><td><math>\frac{5+9}{2} = \frac{14}{2} = 7</math></td><td>5</td></tr><tr><td>10 – 14</td><td><math>\frac{10+14}{2} = \frac{24}{2} = 12</math></td><td>6</td></tr><tr><td>15 – 19</td><td><math>\frac{15+19}{2} = \frac{34}{2} = 17</math></td><td>5</td></tr><tr><td>20 – 24</td><td><math>\frac{20+24}{2} = \frac{44}{2} = 22</math></td><td>6</td></tr><tr><td colspan="2"><b>TOTAL</b></td><td>24</td></tr></table> |  |  | Class interval | mid-point | Frequency | 0 – 4 | $\frac{0+4}{2} = \frac{4}{2} = 2$ | 2 | 5 – 9 | $\frac{5+9}{2} = \frac{14}{2} = 7$ | 5 | 10 – 14 | $\frac{10+14}{2} = \frac{24}{2} = 12$ | 6 | 15 – 19 | $\frac{15+19}{2} = \frac{34}{2} = 17$ | 5 | 20 – 24 | $\frac{20+24}{2} = \frac{44}{2} = 22$ | 6 | <b>TOTAL</b> |  | 24 |
|                                                                                                                                                                                                                                                                                                                                                                       |                                       | Class interval | Frequency |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
|                                                                                                                                                                                                                                                                                                                                                                       |                                       | 0 – 4          | 2         |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
|                                                                                                                                                                                                                                                                                                                                                                       |                                       | 5 – 9          | 5         |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
|                                                                                                                                                                                                                                                                                                                                                                       |                                       | 10 – 14        | 6         |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
|                                                                                                                                                                                                                                                                                                                                                                       |                                       | 15 – 19        | 5         |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
|                                                                                                                                                                                                                                                                                                                                                                       |                                       | 20 – 24        | 6         |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| <b>TOTAL</b>                                                                                                                                                                                                                                                                                                                                                          | 24                                    |                |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| Class interval                                                                                                                                                                                                                                                                                                                                                        | mid-point                             | Frequency      |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| 0 – 4                                                                                                                                                                                                                                                                                                                                                                 | $\frac{0+4}{2} = \frac{4}{2} = 2$     | 2              |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| 5 – 9                                                                                                                                                                                                                                                                                                                                                                 | $\frac{5+9}{2} = \frac{14}{2} = 7$    | 5              |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| 10 – 14                                                                                                                                                                                                                                                                                                                                                               | $\frac{10+14}{2} = \frac{24}{2} = 12$ | 6              |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| 15 – 19                                                                                                                                                                                                                                                                                                                                                               | $\frac{15+19}{2} = \frac{34}{2} = 17$ | 5              |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| 20 – 24                                                                                                                                                                                                                                                                                                                                                               | $\frac{20+24}{2} = \frac{44}{2} = 22$ | 6              |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| <b>TOTAL</b>                                                                                                                                                                                                                                                                                                                                                          |                                       | 24             |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |
| <p><u>STEP 2:</u> For each interval multiply together the <b>frequency</b> and the <b>midpoint</b></p> <p>Estimate of <math>\bar{x}</math></p> $= \frac{(2 \times 2) + (5 \times 7) + (6 \times 12) + (5 \times 17) + (6 \times 22)}{2 + 5 + 6 + 5 + 6}$ $= \frac{4 + 35 + 72 + 85 + 132}{24} = \frac{328}{24} = 13,67$                                               |                                       |                |           |       |   |       |   |         |   |         |   |         |   |              |    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |  |                |           |           |       |                                   |   |       |                                    |   |         |                                       |   |         |                                       |   |         |                                       |   |              |  |    |

### Exercise 4.8

The following table of grouped data gives the ages (in years) of a number of people:

| Age group in years | Mid point of age group | Number of people |
|--------------------|------------------------|------------------|
| 0 – 9              |                        | 5                |
| 10 – 19            |                        | 4                |
| 20 – 29            |                        | 10               |
| 30 – 39            |                        | 5                |
| 40 – 49            |                        | 3                |
| <b>TOTAL</b>       |                        |                  |

- 1) Complete the table
- 2) Use the table to find:
  - a) The modal class .....
  - b) The median class .....
  - c) The estimate of the mean, correct to the nearest whole year  
 .....  
 .....

## § 4.9 FINDING QUARTILES

- ✦ In many experimental situations, data extremes are often discarded as inaccurate measurements. The middle section of the data set, where most of the data lies, gives the best picture of the group they represent.
- ✦ The **median** is the value that divides the data into two halves.
- ✦ **Quartiles** are the values that divide the data into four quarters, each quarter containing the same number of data items
  - The **median** is the middle quartile.
  - The **lower quartile ( $Q_1$ )** is the median of the lower half of the values.
  - The **upper quartile ( $Q_3$ )** is the median of the upper half of the values.

| EXAMPLE                                                                                       | SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-----------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| For the following set of data<br>7 1 3 9 8 4 4 3 2 7 9<br>Find                                | First write the data in ascending order:<br>1 2 <span style="border: 1px solid black; padding: 0 2px;">3</span> 3 4 <span style="border: 1px solid black; padding: 0 2px;">4</span> 7 7 <span style="border: 1px solid black; padding: 0 2px;">8</span> 9 9                                                                                                                                                                                                                                                                                                                                                                                            |
| 1) The range<br>2) The median<br>3) The lower quartile, $Q_1$<br>4) The upper quartile, $Q_3$ | <div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <span><math>Q_1</math></span> <span>Median</span> <span><math>Q_3</math></span> </div> 1) <b>Range</b> = $9 - 1 = 8$<br>2) There are 11 data items, an <b>odd number</b> .<br>The median is the 6 <sup>th</sup> data item.<br>The <b>median = 4</b> , since there are 5 data items below it and 5 above.<br>3) <b><math>Q_1 = 3</math></b> is the median of the first 5 data items, since there are 2 data items on either side of it.<br>4) <b><math>Q_3 = 8</math></b> is the median of the upper 5 data values, since there are 2 data items on either side of it. |

### Exercise 4.9

For the following sets of data:

- 1) a) Arrange the following set of data in ascending order:

4; 2; 3; 4; 1; 5; 5; 1; 6; 6; 1; 4; 3; 4; 2

.....

