CHAPTER 7
Properties of shapes

In this chapter you will:

- Calculate the area and perimeter of 2 dimensional shapes
- Calculate the volume and surface area of 3 dimensional shapes
- Calculate the sizes of angles
- Calculate the sizes of angles using the straight line relationships
- Name an angle according to its position
- Use parallel lines to calculate the sizes of angles
- Prove lines parallel
- Calculate the sum and the size of the interior angles of polygons
- Distinguish between congruent and similar polygons
- Name triangles according to the length of their sides
- Calculate the sizes of angles in isosceles triangles
- Name congruent triangles
- Prove triangles congruent
- Prove triangles similar
- Use the definitions of quadrilaterals to look at the relationship between them

This chapter covers material from Topic 3: Space, Shape and Orientation

SUBJECT OUTCOME 3.1:

Describe, represent, analyse and explain properties of shape in two-and three-dimensional space with justification

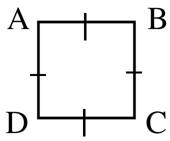
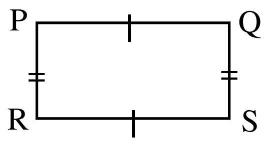
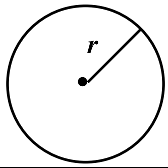
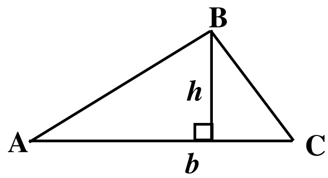
Learning Outcome 2:	Investigate and generalise formula for calculating the volume and surface area of right prisms and cylinders.

SUBJECT OUTCOME 3.2:

Describe and represent the properties of geometric shapes

Learning Outcome 1:	Investigate polygons through Euclidian, co-ordinate and/or transformation geometry
Learning Outcome 2:	Generalise the properties of different polygons
Learning Outcome 3:	Expose alternate definitions of polygons

§ 7.1 AREA AND PERIMETERS OF 2 DIMENSIONAL FIGURES

NAME	SHAPE	FORMULA FOR AREA	FORMULA FOR PERIMETER
square		Area ABCD = side^2 = AB^2	Perimeter = 4 × side = 4AB
rectangle		Area PQRS = length × breadth = PQ × QS	Perimeter = 2 length + 2 breadth = 2PQ + 2QS
circle		Area = πr^2 • r is the radius • d (diameter) = $2r$	Circumference = $2\pi r$
triangle		Area Δ ABC = $\frac{1}{2} \times b \times h$ • ***b*** is the length of the **base** • ***h*** is the length of the **perpendicular height**	Perimeter = AB + BC + AC

EXAMPLE	SOLUTION	
Find the area (A) and perimeter (P) of: 1) a square: side = 4 cm	A = 4^2 = 16 cm^2	P = 4 × 4 = 16 cm
2) a rectangle: length = 3 cm and breadth = 6 cm	A = 3 × 6 = 18 cm^2	P = 2(3) + 2(6) = 6 + 12 = 18 cm
3) a circle: diameter = 6 cm	A = $\pi \times 3^2$ = 28,3 cm^2	C = $2 \times \pi \times 3$ = 18,8 cm
4) a triangle: base = 8 cm, height = 3 cm, sides = 5 cm	A = $\frac{1}{2} \times 8 \times 3$ = 12 cm^2	P = 8 + 5 + 5 = 18 cm

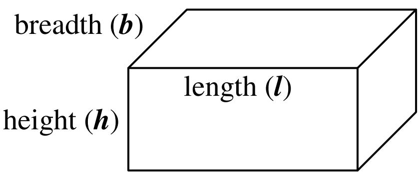
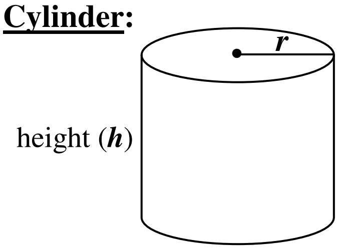
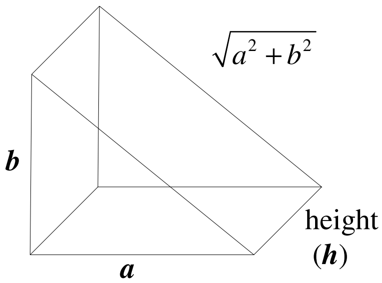
Exercise 7.1

Find a) the area (A) and b) perimeter (P), correct to 1 decimal place, of:

	(a) Area	(b) Perimeter
1) a square: side = 8 cm		
2) a rectangle: length = 7 cm and width = 5 cm		
3) a circle: diameter = 8 cm		
4) a triangle: base = 24 cm, height = 5 cm, other two sides = 13 cm		

§ 7.2 VOLUME AND SURFACE AREA OF 3 DIMENSIONAL SHAPES

✧ To find the volume of a prism, multiply the area of the base by the height.

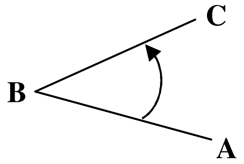
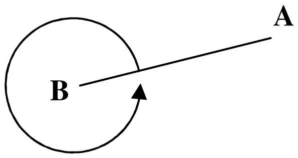
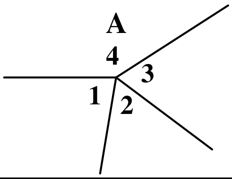
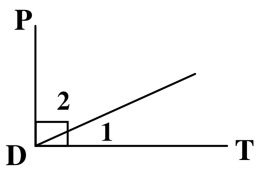
3-dimensonal figure	Volume (V)	Surface area (SA)
Rectangular prism:  	The base is a rectangle. Area of base = $l \times b$ $V = l \times b \times h$	6 rectangular faces $SA = 2lb + 2lh + 2bh$
Cylinder:  	The base is a circle. Area of base = πr^2 $V = \pi r^2 h$	2 circular faces and a curved surface. $SA = 2\pi r^2 + 2\pi rh$
Triangular prism:  	The base is a right-angled triangle. Area of base $= \frac{1}{2} \times a \times b$ $V = \frac{1}{2} \times a \times b \times h$	2 triangular faces and 3 rectangular faces. Area of 2 triangular faces $= 2(\frac{1}{2}ab) = ab$ Surface Area $= ab + bh + ah + \sqrt{a^2+b^2}\,.h$

Exercise 7.2

Find (a) the volume (V) and (b) surface area (SA), correct to 1 decimal place, of a:

	(a) Volume	(b) Surface Area
1) A rectangular prism having length = 14 cm, breadth = 5 cm and height = 4 cm		
2) A closed cylinder of diameter = 12 cm and height = 20 cm		
3) A right-angled triangular prism with side = 5cm, side = 12 cm (not the hypotenuse) and height = 10 cm		

§ 7.3 ANGLES

	• Turning BA about B to position BC forms an angle. We name the angle $A\hat{B}C$ or $\hat{B}$. It is measured in degrees. • B is the vertex of the angle • BC and BA are arms of the angle
	• Arm BA is rotated about B through a full circle, back to its original position • the size of the angle formed is **1 revolution = 360°**
	• **Angles about point A add to 360°** • $\hat{A}_1 + \hat{A}_2 + \hat{A}_3 + \hat{A}_4 = 360°$
	• Angles adding to 90° are called **complementary angles** • $\hat{D}_1$ and $\hat{D}_2$ are complementary since $\hat{D}_1 + \hat{D}_2 = 90°$ • $\hat{D}_1$ is the **complement** of $\hat{D}_2$ • DP and DT are **perpendicular lines**, written DP ⊥ DT

Exercise 7.3

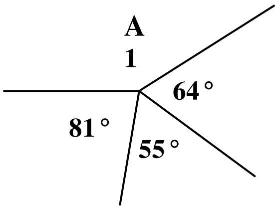
1) What is the complement of 54°? …………………………………………………………….

2) What is the size of 1 revolution? …………………………………………………………..

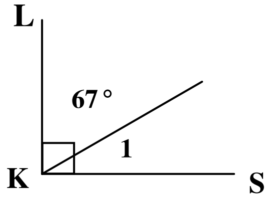
3) What do the angles about a point add up to? ……………………………………………

4)

a) Calculate the size of $\hat{A}_1$.

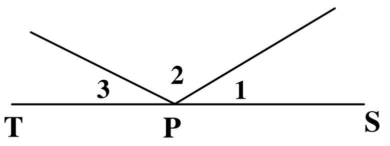
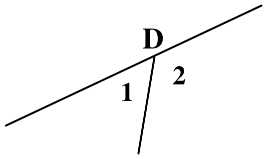
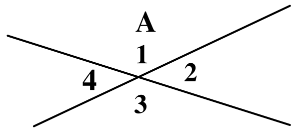


b) Calculate the size of $\hat{K}_1$.



KL …... KS

§ 7.4 STRAIGHT LINE RELATIONSHIPS

	TPS is a straight line. • **Adjacent angles on a line add to 180°** • $\hat{P}_1 + \hat{P}_2 + \hat{P}_3 = 180°$
	• Angles adding to 180° are **supplementary angles** • $\hat{D}_1$ and $\hat{D}_2$ are supplementary since $\hat{D}_1 + \hat{D}_2 = 180°$ • $\hat{D}_1$ is the **supplement** of $\hat{D}_2$
	• **Vertically opposite** angles are equal in size • $\hat{A}_1 = \hat{A}_3$ and $\hat{A}_2 = \hat{A}_4$

<u>Exercise 7.4</u>

1) What is the supplement of 98°? ……………………………………………………..

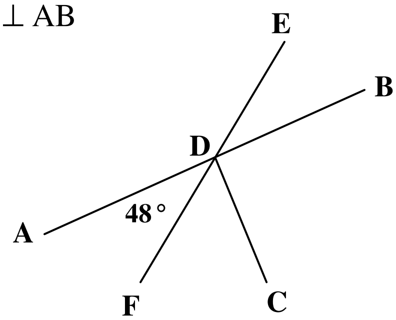
2) What is the supplement of 129°? ……………………………………………………

3) What is the sum of adjacent angles on a line? ……………………………………………

4) What is the size of an angle vertically opposite to an angle of 136°? …………………….

5) Calculate:

a) FE and AB are straight lines. CD ⊥ AB



$C\hat{D}B$ = ……………………………..….

$E\hat{D}B$ = …………………..……….

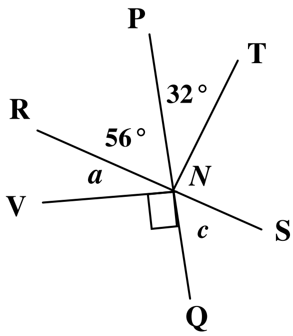
$C\hat{D}E$ = ………………..……………

$C\hat{D}F$ = ……………………………...

$F\hat{D}B$ = ……………………………...

$A\hat{D}E$ = ………………………………

b) PQ and RS are straight lines.



c = …………………………………..…

a = ………………………………….…

$T\hat{N}S$ = ………………….……….…

$R\hat{N}T$ = ...

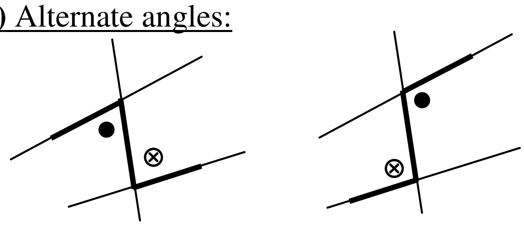
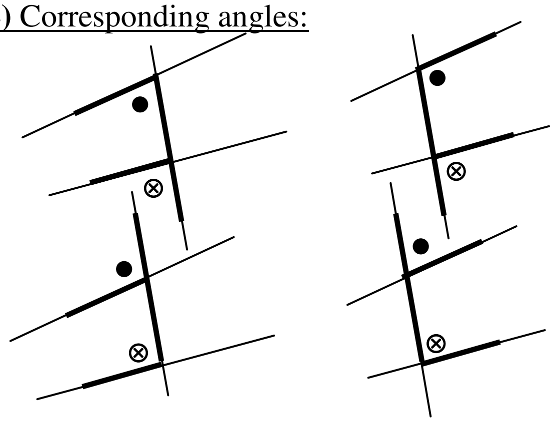
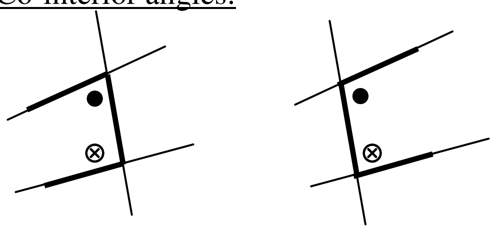
$V\hat{N}T$ = ………………………….……

Reflex $R\hat{N}Q$ = ………………………..

$S\hat{N}V$ = ……………………………...

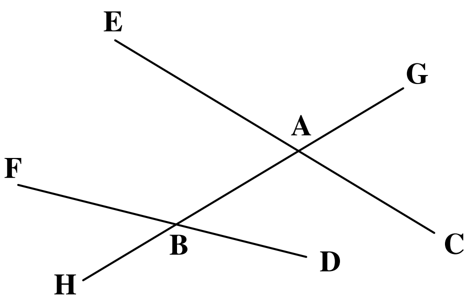
§ 7.5 ANGLE POSITION

✧ In each sketch below, two lines are cut by a transversal. The marked pairs of angles have a name according to their position. The angle pairs are alternate or corresponding or co-interior.

1) Alternate angles: 	• Alternate angles are recognised by their position in a "**Z**" shape • The "Z" shape may be back to front • There are 2 pairs of alternate angles
2) Corresponding angles: 	• Corresponding angles lie in an "**F**" shape • The "F" shape may be back to front or upside down • There are 4 pairs of corresponding angles
3) Co-interior angles: 	• Co-interior angles lie within a "**C**" shape • The "C" shape may be back to front • There are two pairs of co-interior angles

Exercise 7.5

1) Use the sketch to write down the names of:



a) 4 pairs of corresponding angles:

b) 2 pairs of co-interior angles:

c) 2 pairs of alternate angles:

2) a) The shape associated with corresponding angles is ……………………………..……….

b) The shape associated with alternate angles is ……………………………………………….

c) The shape associated with co-interior angles is …………………………………….………

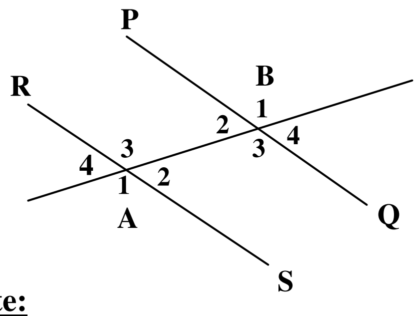
§ 7.6 PARALLEL LINES

When two lines are parallel and are cut by a transversal, then:
- The alternate angles are equal
- The corresponding angles are equal
- The co-interior angles add to $180°$ (they are supplementary)

<table>
<tr><th>EXAMPLE</th><th>SOLUTION</th></tr>
<tr><td>

<u>Given:</u> PQ $\parallel$ RS and $\hat{B}_4 = 72°$

Calculate the marked angles. Give reasons for your answers.



<u>Note:</u>

- There are a number of different ways to find the size of an angle.

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$\hat{B}_4 = \hat{B}_2 = 72°$ vertically opposite angles

$\hat{B}_2 + \hat{A}_3 = 180°$ co-interior angles

$\hat{A}_3 = 108°$

$\hat{B}_1 = 180° - 72°$ adjacent angles on a line

$= 108°$

$\hat{B}_1 = \hat{B}_3 = 108°$ vertically opposite angles

$\hat{B}_2 = \hat{A}_2 = 72°$ alternate angles

$\hat{B}_3 = \hat{A}_1 = 108°$ corresponding angles

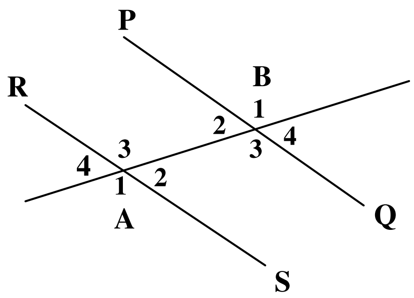
$\hat{A}_2 = \hat{A}_4$ vertically opposite angles

</td></tr>
</table>

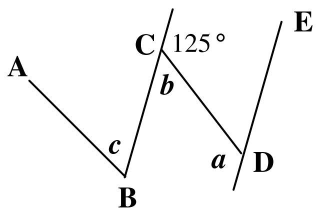
<u>Exercise 7.6</u>

Calculate the marked angles. Give reasons for your answers.

1) <u>Given:</u> PQ $\parallel$ RS and $\hat{A}_1 = 112°$



2) <u>Given</u>: AB $\parallel$ CD and CB $\parallel$ ED

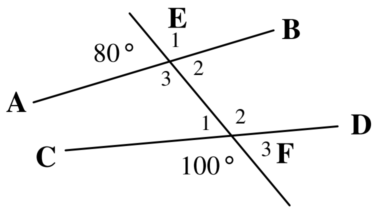
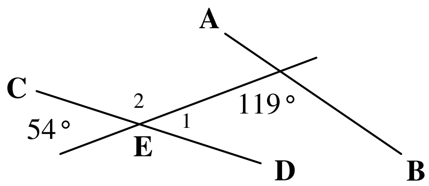


§ 7.7 PROVING LINES PARALLEL

If two lines are cut by a transversal, and

- one pair of alternate angles is equal **OR**
- one pair of corresponding angles is equal **OR**
- one pair of co-interior angles adds to $180°$ (they are supplementary)

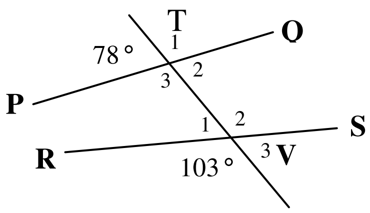
then the lines are parallel.

EXAMPLE	SOLUTION
Is AB $\parallel$ CD? Give a reason for your answer. 1) 	**Yes.** $\hat{F}_1 = 80°$ angles on a line AB $\parallel$ CD corresponding angles equal
2) 	**No.** $\hat{E}_1 = 54°$ vertically opposite angles $54° + 119° = 173° \neq 180°$ co-interior angles are not supplementary.

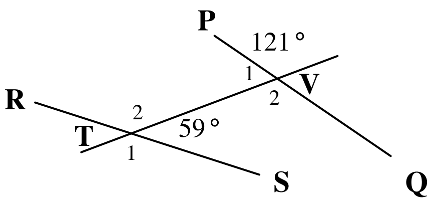
Exercise 7.7

Is PQ $\parallel$ RS? Give a reason for your answer.

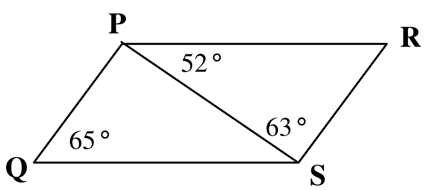
1)



2)

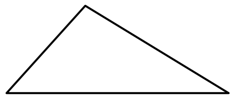
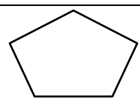
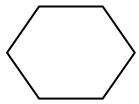


3) Given: PR $\parallel$ QS



§ 7.8 POLYGONS

- A **polygon** is a closed figure having three or more sides which are straight lines.
- A **regular polygon** has sides equal in length and angles equal in size.

Name	Number of sides	Polygon
triangle	3	
quadrilateral	4	
pentagon	5	
hexagon	6	

Note:

- The **sum of the interior angles** of a polygon of n sides is $180° \times (n-2)$
- The **size of an interior angle** of a regular polygon of n sides is $\frac{180°(n-2)}{n}$
- Other polygons are: heptagon (7 sides), octagon (8 sides), nonagon (9 sides), decagon (10 sides)

EXAMPLE	SOLUTION
1) Calculate the sum of the interior angles of a regular pentagon	A pentagon has 5 sides, so $n = 5$. Sum of interior angles = $180°(5-2) = 180°(3) = 540°$
2) Calculate the size of each interior angle.	The size of each interior angle = $540° \div 5 = 108°$.

Exercise 7.8

Calculate:

a) the sum of the interior angles of the regular polygon
b) the size of each interior angle.

1) Hexagon …………………………………………………………………………………

2) Octagon …………………………………………………………………………………

3) Triangle …………………………………………………………………………………

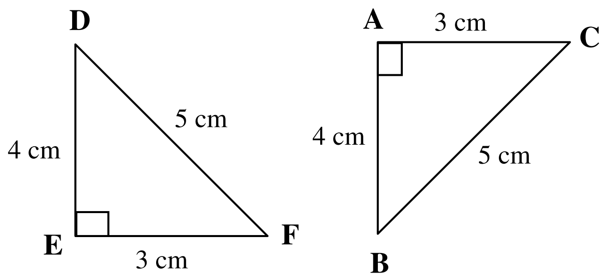
4) Quadrilateral ……………………………………………………………………………

5) Decagon …………………………………………………………………………………

§ 7.9 SIMILAR AND CONGRUENT POLYGONS

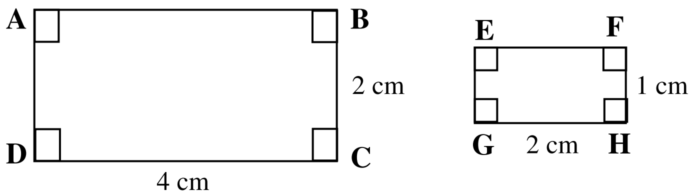
Congruent polygons:

- Congruent polygons are **identical** to each other. Their:
  - corresponding sides are equal in length and
  - corresponding angles are equal in size
- A congruent polygon is obtained when the given polygon is translated, reflected or rotated.

EXAMPLE	**SOLUTION**
	ΔDEF has been reflected to form ΔBAC, but its sides and angles are unchanged. ΔDEF ≡ ΔBAC

Similar polygons:

- Polygons are similar if:
  - they are equiangular (angles equal in size) **and**
  - their corresponding sides are in proportion.
- Similar polygons are obtained when a given polygon is enlarged or reduced.

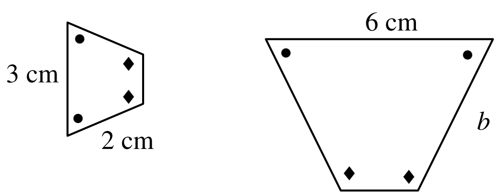
EXAMPLE	SOLUTION
	ABCD and EFGH are equiangular The sides of ABCD and EFGH are in the proportion 2 : 1 So ABCD ||| EFGH

Exercise 7.9

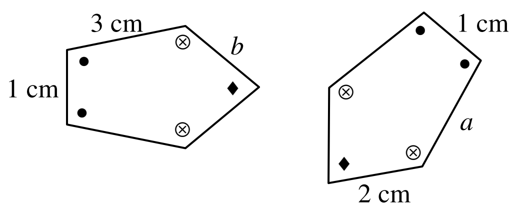
1) Are the following figures congruent or only similar?