- b) Find

i) The range .....

ii) The median .....

iii) The lower quartile  $Q_1$  .....

iv) The upper quartile  $Q_3$  .....

- 2) a) Arrange the following set of data in ascending order:

13; 14; 3; 6; 23; 78; 20; 22; 21; 35; 62; 93; 4; 67; 54; 4; 21; 30; 44; 54

.....

- b) Find

i) The range .....

ii) The median .....

iii) The lower quartile  $Q_1$  .....

iv) The upper quartile  $Q_3$  .....

### § 4.10 THE INTERQUARTILE RANGE (IQR)

✦ The **interquartile range** (IQR) is a measure of spread of a set of statistical data. It is the difference between the upper and lower quartiles. We write:  $\text{IQR} = Q_3 - Q_1$

✦ The **semi-interquartile range** (Semi-IQR) is one half of the interquartile range.

We write:  $\text{Semi-IQR} = \frac{\text{IQR}}{2} = \frac{Q_3 - Q_1}{2}$

#### EXAMPLE

The following set of data is a set of marks, out of 10, of a class of learners:

3 6 4 4 5 9 1 3 2 5 3 7 2 7 3 1 9 1

- 1) For this set of marks find
  - a) The median
  - b) The lower quartile,  $Q_1$
  - c) The upper quartile,  $Q_3$
  - d) The interquartile range, the IQR
  - f) The semi-IQR range
- 2) Explain what each of these statistics mean practically with respect to the class.

#### SOLUTION

- |                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>1) Data arranged in order:<br/>1;1;1;2;2;3;3;3;3;4;4;5;5;6;7;7;9;9</p> <p>a) There are 18 data items, so<br/>The median = <math>\frac{3+4}{2} = \frac{7}{2} = 3,5</math></p> <p>b) <math>Q_1 = 2</math></p> <p>c) <math>Q_3 = 6</math></p> <p>d) <math>\text{IQR} = Q_3 - Q_1 = 6 - 2 = 4</math></p> <p>e) <math>\text{Semi-IQR} = \frac{\text{IQR}}{2} = \frac{4}{2} = 2</math></p> | <p>2) Nobody got full marks</p> <p>a) The middle mark of the group is 3,5 out of 10, even though nobody actually got this mark. The learners as a class have not done well.</p> <p>b) <math>Q_1</math> shows that <math>\frac{1}{4}</math> of the learners scored less than 2 marks. They scored 1's and 2's. <math>\frac{1}{4}</math> of the class did badly.</p> <p>c) <math>Q_3</math> shows that <math>\frac{3}{4}</math> of the learners scored less than 6 marks.</p> <p>d) The <b>IQR</b> shows that the marks of half the learners lie within 4 marks of each other.</p> <p>e) The <b>semi-IQR</b> shows that the marks of half the learners lie within 2 marks of the median.</p> |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

#### Exercise 4.10

The class wrote another test out of 10 marks, covering the same work. Their marks were:

2; 9; 0; 2; 3; 10; 2; 1; 9; 8; 4; 1; 8; 8; 10; 8; 3; 9

- 1) Find the range, median,  $Q_1$ ,  $Q_3$ , IQR, and the semi-IQR range of this set of marks.

- 2) Use these statistics to determine whether the class did better in this test.

### § 4.11 QUARTILES AND FREQUENCY TABLES

✦ We can find the **Interquartile Range (IQR)** of data listed in a **frequency table**.

| EXAMPLE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | SOLUTION  |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----|---|--------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>The test scores, out of 10 marks, of 29 learners is listed in the following frequency table:</p> <table border="1"> <thead> <tr> <th>SCORE</th><th>FREQUENCY</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>4</td></tr> <tr><td>3</td><td>0</td></tr> <tr><td>4</td><td>5</td></tr> <tr><td>5</td><td>2</td></tr> <tr><td>6</td><td>6</td></tr> <tr><td>7</td><td>3</td></tr> <tr><td>8</td><td>3</td></tr> <tr><td>9</td><td>2</td></tr> <tr><td>10</td><td>1</td></tr> <tr> <td><b>TOTAL</b></td><td><b>29</b></td></tr> </tbody> </table> <p>For this set of marks, calculate:</p> <p>1) The range                      2) The median<br/> 3) <math>Q_1</math>                              4) <math>Q_3</math><br/> 5) The IQR                        6) The semi-IQR range</p> | SCORE     | FREQUENCY | 0 | 0 | 1 | 2 | 2 | 4 | 3 | 0 | 4 | 5 | 5 | 2 | 6 | 6 | 7 | 3 | 8 | 3 | 9 | 2 | 10 | 1 | <b>TOTAL</b> | <b>29</b> | <p>1) The range = <math>10 - 1 = 9</math></p> <p>2) There are 29 data items. The median is the 15th item. Add the frequencies to find this item. Median = 6, since there are 14 items on either side of it.</p> <p>3) <math>Q_1</math> is the mean of the 7th and 8th data items, since there are 6 items on either side of these. Both of these items are 4, so <math>Q_1 = 4</math>.</p> <p>4) <math>Q_2</math> is the mean of the 22nd and 23rd data items, since there are 6 items on either side of these. Both of these items are 7, so <math>Q_2 = 7</math></p> <p>5) <math>IQR = Q_3 - Q_1 = 7 - 4 = 3</math>.</p> <p>6) <math>\text{Semi-IQR} = \frac{IQR}{2} = \frac{3}{2} = 1,5</math>.</p> |
| SCORE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | FREQUENCY |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 0         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 2         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 4         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 0         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 5         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 2         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 6         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 3         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 3         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 2         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 10                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 1         |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <b>TOTAL</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <b>29</b> |           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |              |           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