2) Calculate the lengths of the marked sides.

a)

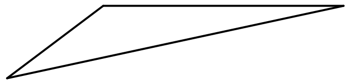
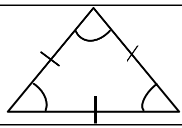
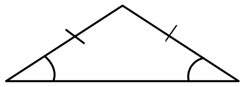


b)



§ 7.10 TRIANGLES

✧ A triangle is a thee sided polygon.

Type	Figure	Properties
Scalene triangle		• No equal sides • No equal angles
Equilateral triangle		• Three equal sides • Three equal angles • All angles $60°$
Isosceles triangle		• Two equal sides • Two equal angles

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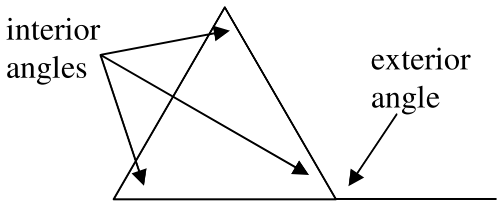

</td>
<td>
✧ Every triangle has three interior angles. 'interior' means 'inside'.
✧ The exterior angle of a triangle lies between one side and an extended side of the triangle.
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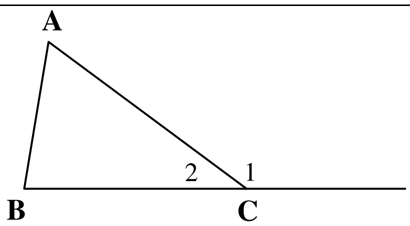
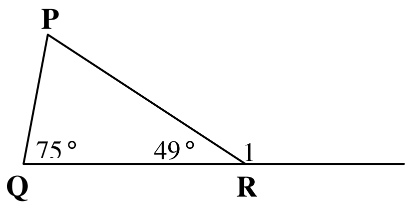


</td>
<td>
✧ The exterior angle of a triangle is equal to the sum of the two interior opposite angles.
$\hat{C}_1 = \hat{A} + \hat{B}$
✧ The 3 interior angles of a triangle add to $180°$.
$\hat{A} + \hat{B} + \hat{C}_2 = 180°$
</td>
</tr>
</table>

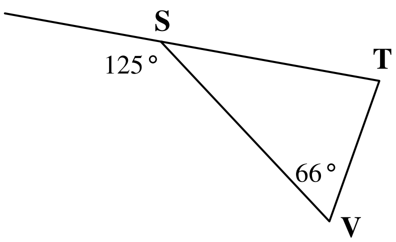
Exercise 7.10

1)



Calculate $\hat{P}$ and $\hat{R}_1$

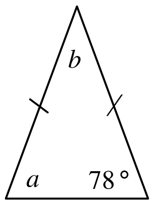
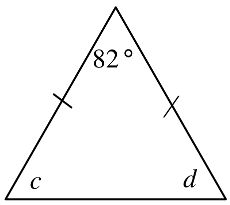
2)



Calculate $\hat{T}$ and $T\hat{S}V$.

§ 7.11 ISOSCELES TRIANGLES

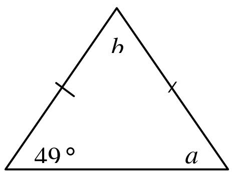
- If **two sides of a triangle are equal in length**, the triangle is isosceles, and the angles opposite those sides are equal in size
- If **two angles of a triangle are equal in size**, the triangle is isosceles, and the sides opposite those angles are equal in length

EXAMPLE	SOLUTION
Calculate the size of the marked angles: 1) 	$a = 78°$ angles opposite equal sides $b + 78° + 78° = 180°$ angle sum of triangle $b + 156° = 180°$ $b + 156° - 156° = 180° - 156°$ $b = 24°$
2) 	$82° + c + d = 180°$ angle sum of triangle $82° + c + d - 82° = 180° - 82°$ $c + d = 98°$ $c = d$ angles opposite equal sides $c = d = 49°$

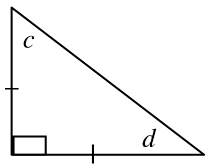
Exercise 7.11

Calculate the size of the angles marked in the figure. Give reasons for tour answer.

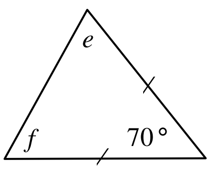
1)



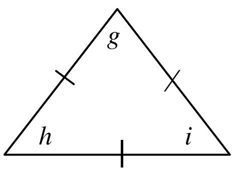
2)



3)

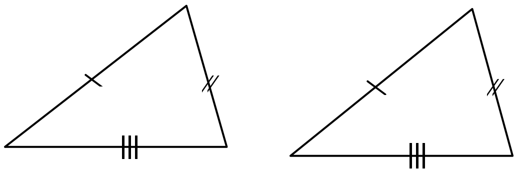
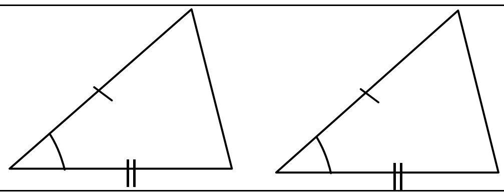
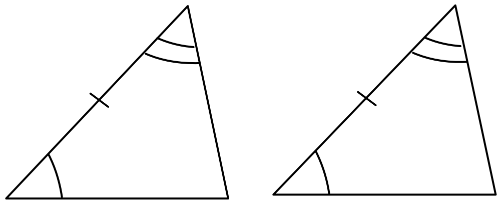
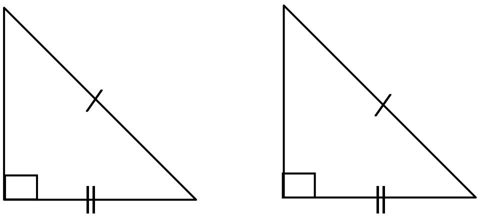


4)



§ 7.12 CONGRUENT TRIANGLES

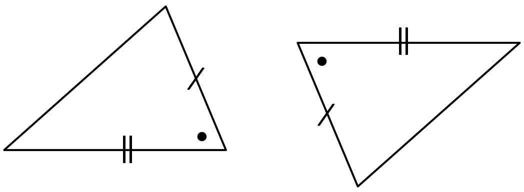
✧ Two triangles are congruent if **one** of these four conditions is true:

1)		• Three sides of one triangle equal in length to three sides of the other triangle • Notation: **SSS**
2)		• Two sides of one triangle equal to two sides of the other AND • The included angles equal in size • Notation: **SAS**
3)		• Two angles of one triangle equal in size to two angles of the other triangle AND • One side of one triangle equal in length to the corresponding side of the other • Notation: **SAA**
4)		• Both triangles are right-angled AND • Hypotenuses are equal AND • One pair of corresponding sides is equal • Notation: **RHS**

Exercise 7.12

1) Are the following pairs of triangles congruent? If yes, give a reason for your answer.

a)



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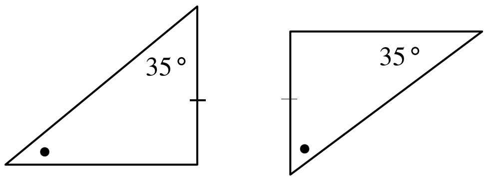
b)



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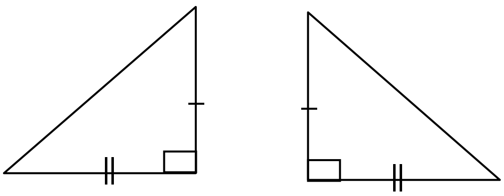
c)



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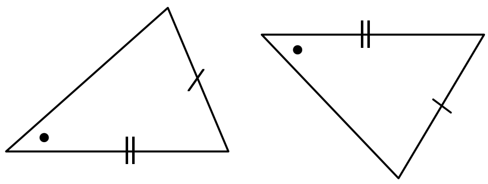
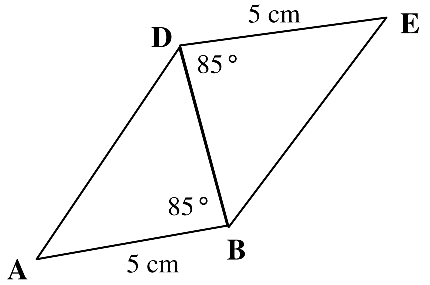
d)



....

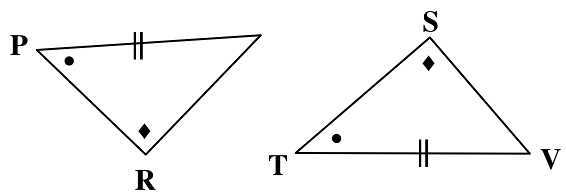
2) Is AAA a condition for congruency? ....

§ 7.13 PROVING TRIANGLES CONGRUENT

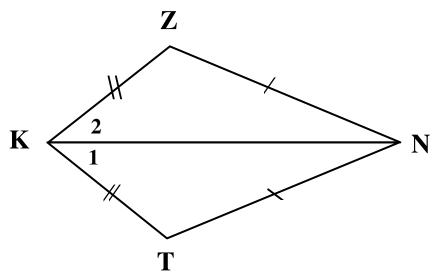
EXAMPLE	SOLUTION
a) Prove: $\triangle$ DEB $\equiv$ $\triangle$ BAD **b)** Write down the other equal angles and sides 	**a)** $E\hat{D}B = D\hat{B}A = 85°$ (given) DE = AB = 5 cm (given) DB = DB (common side) $\triangle$ DEB $\equiv$ $\triangle$ BAD (SAS) **b)** BE = AD (congruency) $D\hat{B}E = B\hat{D}A$ (congruency) $\hat{E} = \hat{A}$ (congruency)

Exercise 7.13

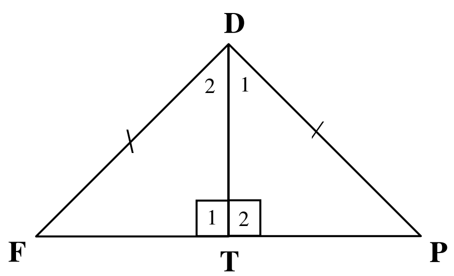
1) a) Prove: $\triangle$ PQR $\equiv$ $\triangle$ STV
b) Write down other equal angles and sides



2) a) Prove $\triangle$ KZN $\equiv$ $\triangle$ KTN **b)** $\hat{K}_1 = \hat{K}_2$

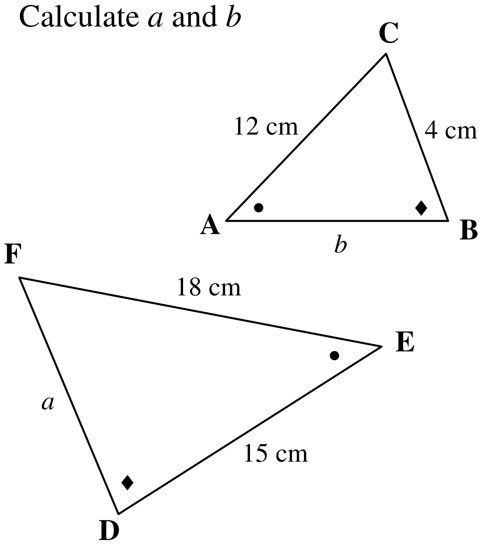


3) a) Prove $\triangle$ DTF $\equiv$ $\triangle$ DTP
b) $\hat{D}_2 = \hat{D}_1$ and $\hat{F} = \hat{P}$



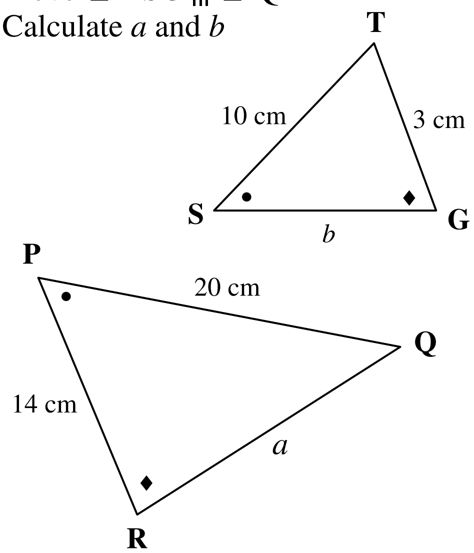
§ 7.14 SIMILAR TRIANGLES

- If two triangles are **equiangular,** their corresponding sides are in proportion.
- If two triangles have their **corresponding sides in proportion**, they are equiangular.
- 'Equiangular' means 'equal angles' which means that the three angles of the one triangle are equal in size to the three angles of the other triangle
- Triangles are **similar** if:
  - they are equiangular **OR**
  - **t**he lengths of their corresponding sides are in proportion

EXAMPLE	SOLUTION
1) Prove △ ABC ||| △ DEF 2) Calculate *a* and *b* 	$\hat{A} = \hat{E}$ $\hat{B} = \hat{D}$ $\hat{C} = \hat{F}$ (angle sum of triangle) △ ABC ||| △ EDF (AAA) $\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$ $\frac{b}{15} = \frac{12}{18} = \frac{4}{a}$ $\frac{b}{15} \times 15 = \frac{12}{18} \times 15$ $b = 10$ cm $\frac{12}{18} \times a \times \frac{18}{12} = \frac{4}{a} \times a \times \frac{18}{12}$ $a = 6$ cm

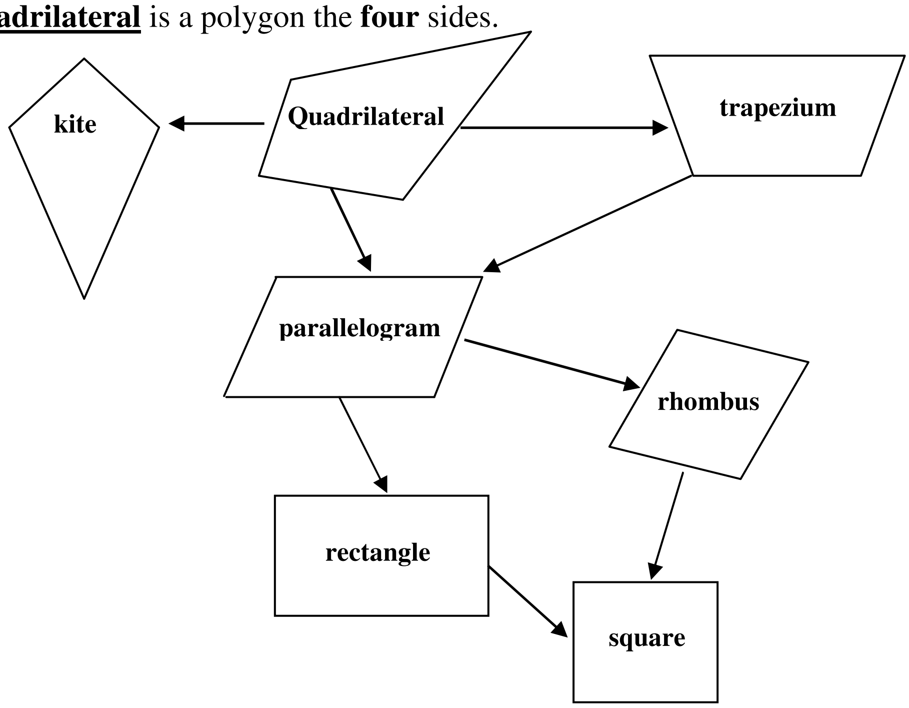
Exercise 7.14

1) Prove △ TSG ||| △ QPR
2) Calculate *a* and *b*



§ 7.15 TYPES OF QUADRILATERALS

A **quadrilateral** is a polygon the **four** sides.



The chart above helps to understand the definitions of the different quadrilaterals.

- A **kite** is a quadrilateral with two pairs of adjacent sides equal.
- A **trapezium** is a quadrilateral with one pair of parallel sides.
- A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- A **rectangle** is a parallelogram with right angles.
- A **square** is a rectangle with four equal sides.
- A **rhombus** is a parallelogram with four equal sides.

Exercise 7.15

1) Define a square in terms of a rhombus. ……………………………………………………..

....

2) Define a parallelogram in terms of a trapezium. ……………………………………………..

....

3) Answer Sometimes (S) or Always (A):

a) A rhombus is a square ……………….

b) A square is a rectangle ……………………

c) A trapezium is a parallelogram ……..

d) A quadrilateral is a trapezium ……………

e) A kite is a quadrilateral ……………..

f) All parallelograms are quadrilaterals ……..

g) A rhombus is a rectangle …………….

h) A kite is a parallelogram ………………….

i) A square is a rhombus ………………

j) All squares are parallelograms ……………

CHAPTER 8
Analytical and transformation geometry

In this chapter you will:

- Use the formula for the gradient of a line and the distance formula
- Use the formula for the midpoint of a line segment
- Find the equation of a line given the co-ordinates of two points on the line
- Change the form of the equation of a straight line
- Translate polygons
- Reflect a polygon about the x-axis and about the y-axis.
- Reflect a polygon about the line $y = x$
- Reflect a polygon about the line $y = -x$

This chapter covers material from Topic 3: Space, Shape and Orientation

SUBJECT OUTCOME 3.2:

Describe and represent the properties of geometric shapes

Learning Outcome 1	Investigate polygons through Euclidian, co-ordinate and/or transformation geometry

SUBJECT OUTCOME 3.3

Represent geometric figures on a Cartesian co-ordinate system

Learning Outcome 1:	Plot points, lines and polygons on a Cartesian plane
Learning Outcome 2:	Calculate the distance between two points
Learning Outcome 3:	Calculate the gradient of a line segment joining two points
Learning Outcome 4:	Calculate the midpoint of a line segment joining two points on a graph
Learning Outcome 5:	Estimate distance, slope and midpoint from a graphical representation of a straight line

SUBJECT OUTCOME 3.4

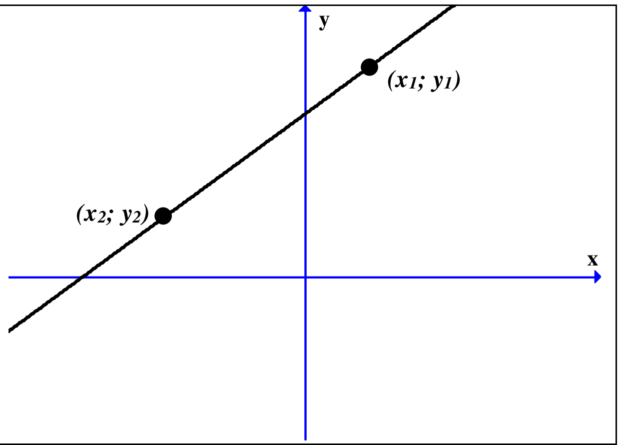
Solve problems by constructing and interpreting geometrical and trigonometric models

Learning Outcome 1:	Use transformation geometry to translate p units horizontally and q units vertically
Learning Outcome 2:	Use transformation geometry to reflect graphs about the x-axis, the y-axis and the line $y = x$

§ 8.1 GRADIENT AND DISTANCE FORMULAE

- The **gradient** of a straight line passing through the points (x_1; y_1) and (x_2; y_2) is $a = \dfrac{y_2 - y_1}{x_2 - x_1}$
- The **distance** between the points (x_1; y_1) and (x_2; y_2) is $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$.

Note:
- Points that lie on the same straight line are called **collinear** points
- The gradient is the same between any two points on a straight line



EXAMPLE	SOLUTION
1) Determine whether the points A(1; –1), B(–1; –5) and C(2; 1) are collinear.	Gradient $_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-5-(-1)}{-1-1} = \dfrac{-4}{-2} = 2$ Gradient $_{BC} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-(-5)}{2-(-1)} = \dfrac{1+5}{2+1} = \dfrac{6}{3} = 2$ Gradient $_{AB}$ = Gradient $_{BC}$ $\therefore$ A, B and C are collinear (gradients are the same)
2) Prove that $\triangle$ PQR with vertices P(1; 1), Q(1; 4) and R(5; 1) is right-angled. Where is the right-angle in the triangle?	Distance $_{PQ} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(4-1)^2 + (1-1)^2} = \sqrt{3^2} = 3$ Distance $_{QR} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(1-4)^2 + (5-1)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ Distance $_{PR} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(1-1)^2 + (5-1)^2} = \sqrt{4^2} = 4$ • PQ : PR : QR = 3 : 4 : 5, so the triangle is right-angled. • QR is the hypotenuse, so the right angle is at P.

Exercise 8.1

1) Prove that $\triangle$ ABC having vertices A(2; 4), B(0; 3) and C(6; –4) is right-angled. Where is the right-angle in the triangle?

2) Determine whether the points P(2; –4), R(–2; –8) and S(0; –2) are collinear.