#### Exercise 4.11

The test scores, out of 10 marks, of 29 learners are recorded in the following frequency table:

| Score        | Frequency |
|--------------|-----------|
| 0            | 1         |
| 1            | 2         |
| 2            | 2         |
| 3            | 1         |
| 4            | 3         |
| 5            | 6         |
| 6            | 7         |
| 7            | 3         |
| 8            | 2         |
| 9            | 1         |
| 10           | 1         |
| <b>TOTAL</b> |           |

For this set of marks, calculate

- 1) The total number of scores and write it on the table
- 2) The range .....
- 3) The median .....
- 4)  $Q_1$  .....
- 5)  $Q_3$  .....
- 6) The IQR .....
- 7) The semi-IQR range .....

## § 4.12 DRAWING BAR GRAPHS

- ✦ We can represent numeric or non-numeric discrete data on a bar graph. A bar graph gives a better visual picture than a frequency table.
- ✦ The bar length represents the frequency of the data item.
- ✦ The bars are not joined together as the data is discrete.

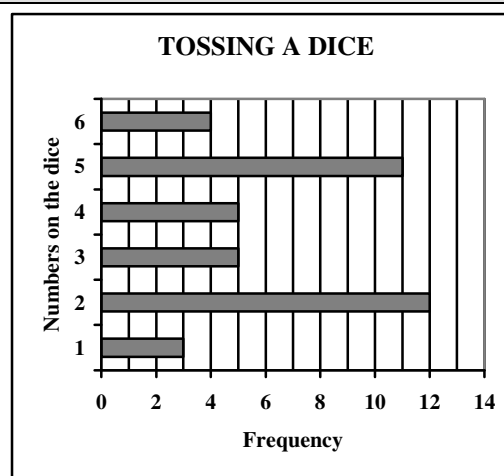
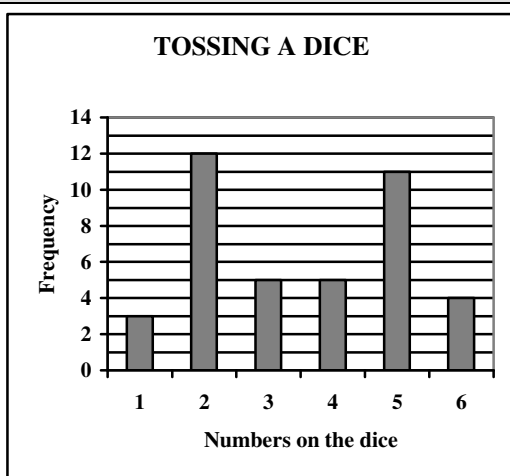
### EXAMPLE

A dice is thrown 40 times and the number is recorded after each throw.

Represent the given data: **a)** as a vertical bar graph **b)** as a horizontal bar graph

| Number on dice | 1 | 2  | 3 | 4 | 5  | 6 |
|----------------|---|----|---|---|----|---|
| Frequency      | 3 | 12 | 5 | 5 | 11 | 4 |

### SOLUTION

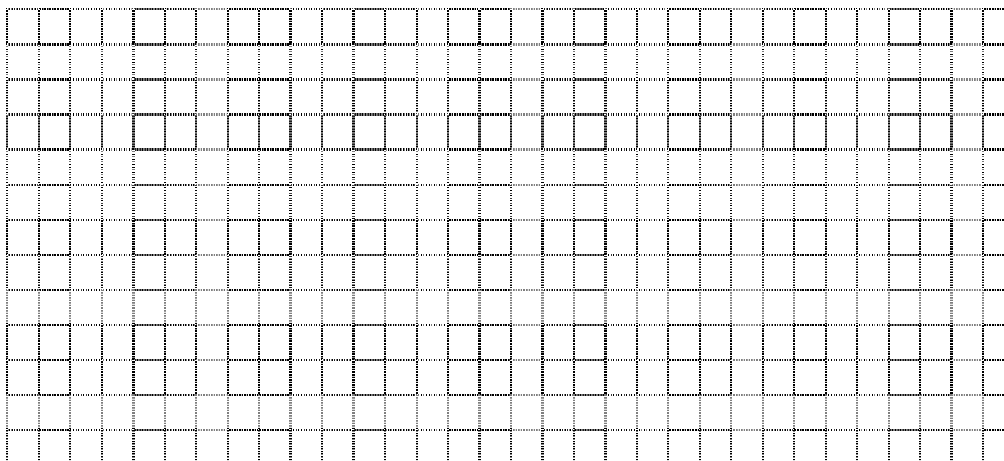


### Exercise 4.12

A bag contains red, black, yellow and white balls that are all identical in size. A ball is chosen at random, its colour recorded and the ball is returned to the bag, which is shaken. This process is repeated 34 times, and the following results are obtained:

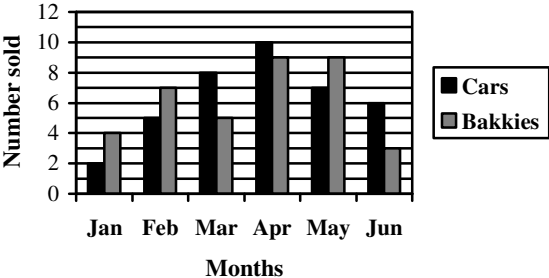
| Ball colour | Red (R) | Black (B) | Yellow (Y) | White (W) |
|-------------|---------|-----------|------------|-----------|
| Frequency   | 7       | 9         | 10         | 8         |

Represent the data: **a)** as a vertical bar graph **b)** as a horizontal bar graph.