§ 8.2 FORMULA FOR THE MIDPOINT OF A LINE SEGMENT

✧ The formula for finding the co-ordinates of the midpoint of a line segment is $\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ where $(x_1; y_1)$ and $(x_2; y_2)$ are points on the graph

EXAMPLE	SOLUTION
1) Calculate the mid-point C of the line joining A(–2; –3) to B(4; –9).	$C(\frac{-2+4}{2};\ \frac{-3+(-9)}{2}) = (\frac{2}{2};\ \frac{-12}{2}) = (1; -6)$
2) Calculate a and b if R(4; –3) is the mid-point of the line joining P(a; –8) to Q(10; b).	$4 = \frac{a+10}{2}$ $4 \times 2 = \frac{a+10}{2} \times 2$ $8 = a + 10$ $8 - 10 = a + 10 - 10$ $-2 = a$ $-3 = \frac{-8+b}{2}$ $-3 \times 2 = \frac{-8+b}{2} \times 2$ $-6 = -8 + b$ $-6 + 8 = 8 + b + 8$ $2 = b$

<u>Exercise 8.2</u>

Calculate x and y:

1) if P(x; y) is the mid-point of the line joining Q(–1; 4) to R(5; –6)

2) if N(0; 2) is the mid-point of the line joining C(x; y) to D(6; 0)

3) if R(6; –2) is the mid-point of the line joining P(–3; y) to Q(x; –8)

§ 8.3 FINDING THE EQUALTION OF A STRAIGHT LINE

✧ Suppose a straight line passes through the points (x; y) and (x_1; y_1)

The gradient of the line is $a = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Then $a(x - x_1) = \dfrac{y - y_1}{(x - x_1)} \times (x - x_1)$

So $\boldsymbol{a(x - x_1) = y - y_1}$ **is the equation of a straight line graph.**

EXAMPLE	SOLUTION
1) Find the equation of the line passing through the points D(–3; –7) and F(–5; 3). **<u>Note:</u>** • It does not matter which point is used to substitute into the equation.	$a = \dfrac{y - y_1}{x - x_1} = \dfrac{3-(-7)}{-5-(-3)} = \dfrac{3+7}{-5+3} = \dfrac{10}{-2} = -5$ Use point D(–3; –7) and substitute into $a(x - x_1) = y - y_1$ $-5(x - (-3)) = y - (-7)$ $-5(x + 3) = y + 7$ $-5x - 15 - 7 = y + 7 - 7$ $-5x - 22 = y$ **<u>OR</u>**, using point F(–5; 3): $-5(x - (-5)) = y - 3$ $-5(x + 5) = y - 3$ $-5x - 25 + 3 = y - 3 + 3$ $-5x - 22 = y$
2) Does the point G(–2; 3) lie on the line having equation $y = \frac{3}{2}x - 4$.? **<u>Note:</u>** • Substitute $x = -2$ into the RHS of the equation and $y = 3$ into the LHS of the equation.	LHS = $y = 3$ RHS = $\frac{3}{2}x - 4 = \frac{3}{2}(-2) - 4 = -3 - 4 = -7$ LHS $\neq$ RHS, So G does not lie on the line.

<u>Exercise 8.3</u>

1) Find the equation of the line passing through the points B(–4; 9) and T(–6; 5).

2) Does the point G(–2; 3) lie on the line having equation $y = \frac{3}{2}x - 4$.

§ 8.4 CHANGING THE FORM OF A LINEAR EQUATION

✧ Equations of straight line graphs can be written in different forms. These can be converted from one form to another.

EXAMPLE	SOLUTION
1) Write the equation $y = \frac{-1}{2}x - \frac{1}{4}$ in the form $Ax + By + C = 0$	The HCF of 2 and 4 is 4. So multiply both sides by 4. $4y = 4(\frac{-1}{2}x - \frac{1}{4})$ $4y = -2x - 1$ $4y + 2x + 1 = -2x - 1 + 2x + 1$ $4y + 2x + 1 = 0$
2) Write the equation $3y - 2x + 9 = 0$ in the form $y = ax + q$	To have y alone on one side, we must divide each term on both sides by 3. $\frac{3y}{3} - \frac{2x}{3} + \frac{9}{3} = \frac{0}{3}$ $y - \frac{2x}{3} + 3 = 0$ $y - \frac{2x}{3} = -3$ $y - \frac{2x}{3} + \frac{2x}{3} = -3 + \frac{2x}{3}$ $y = \frac{2x}{3} - 3$

<u>Exercise 8.4</u>

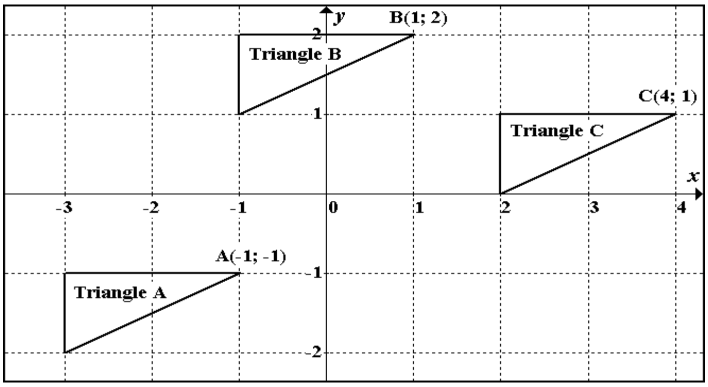
1) Write the equation $y = \frac{2}{3}x - \frac{5}{6}$ in the form $Ax + By + C = 0$

2) Write the equation $6x - 4y - 8 = 0$ in the form $y = ax + q$.

§ 8.5 TRANSLATING POLYGONS

- Translating a polygon changes its position, but not its size or shape.
- A translation is written (x; y), and acts on every point in the polygon.

EXAMPLE:

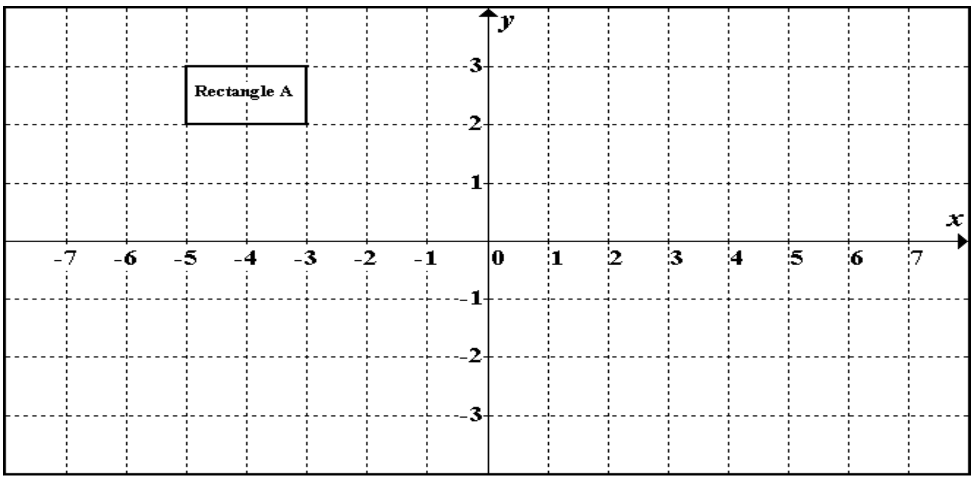


In the above graph:
Triangle A has moved 2 units to the right and 3 units up to triangle B. Translation (2; 3).
The point (–1; –1) moved to the point (–1 + 2; –1 + 3) = (1; 2).
Triangle B moved 3 units to the right and 1 unit down to triangle C. Translation (3; –1).
The point (1; 2) has moved to the point (1 + 3; 2 – 1) = (4; 1)

EXAMPLE	SOLUTION
Find the translation as triangle A in the above diagram moves to triangle C. A(–1; –1) moves to C(4; 1).	Let the translation be (x; y). $-1 + x = 4$ and $-1 + y = 1$ $-1 + x + 1 = 4 + 1$ $-1 + y + 1 = 1 + 1$ $x = 5$ $y = 2$ The translation is (5; 2)

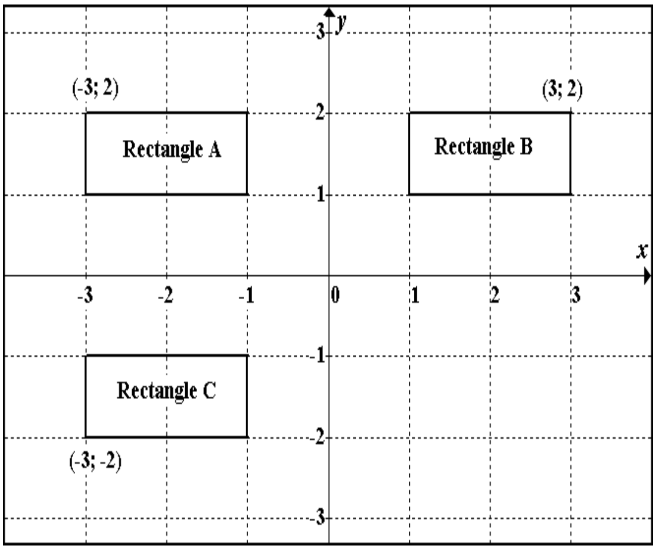
Exercise 8.5

On the grid, draw the image of Rectangle A after each of the following translations:
1) Rectangle B after the translation (6; –4)
2) Rectangle C after the translation (4; –2)
3) Rectangle D after the translation (–2; –5)
4) Rectangle E after the translation (8; –6)



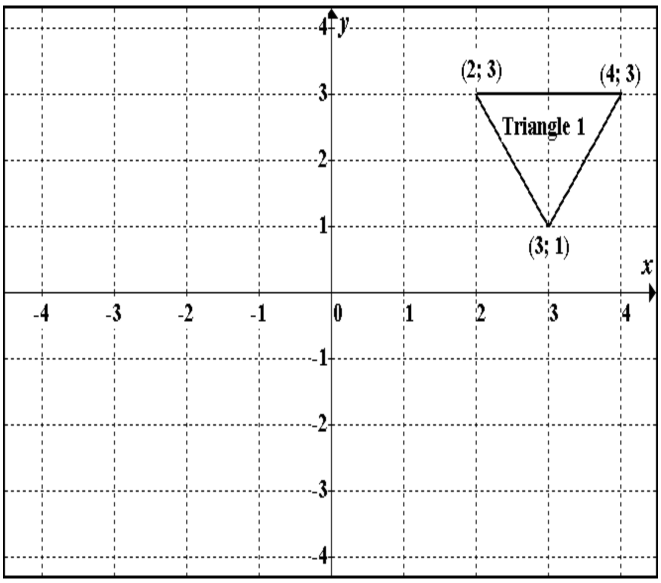
§ 8.6 REFLECTION OF A POLYGON ABOUT THE x-AXIS AND ABOUT THE y-AXIS

- Reflecting a polygon about the ***x*-axis** changes **the sign of the *y*-coordinate** of each point on the polygon. In the graph below, (–3; **2**) on Rectangle A reflects about the *x*-axis to (–3; **–2**) on Rectangle B.
- Reflecting a polygon about the ***y*-axis** changes **the sign of the *x*-coordinate** of each point on the polygon. In the graph, (**–3**; 2) on Rectangle A reflects about the *y*-axis to (**3**; 2) on Rectangle B.



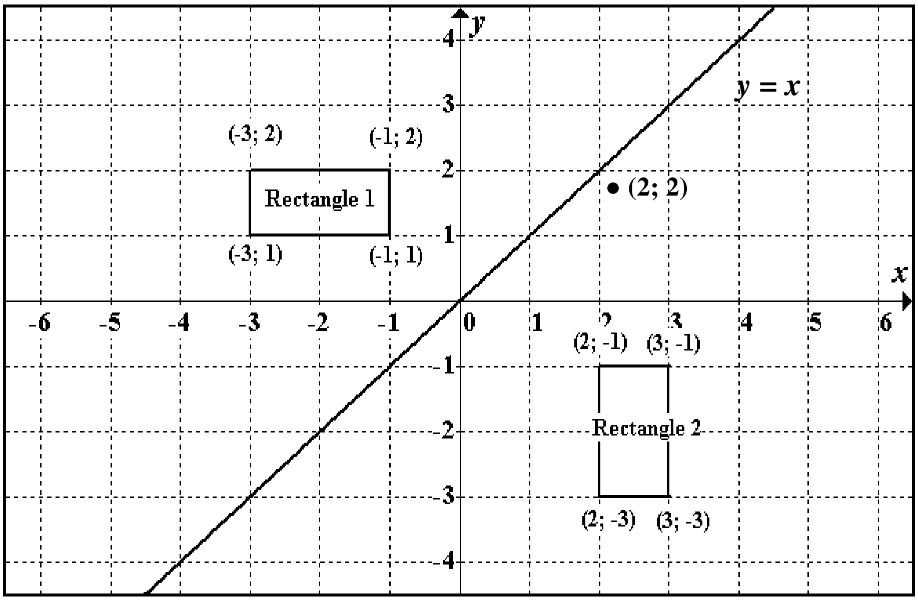
Exercise 8.6

1) On the grid:
   a) draw the reflection of Triangle 1 in the *x*-axis (Triangle 2)
   b) the reflection of Triangle 1 in the *y*-axis (Triangle 3).
2) Show the coordinates of all the vertices.



§ 8.7 REFLECTION OF A POLYGON ABOUT THE LINE $y = x$.

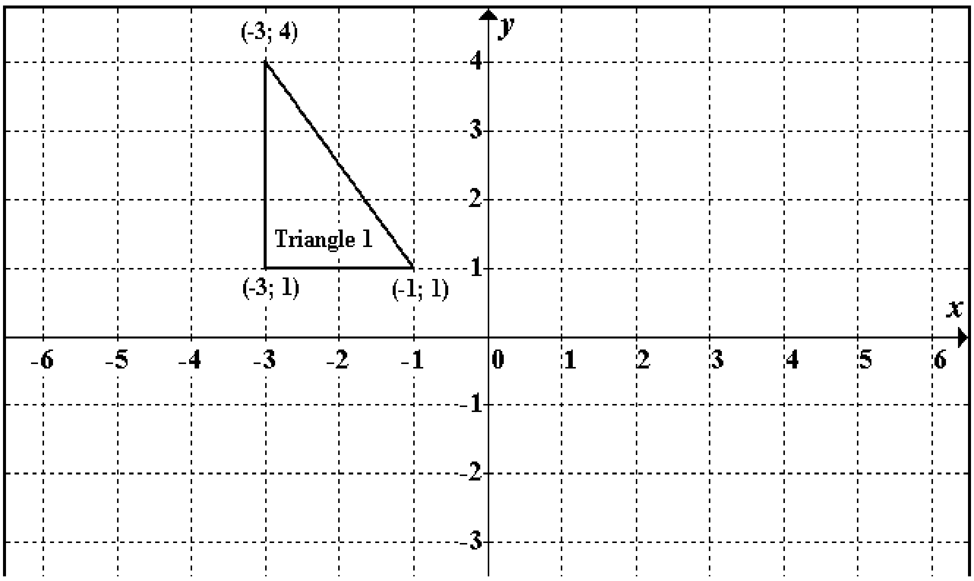
✧ Reflecting a polygon in the line $y = x$, reverses the x and y-coordinates of each point on the polygon. In the graph, (–3; 2) on Rectangle 1 reflects in the x-axis to (2; –3) on Rectangle 2.



Exercise 8.7

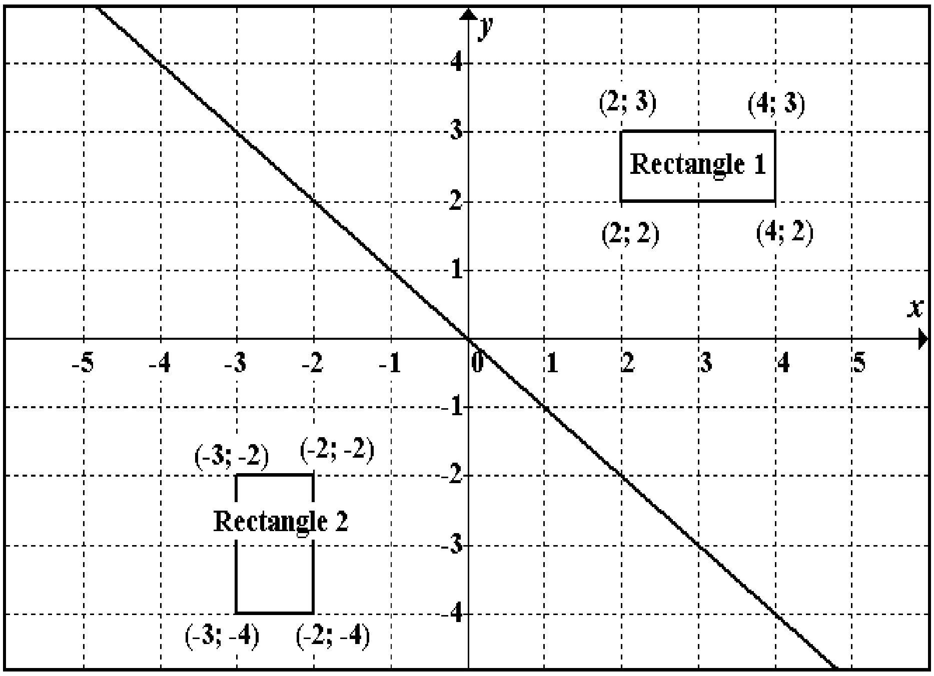
1) Draw the line $y = x$.
2) Reflect triangle 1 in the line $y = x$ and draw it on the grid (Triangle 2). Show the coordinates of each of the vertices.

b



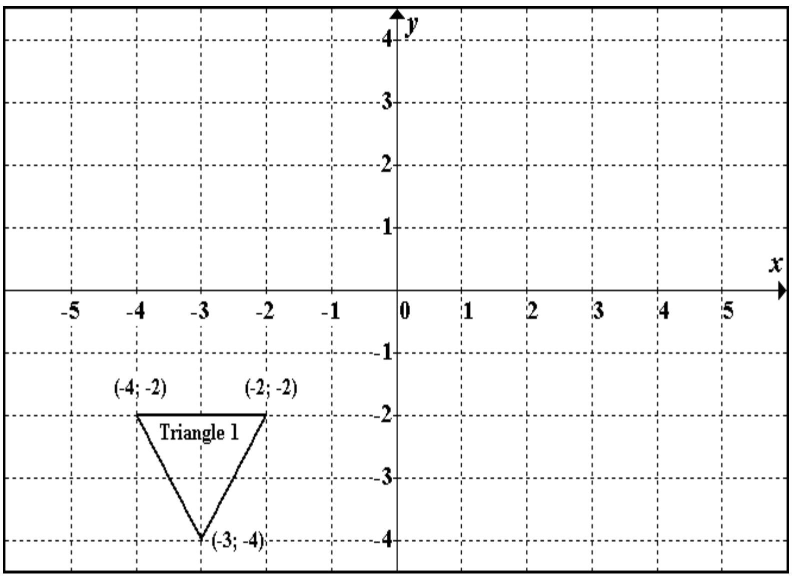
§ 8.8 REFLECTION OF A POLYGON ABOUT THE LINE $y = -x$

✧ Reflecting a polygon in the line $y = -x$, reverses the x and y-coordinates of each point on the polygon and changes the signs of both. In the graph, (2; 3) on Rectangle 1 reflects in the line $y = -x$ to (–3; –2) on Rectangle 2.



Exercise 8.8

1) Draw the line $y = -x$.
2) Reflect the polygon (Triangle 1) in the line $y = -x$ and draw it on the grid (Triangle 2). Show the coordinates of each of the vertices.



CHAPTER 9
Trigonometry

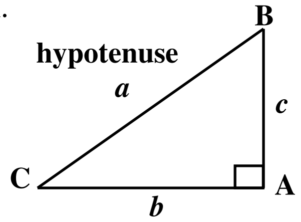
In this chapter you will:

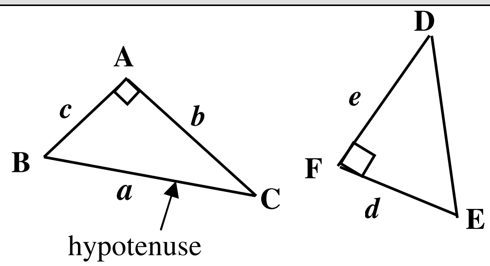
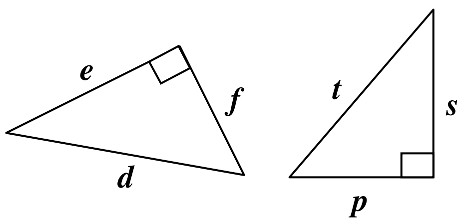
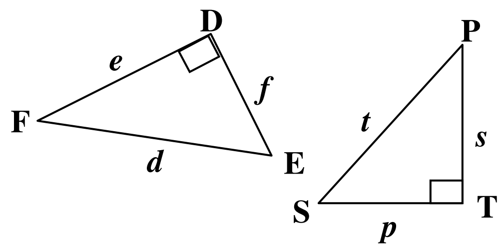
- Name sides and angles in a right-angled triangle
- Practise using the theorem of Pythagoras
- Use the definitions for the three trig ratios
- Use the sin and cos ratios to find the length of a side
- Use the sin and cos ratios to find the length of the hypotenuse
- Use the tan ratio to find the length of a side
- Use the sin ratio to find the size of an angle
- Use the cos ratio to find the size of an angle
- Use the tan ratio to find the size of an angle
- Draw the graph of the sin function
- Draw the graph of y = a sin x
- Draw the graph of y = sin x + q
- Draw the graph of y = a sin x + q
- Draw the graph of the cos function
- Draw the graph of y = a cos x + q
- Draw the graph of the tan function.

This chapter covers material from Topic 3: Space, Shape and Orientation
SUBJECT OUTCOME 3.4:
Solve problems by constructing and interpreting geometrical and trigonometric models
Learning Outcome 5: Know and use the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$
Learning Outcome 6: Sketch the sin, cos and tan functions and discuss their periodicity.

§ 9.1 NAMING SIDES AND ANGLES IN A RIGHT-ANGLED TRIANGLE

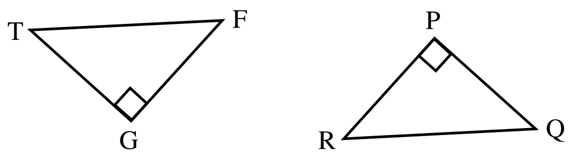
- The sides of a triangle are named from the angles opposite them.
  In Δ ABC, BC, the side BC opposite angle A ($\hat{A}$) is called *a*.
  the side AC opposite angle B ($\hat{B}$) is called *b*.
  the side AB opposite angle C ($\hat{C}$) is called *c*.
- The side opposite the right angle, $\hat{A}$, is the **hypotenuse**.



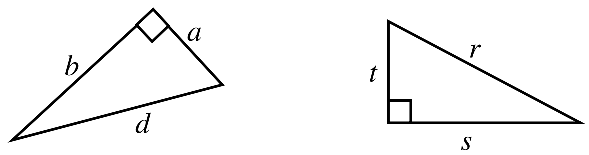
EXAMPLE	SOLUTION
1) Label the sides in Δ ABC which is right-angled at A, and Δ DEF which is right-angled at F. Mark the hypotenuse in each triangle.	
2) Label each vertex and hypotenuse on the following two triangles:  **NOTE:** One vertex, many vertices.	

Exercise 9.1

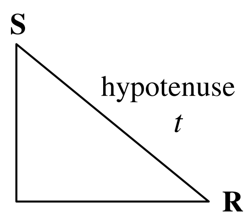
1) Label
   a) Each of the sides in each of the triangles
   b) Each hypotenuse.