### § 4.13 COMPOUND BAR GRAPHS

- ✦ Two or more sets of data can be compared by drawing them side by side on a compound bar graph.

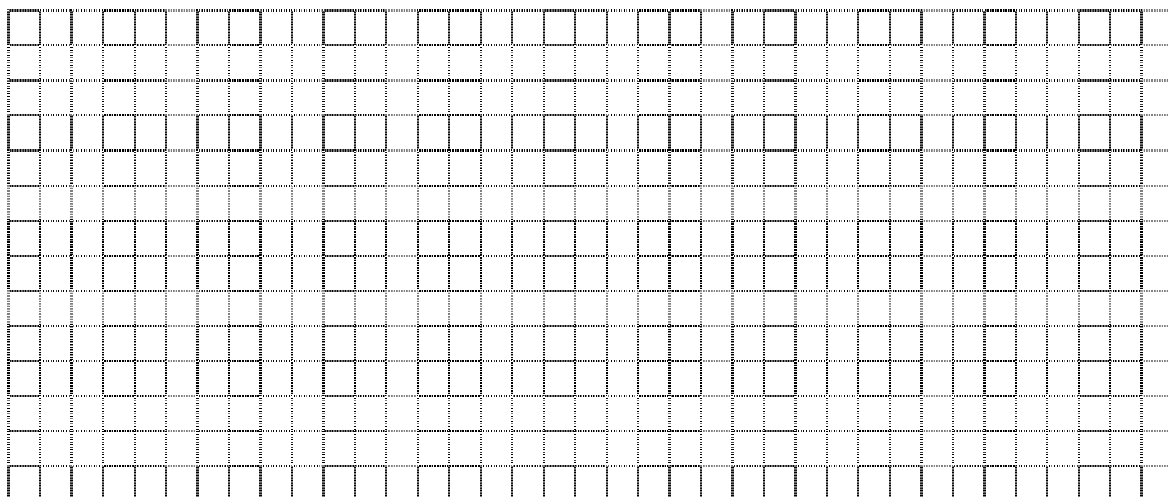
| EXAMPLE                                                                                                                                                                                                                                                                                |       |     |     |     |     |     | SOLUTION                                                                                                                                                      |     |      |   |   |   |    |   |   |         |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                           |  |  |  |  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-----|-----|-----|-----|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|------|---|---|---|----|---|---|---------|---|---|---|---|---|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|
| The table shows the number of cars and bakkies sold by a garage in each of the first six months of 2008:                                                                                                                                                                               |       |     |     |     |     |     | <div><p>NUMBER OF CARS AND BAKKIES SOLD FROM JANUARY TO JUNE 2008</p></div> |     |      |   |   |   |    |   |   |         |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                           |  |  |  |  |
| <table><tr><th>Month</th><th>Jan</th><th>Feb</th><th>Mar</th><th>Apr</th><th>May</th><th>Jun</th></tr><tr><td>Cars</td><td>2</td><td>5</td><td>8</td><td>10</td><td>7</td><td>6</td></tr><tr><td>Bakkies</td><td>4</td><td>7</td><td>5</td><td>9</td><td>9</td><td>3</td></tr></table> | Month | Jan | Feb | Mar | Apr | May |                                                                                                                                                               | Jun | Cars | 2 | 5 | 8 | 10 | 7 | 6 | Bakkies | 4 | 7 | 5 | 9 | 9 | 3 | <div><p>1) Draw a compound bar graph to illustrate the data</p><p>2) Use the graph to answer the following:</p><div><p>a) Which was the best month for car sales?<br/>How many were sold?</p><p>b) Which was the worst month for bakkie sales?</p><p>c) In which months were more cars than bakkies sold?</p></div></div> |  |  |  |  |
| Month                                                                                                                                                                                                                                                                                  | Jan   | Feb | Mar | Apr | May | Jun |                                                                                                                                                               |     |      |   |   |   |    |   |   |         |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                           |  |  |  |  |
| Cars                                                                                                                                                                                                                                                                                   | 2     | 5   | 8   | 10  | 7   | 6   |                                                                                                                                                               |     |      |   |   |   |    |   |   |         |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                           |  |  |  |  |
| Bakkies                                                                                                                                                                                                                                                                                | 4     | 7   | 5   | 9   | 9   | 3   |                                                                                                                                                               |     |      |   |   |   |    |   |   |         |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                           |  |  |  |  |
|                                                                                                                                                                                                                                                                                        |       |     |     |     |     |     | <div><p>a) April was the best month. 10 cars were sold.</p><p>b) June was the worst month</p><p>c) March, April and June</p></div>                            |     |      |   |   |   |    |   |   |         |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                                           |  |  |  |  |

#### Exercise 4.13

The number of cars and bakkies sold by a garage in each of the last 6 months of 2007:

| Month   | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---------|------|------|-------|------|------|------|
| Cars    | 6    | 9    | 8     | 8    | 12   | 6    |
| Bakkies | 4    | 3    | 9     | 10   | 11   | 4    |

- 1) Represent the data as a compound bar graph.



- 2) Use the graph to answer the questions:
- a) Which was the best month for car sales? How many were sold? .....
- b) Which was the worst month for bakkie sales? .....
- c) In which months were more cars than bakkies sold? .....

## § 4.14 PIE CHARTS

- ✦ To represent data in a pie chart, we divide a circle into 'slices' like a pie.
- ✦ Each slice or sector represents the frequency of the data item.