2) For each triangle label the vertices and the hypotenuse:

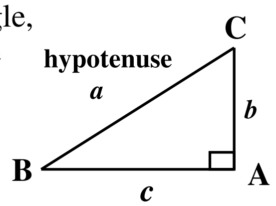


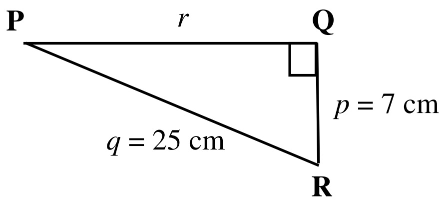
3) For following triangle fill in the labels for the angles and sides that are missing.



§ 9.2 THE THEOREM OF PYTHAGORAS

✧ The **Theorem of Pythagoras** states that in a right-angled triangle, the hypotenuse squared is equal to the sum of the squares of the other two sides.
In other words, $a^2 = b^2 + c^2$ **only in a right - angled triangle**.

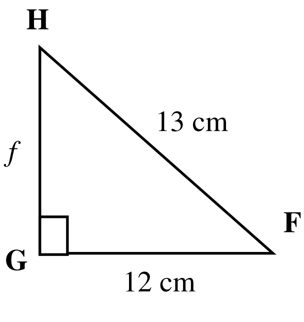


EXAMPLE	SOLUTION
Calculate r in ΔPQR:  **Note:** • this triangle is **right - angled.**	Because ΔPQR is right angled, $q^2 = p^2 + r^2$ (Pythagoras) $25^2 = 7^2 + r^2$ $625 = 49 + r^2$ $625 - 49 = r^2$ $576 = r^2$ $\sqrt{576} = r$ $24 \text{ cm} = r$

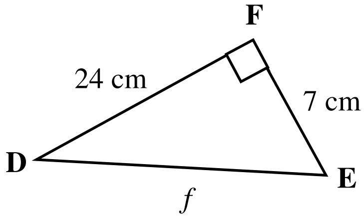
Exercise 9.2

Use the theorem of Pythagoras to:

1) find the length of side f in ΔFGH

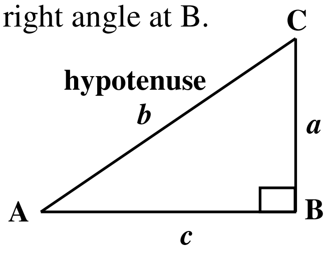


2) calculate the length of side f in ΔDEF



§ 9.3 THREE TRIG RATIOS IN A RIGHT-ANGLED TRIANGLE

- In triangle ABC, ***b*** is the **hypotenuse**, since it is opposite the right angle at B.
- For $\hat{A}$: *a* is the **opposite** side (opposite $\hat{A}$)
  c is the **adjacent** side (next to $\hat{A}$)
- For $\hat{C}$: *c* is the **opposite** side (opposite $\hat{C}$)
  a is the **adjacent** side (next to $\hat{C}$)

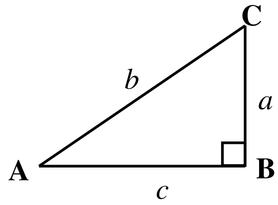


- We define the three trigonometric ratios *sin*, *cos* and *tan* as follows:

$\sin \hat{A} = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{a}{b}$ $\quad\sin \hat{C} = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{c}{b}$

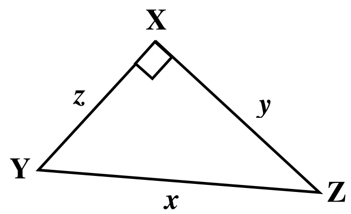
$\cos \hat{A} = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{c}{b}$ $\quad\cos \hat{C} = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{a}{b}$

$\tan \hat{A} = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{a}{c}$ $\quad\tan \hat{C} = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{c}{a}$



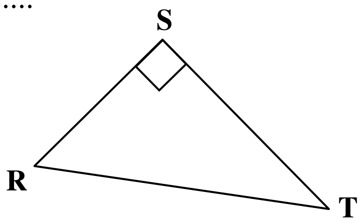
Exercise 9.3

1) Use $\triangle$ XYZ to write down:



   a) the side adjacent to $\hat{Z}$ ....
   b) the side opposite $\hat{Y}$ ....
   c) the hypotenuse ....
   d) the side adjacent to $\hat{Y}$ ....
   e) the side opposite $\hat{Z}$ ....

2)



   a) Label the sides of $\triangle$ RST.
   b) Use $\triangle$ RST to write down:
      i) $\tan \hat{R}$ = ... ii) $\tan \hat{T}$ = ...
      iii) $\sin \hat{R}$ = ... iv) $\sin \hat{T}$ = ...
      iv) $\cos \hat{R}$ = ... v) $\cos \hat{T}$ = ...

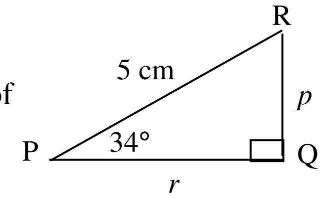
3) Draw a triangle in which $\cos \hat{P} = \dfrac{b}{t}$

§ 9.4 USING THE SIN AND COS RATIOS TO FIND THE LENGTH OF A SIDE

EXAMPLE

Given: Hypotenuse = 5 cm $\hat{P} = 34°$

Use Δ PQR to find the length, correct to 2 decimal places of
1) sides p and
2) side r,



SOLUTION

1) Side p is opposite $\hat{P}$, so we use the sin ratio which contains both opposite and hypotenuse:

$$\sin 34° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{p}{5}$$

$$5 \times \sin 34° = 5 \times \frac{p}{5}$$

2,795 …. cm = p
So p = 2,80 (correct to 2 decimal places)

Key sequence	Display
5 sin 34 =	2.795 ….

2) Side r is adjacent to $\hat{P}$, so use the cos ratio which contains both adjacent and hypotenuse:

$$\cos 34° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{r}{5}$$

$$5 \times \cos 34° = 5 \times \frac{r}{5}$$

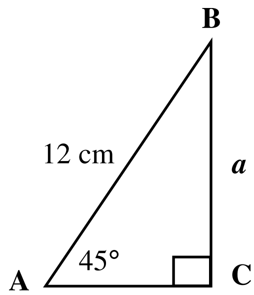
So r = 4,15 (correct to 2 decimal places)

Key sequence	Display
5 cos 34 =	4.145 ….

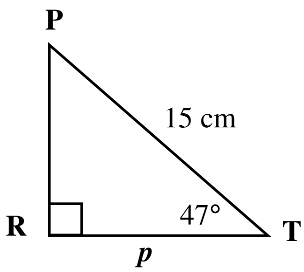
Exercise 9.4

Give answers correct to 2 decimal places:

1) Use Δ ABC to find the length of side a.



2) In Δ PTR, calculate the length of side p.

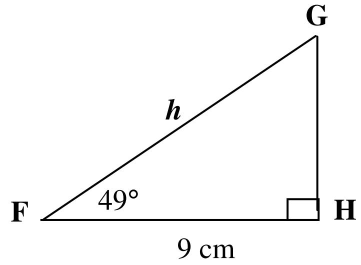


§ 9.5 USING THE SIN AND COS RATIOS TO FIND THE LENGTH OF THE HYPOTENUSE

EXAMPLE

Given: $\hat{F} = 49°$ and $g = 9$ cm, adjacent to $\hat{F}$

Find the length of the hypotenuse h of Δ FGH, correct to the nearest whole cm



SOLUTION

Side h is the hypotenuse. Use the cos ratio which contains both adjacent and hypotenuse:

$$\cos 49° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{9}{h}$$

$$h \times \cos 49° = h \times \frac{9}{h}$$

$$h \cos 49° = 9$$

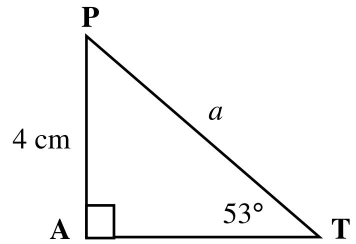
$$\frac{h \cos 49°}{\cos 49°} = \frac{9}{\cos 49°}$$

$h = 14$ cm (correct to the nearest cm)

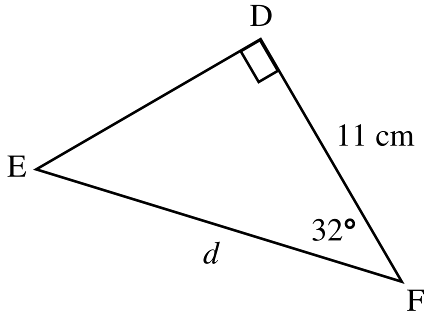
Key sequence	Display
9 $\boxed{\div}$ $\boxed{\cos}$ 49 $\boxed{=}$	13.71….

Exercise 9.5

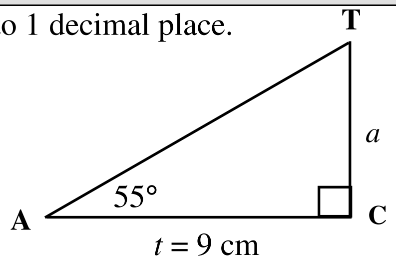
1) Calculate the length of g, the hypotenuse of ΔPTA. Answer to the nearest whole cm.



2) Calculate the length of hypotenuse d in Δ DEF, correct to the nearest whole cm.



§9.6 USING THE TAN RATIO TO FIND THE LENGTH OF A SIDE

EXAMPLE
Use Δ CAT to find the length of side a, correct to 1 decimal place. 

SOLUTION

Side a is opposite to the given $\hat{A}$ and side t is adjacent to the given $\hat{A}$, so we use the tan ratio which contains both opposite and adjacent.

$$\tan 55° = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{a}{9}$$

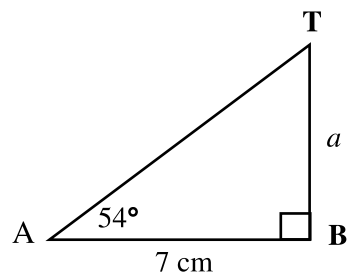
$$9 \times \tan 55° = 9 \times \frac{a}{9}$$

$$12{,}9 \text{ cm} = a$$

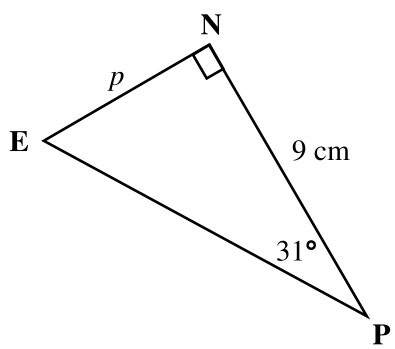
Key sequence	Display
9 $\boxed{\tan}$ 55 $\boxed{=}$	12.853….

Exercise 9.6

1) Find the length of side a in Δ BAT, correct to one decimal place.



2) Calculate the length of side p in Δ PEN, correct to one decimal place.



§ 9.7 USING THE SIN RATIO TO FIND THE SIZE OF AN ANGLE

✧ We can use the three trig ratios to calculate the size of an angle in a right-angled triangle, by using the keys on the calculator labelled sin $^{-1}$, cos $^{-1}$ and tan $^{-1}$

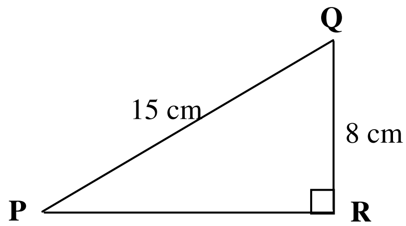
<u>Note:</u>

- to use $\boxed{\sin^{-1}}$ key in $\boxed{\text{shift}}$ $\boxed{\sin}$
- to use $\boxed{\cos^{-1}}$ key in $\boxed{\text{shift}}$ $\boxed{\cos}$
- to use $\boxed{\tan^{-1}}$ key in $\boxed{\text{shift}}$ $\boxed{\tan}$

EXAMPLE

In the right-angled Δ PQR calculate:

1) the size of $\hat{P}$, correct to one decimal place
2) the size of $\hat{Q}$.