### EXAMPLE

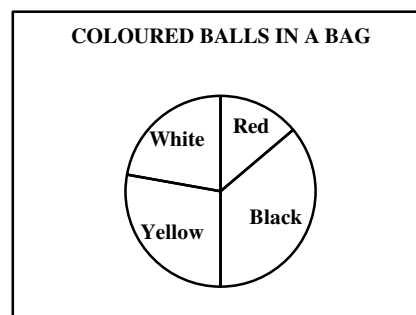
Suppose there are 36 different coloured balls of identical size in a bag.

| Ball colour | Red | Black | Yellow | White | TOTAL |
|-------------|-----|-------|--------|-------|-------|
| Frequency   | 5   | 13    | 10     | 8     | 36    |

Draw a pie chart to represent this data

### SOLUTION

- The angle at the centre of a circle (one revolution) is  $360^\circ$
- There is a total of 36 balls in the frequency table, so each ball is represented by  $\frac{360^\circ}{36} = 10^\circ$  at the centre of the pie.
- Red balls are represented by  $5 \times 10^\circ = 50^\circ$ .  
Black balls are represented by  $13 \times 10^\circ = 130^\circ$ .  
Yellow balls are represented by  $10 \times 10^\circ = 100^\circ$ .  
White balls are represented by  $8 \times 10^\circ = 80^\circ$ .
- Note that these angles add up to  $360^\circ$   
( $50^\circ + 130^\circ + 100^\circ + 80^\circ = 360^\circ$ )



### Exercise 4.14

A dice is thrown 40 times and the number is recorded after each throw.

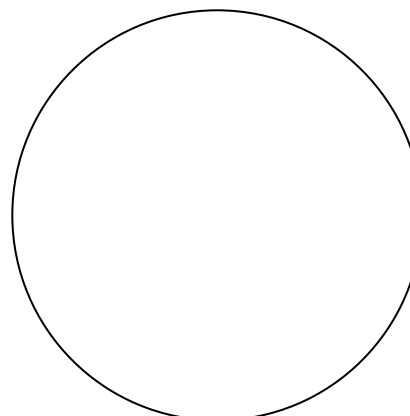
| Number              | 1 | 2  | 3 | 4 | 5  | 6 | TOTAL |
|---------------------|---|----|---|---|----|---|-------|
| Frequency           | 4 | 11 | 6 | 5 | 12 | 2 |       |
| Angle at the centre |   |    |   |   |    |   |       |

- 1) What angle represents each throw?

.....  
 .....

- 2) Calculate the angles at the centre of the circle and then complete the above table. Check that the angles add to  $360^\circ$ .

.....  
 .....  
 .....  
 .....  
 .....  
 .....



- 3) Draw in a radius of the circle. Use it to begin measuring the angles with your protractor. Complete your pie chart by accurately drawing in each sector or slice of the circle.
- 4) Label each sector and include a title for your pie chart.

### § 4.15 DRAWING LINE GRAPHS

✦ To represent data using a line graph, the data on the horizontal axis must be **continuous**.

| EXAMPLE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | SOLUTION       |    |    |    |             |  |    |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                |   |    |    |              |    |    |    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|----|----|----|-------------|--|----|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|---|----|----|--------------|----|----|----|
| <p>One kilogram (kg) is approximately equal to 2,2 pounds (lb)</p> <p>1) Complete the following table and use it to draw a conversion graph.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">Kilograms (kg)</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">15</td> </tr> <tr> <td style="padding: 5px;">Pounds (lb)</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">22</td> <td style="padding: 5px;"></td> </tr> </table> <p>2) Use your graph to:</p> <p style="margin-left: 20px;">a) Convert 9 kg to lb</p> <p style="margin-left: 20px;">b) Convert 24 lb to kg</p> | Kilograms (kg) | 5  |    | 15 | Pounds (lb) |  | 22 |  | <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">Kilograms (kg)</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">10</td> <td style="padding: 5px;">15</td> </tr> <tr> <td style="padding: 5px;">Pounds (lbs)</td> <td style="padding: 5px;">11</td> <td style="padding: 5px;">22</td> <td style="padding: 5px;">33</td> </tr> </table> <div style="text-align: center; margin: 10px 0;"> <b>CONVERSIONS BETWEEN KG AND LBS</b> </div> <p style="margin-top: 10px;">a) 9 kg <math>\approx</math> 20 lb</p> <p style="margin-left: 20px;">b) 24 lb <math>\approx</math> 11 kg</p> | Kilograms (kg) | 5 | 10 | 15 | Pounds (lbs) | 11 | 22 | 33 |
| Kilograms (kg)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 5              |    | 15 |    |             |  |    |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                |   |    |    |              |    |    |    |
| Pounds (lb)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                | 22 |    |    |             |  |    |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                |   |    |    |              |    |    |    |
| Kilograms (kg)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 5              | 10 | 15 |    |             |  |    |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                |   |    |    |              |    |    |    |
| Pounds (lbs)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 11             | 22 | 33 |    |             |  |    |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                |   |    |    |              |    |    |    |

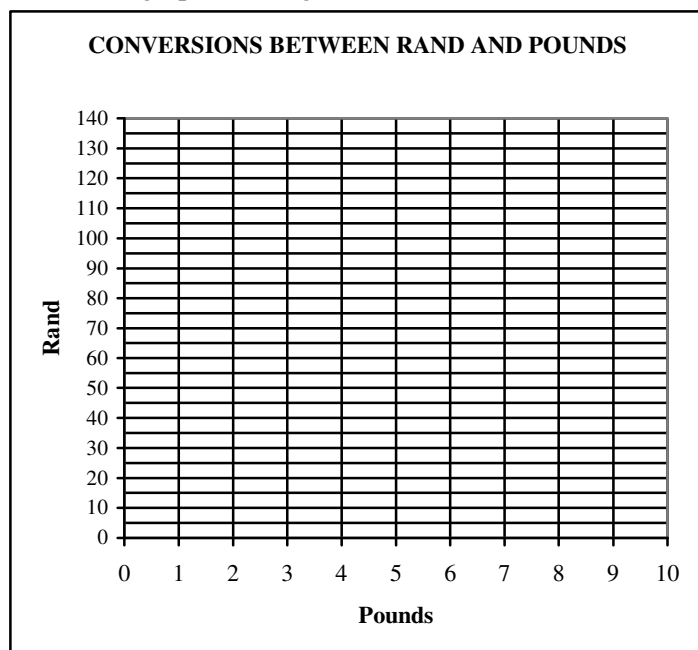
#### Exercise 4.15

On a certain day the conversion rate between the South African rand and the British pound is £1 = R15

- 1) Use the conversion rate to complete the given table:

|                   |          |          |          |
|-------------------|----------|----------|----------|
| <b>Rand (R)</b>   |          |          |          |
| <b>Pounds (£)</b> | <b>1</b> | <b>2</b> | <b>9</b> |

- 2) Draw a graph showing the conversion between rand and pounds on the following grid:



- 3) Use your graph to:

- a) Convert £5 to rand  
.....
- b) Convert £8 to rand  
.....
- c) Convert R60 to £  
.....
- d) Convert R100 to £  
.....



### § 4.16 BROKEN LINE GRAPHS

✦ Broken line graphs are used to represent **discrete** data.

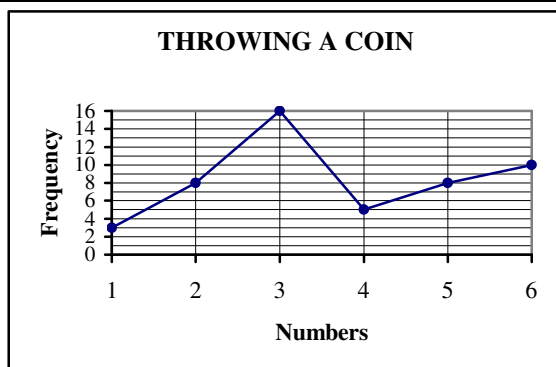
#### EXAMPLE

A dice is thrown 50 times and the numbers are recorded.

| Number    | 1 | 2 | 3  | 4 | 5 | 6  |
|-----------|---|---|----|---|---|----|
| Frequency | 3 | 8 | 16 | 5 | 8 | 10 |

Represent this data on a broken line graph.

#### SOLUTION



#### Note:

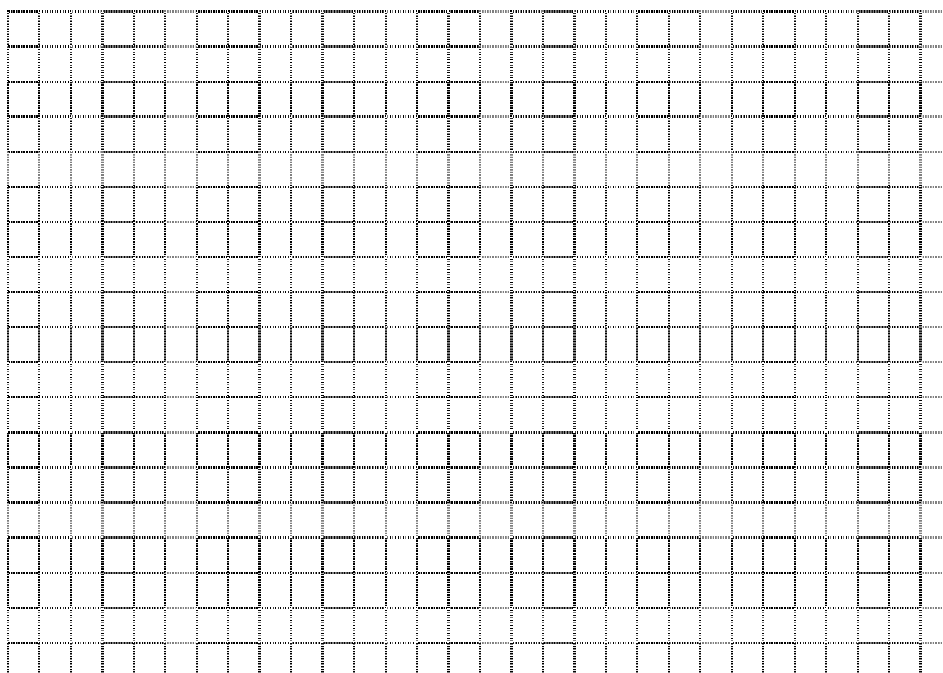
- The data is discrete, so plotted points are joined with a broken line to show the trend.
- There are no values between the plotted points
- It would seem from the frequencies that the die is weighted

#### Exercise 4.16

A class of learners were asked to choose whether they preferred chocolate bar A, B, C or D. The results of the survey are recorded on a frequency table.

| Chocolate bar | A | B  | C  | D  |
|---------------|---|----|----|----|
| Frequency     | 6 | 13 | 16 | 10 |

Draw a broken line graph to represent this data.



### § 4.17 DRAWING A HISTOGRAM

- ✦ Continuous data can be represented by using a histogram.
- ✦ If the width of each column is the same, the height of the column indicates the frequency.
- ✦ A histogram has **no gaps** between the rectangular bars.