SOLUTION

Given: the length of the hypotenuse and of the side opposite to $\hat{P}$, so use the sin ratio

1) $\sin \hat{P} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$= \dfrac{8}{15}$

Key sequence	Display
$\boxed{\sin^{-1}}$ 8 $\boxed{\div}$ 15 $\boxed{=}$	32.23…..

$\hat{P} = 32{,}2°$ (correct to one decimal place)

2) The three angles of a triangle add up to 180°, so $32{,}2° + 90° + \hat{Q} = 180°$

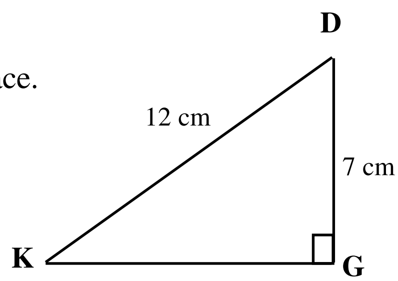
$122{,}2° + \hat{Q} = 180°$

$\hat{Q} = 180° - 122{,}2°$

$= 57{,}8°$

<u>Exercise 9.7</u>

1) Use Δ DKG to find the size of $\hat{K}$ correct to 1 decimal place.



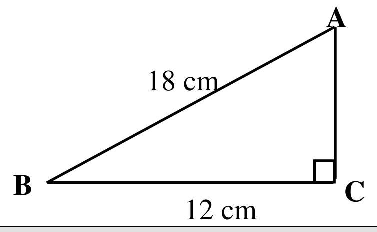
2) Find the size of $\hat{D}$ correct to one decimal place.

§ 9.8 USING THE COS RATIO TO FIND THE SIZE OF AN ANGLE

EXAMPLE

In the right-angled triangle ABC, calculate:

1) the size of $\hat{B}$ correct to the nearest whole degree
2) the size of $\hat{A}$



SOLUTION

1) Given: the lengths of the hypotenuse and the side adjacent to angle B, so use the cos ratio.

$$\cos \hat{B} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{12}{18}$$

Key sequence	Display
$\boxed{\cos^{-1}}$ 12 $\boxed{\div}$ 18 $\boxed{=}$	48.18…..

$\hat{B} = 49°$ (correct to the nearest degree)

2) The three angles of a triangle add to 180°, so $\hat{A} + 90° + 49° = 180°$

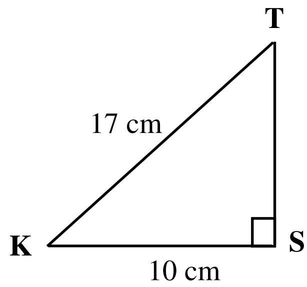
$\hat{A} + 139° = 180°$

$\hat{A} = 180° - 139°$

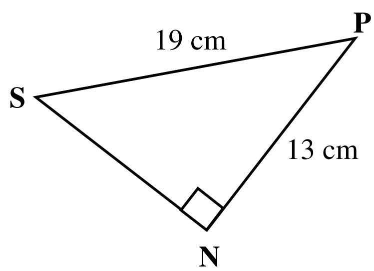
$= 41°$

Exercise 9.8

1) Calculate the sizes of $\hat{K}$ and $\hat{T}$ in triangle KST, correct to the nearest whole degree.



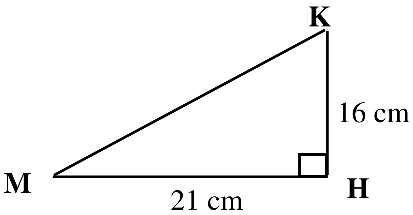
2) Use Δ TRS to find the sizes of $\hat{P}$ and $\hat{S}$, correct to one decimal place.



§ 9.9 USING THE TAN RATIO TO FIND THE SIZE OF AN ANGLE

EXAMPLE

In Δ MHK, calculate the sizes of $\hat{M}$ and $\hat{K}$, correct to the nearest whole degree.



SOLUTION

Given: the length of the opposite and adjacent sides to $\hat{K}$, so use the tan ratio.

$\tan \hat{K} = \dfrac{\text{opposite}}{\text{adjacent}}$

$= \dfrac{21}{16}$

$\hat{K} = 53°$

Key sequence	Display
$\boxed{\tan^{-1}}$ 21 $\boxed{\div}$ 16 $\boxed{=}$	52.69…..

Since the angles of a triangle add up to 180°, $\hat{M} + 90° + 53° = 180°$

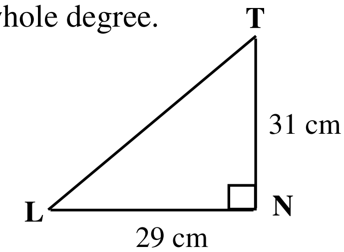
$\hat{M} + 143° = 180°$

$\hat{M} = 180° - 143°$

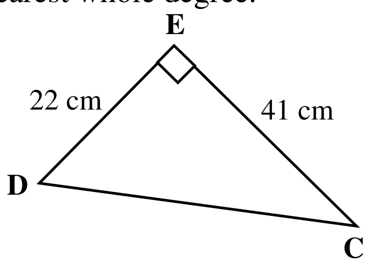
$= 37°$

<u>Exercise 9.9</u>

1) Calculate the size of $\hat{L}$ and $\hat{T}$ in Δ LNT, correct to the nearest whole degree.



2) In Δ CDE, find the size of $\hat{C}$ and $\hat{D}$, both correct to the nearest whole degree.



§ 9.10 THE GRAPH OF THE SIN FUNCTION

- The x-axis is measured in degrees and the y-axis in units.
- The **period** of a trig graph is the number of degrees (measured along the x-axis) necessary to complete one pattern.
- The **amplitude** is the positive distance between the axis and the maximum y value on the graph.

EXAMPLE

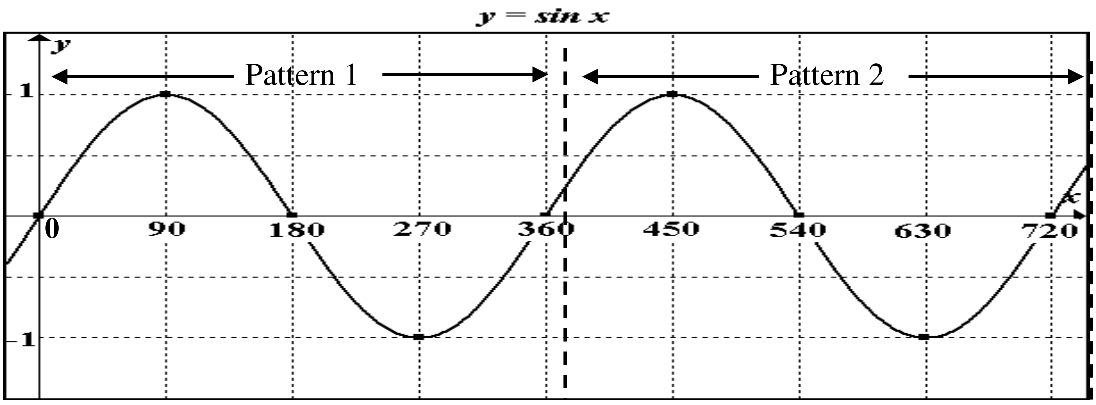
1) Complete the table using a calculator
2) Use the table to draw the graph of $y = \sin x$
3) Write down the period and amplitude of the graph.

SOLUTION

1)

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
$y = \sin x$	0	1	0	–1	0	1	0	–1	0

2)

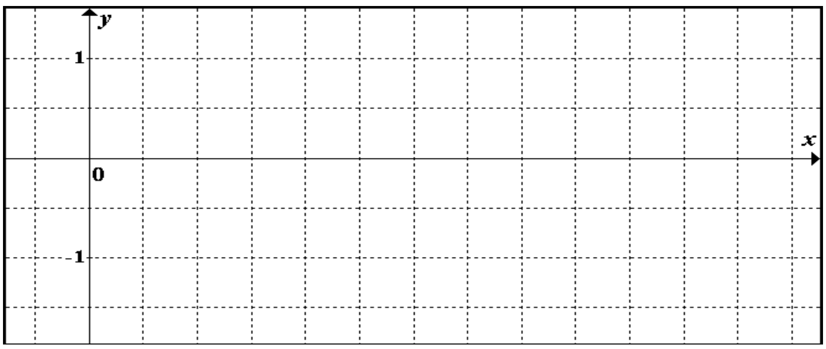


3) One graph pattern is completed in 360°. $y = \sin x$ has a **period = 360°**.
The graph shows two complete patterns. More x values to the left and right would show that this pattern is repeated endlessly, in both the negative and the positive directions.
The distance between the x-axis and the maximum y value is 1, so the **amplitude = 1**.

Exercise 9.10

1) Use a calculator to complete the table. **2)** Draw the graph of $y = \sin x$ for $0° \leq x \leq 180°$.

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
$y = \sin x$									



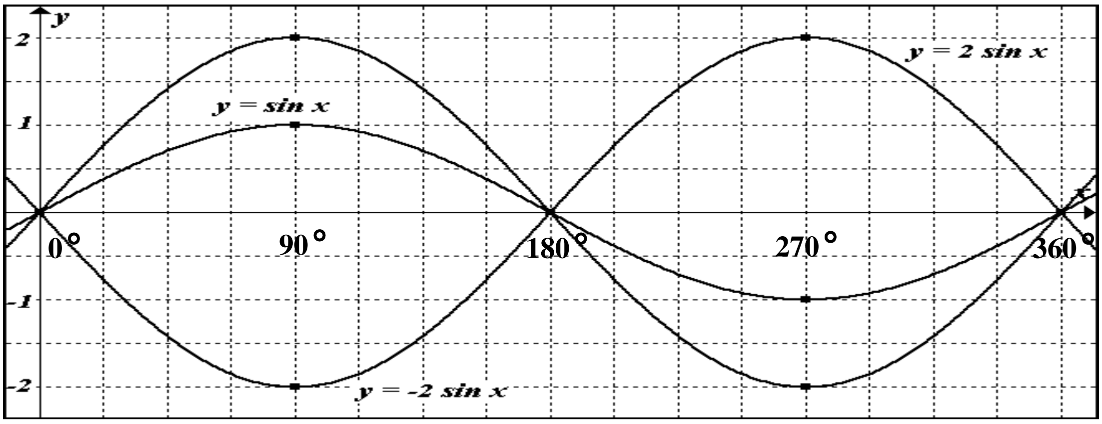
§ 9.11 THE GRAPH OF $y = a \sin x$

EXAMPLE

Complete the table and on the same set of axes draw graphs of $y = 2\sin x$, $y = \sin x$ and $y = -2\sin x$.

SOLUTION

x	0°	90°	180°	270°	360°
$y = 2 \sin x$	0	2	0	–2	0
$y = \sin x$	0	1	0	–1	0
$y = -2 \sin x$	0	–2	0	2	0