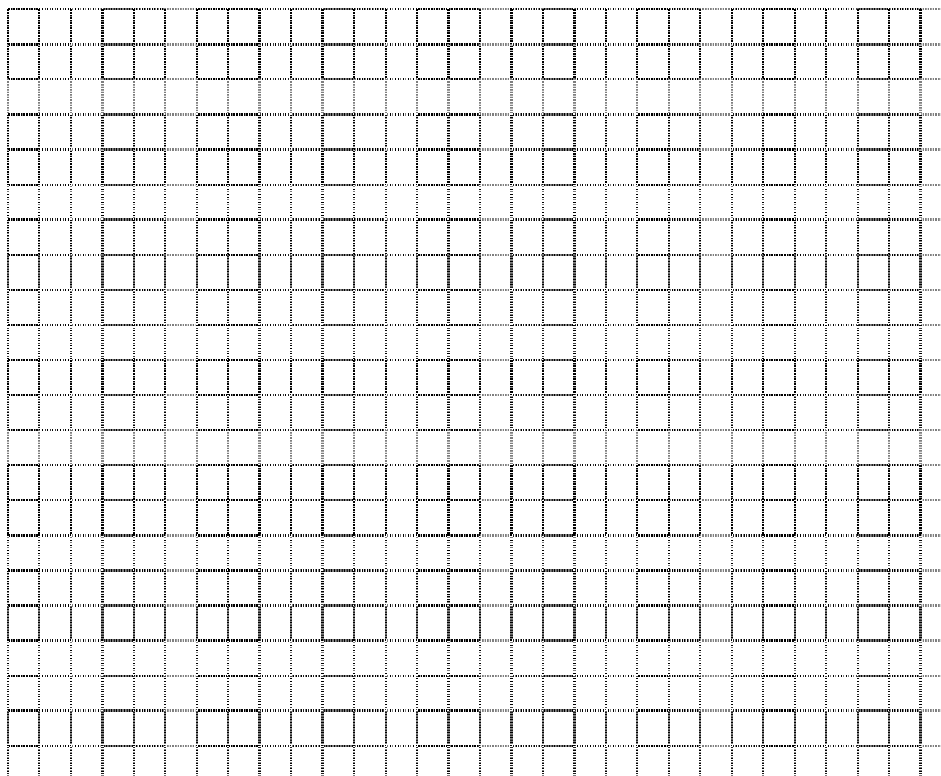
| EXAMPLE                                                                                                                                                                                                                                                                                                                                                         | SOLUTION           |       |       |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|-------|-------|-------|-----------|----|----|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|-----------|--------|----|--------|----|--------|---|
| <p>A dentist recorded the number of fillings in the teeth of a group of college students.</p> <p>He listed the results in a grouped frequency table</p> <table><tr><td>Number of fillings</td><td>0 – 1</td><td>2 – 3</td><td>4 – 5</td></tr><tr><td>Frequency</td><td>22</td><td>18</td><td>4</td></tr></table> <p>Draw a histogram to represent the data.</p> | Number of fillings | 0 – 1 | 2 – 3 | 4 – 5 | Frequency | 22 | 18 | 4 | <p style="text-align: center;"><b>STUDENTS' FILLINGS</b></p> <table><caption>Data for Students' Fillings Histogram</caption><tr><th>Number of fillings</th><th>Frequency</th></tr><tr><td>0 to 1</td><td>22</td></tr><tr><td>2 to 3</td><td>18</td></tr><tr><td>4 to 5</td><td>4</td></tr></table> | Number of fillings | Frequency | 0 to 1 | 22 | 2 to 3 | 18 | 4 to 5 | 4 |
| Number of fillings                                                                                                                                                                                                                                                                                                                                              | 0 – 1              | 2 – 3 | 4 – 5 |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |
| Frequency                                                                                                                                                                                                                                                                                                                                                       | 22                 | 18    | 4     |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |
| Number of fillings                                                                                                                                                                                                                                                                                                                                              | Frequency          |       |       |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |
| 0 to 1                                                                                                                                                                                                                                                                                                                                                          | 22                 |       |       |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |
| 2 to 3                                                                                                                                                                                                                                                                                                                                                          | 18                 |       |       |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |
| 4 to 5                                                                                                                                                                                                                                                                                                                                                          | 4                  |       |       |       |           |    |    |   |                                                                                                                                                                                                                                                                                                    |                    |           |        |    |        |    |        |   |

#### Exercise 4.17

The following frequency table gives the percentage obtained in a test by 38 students in a Level 2 class:

|                  |        |         |         |         |          |
|------------------|--------|---------|---------|---------|----------|
| <b>Marks</b>     | 1 – 20 | 21 – 40 | 41 – 60 | 61 – 80 | 81 – 100 |
| <b>Frequency</b> | 4      | 6       | 9       | 11      | 8        |

Draw a histogram to represent the data:



### § 4.18 DRAWING MORE HISTOGRAMS

- ✦ The horizontal axis of a histogram is a **continuous number line**.
- ✦ The vertical axis show the **frequency**.
- ✦ There are no gaps between the bars of the histogram.
- ✦ The class intervals are all the **same** width, so the bars are the same width.

#### Exercise 4.18

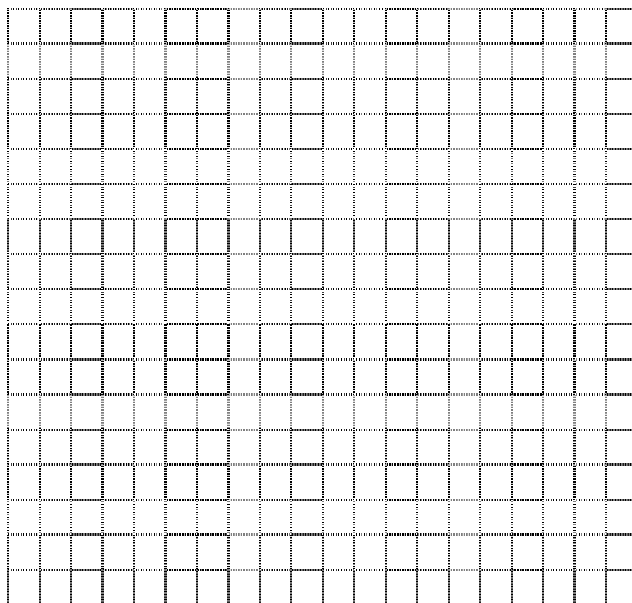
- 1) A farmer checks the masses (in grams) of a number of tomatoes for grading purposes. The table shows the results:

| Mass ( $m$ ) in grams | Frequency |
|-----------------------|-----------|
| $60 \leq m < 65$      | 9         |
| $65 \leq m < 70$      | 20        |
| $70 \leq m < 75$      | 29        |
| $75 \leq m < 80$      | 15        |
| $80 \leq m < 85$      | 7         |

- a) Draw a histogram to illustrate the data
- b) Comment on the distribution of the masses of the tomatoes checked.

.....

.....



- 2) Dungile recorded the following temperatures at noon for one month in whole degrees Celsius:

26 34 24 24 32 18 12 24 12 14  
 12 30 24 20 26 34 16 24 42 16  
 12 24 32 34 16 24 12 20 26 8

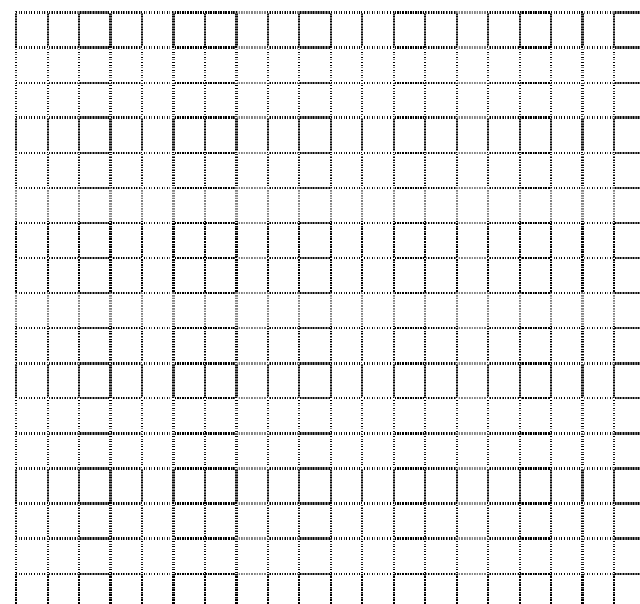
- a) Complete the following grouped frequency table:

| Temperature in $^{\circ}\text{C}$ | Frequency |
|-----------------------------------|-----------|
| $1 \leq t < 10$                   |           |
| $10 \leq t < 20$                  |           |
| $20 \leq t < 30$                  |           |
| $30 \leq t < 40$                  |           |
| $40 \leq t < 50$                  |           |

- b) Draw a histogram to illustrate the information
- c) Comment on any features you notice about this distribution of temperatures.

.....

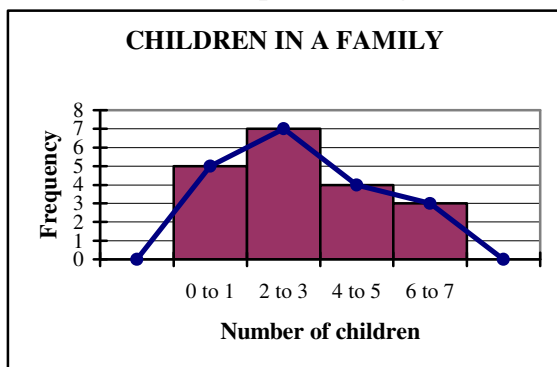
.....



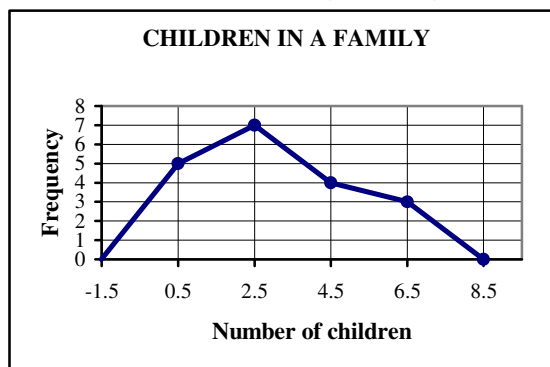
### § 4.19 FREQUENCY POLYGONS

- ✦ A histogram shows frequencies.
- ✦ A frequency polygon is a line graph showing frequencies. It is formed by joining the frequencies at the mid-points of each interval of a histogram with straight lines.

A frequency polygon can be drawn on top of a histogram



A frequency polygon can be drawn without first drawing a histogram.



Notice the following about a frequency polygon

- i. The **midpoints** of the intervals are joined
- ii. An **extra interval** is added to the left and the right of the histogram which has a frequency of 0. These two intervals are necessary to create a **polygon** (a closed shape made up of line segments).
- iii. A frequency polygon can be drawn on top of a histogram or by itself.

#### Exercise 4.19

The following frequency table gives the percentage obtained in a test by 38 students in a Level 2 class:

| Percentages              |   | 1 – 20 | 21 – 40 | 41 – 60 | 61 – 80 | 81 – 100 |   |
|--------------------------|---|--------|---------|---------|---------|----------|---|
| Midpoint of the interval |   |        |         |         |         |          |   |
| Frequency                | 0 | 4      | 6       | 9       | 11      | 8        | 0 |

- 1) Complete the table by finding the midpoint of each interval.
- 2) Draw a frequency polygon to represent the data:

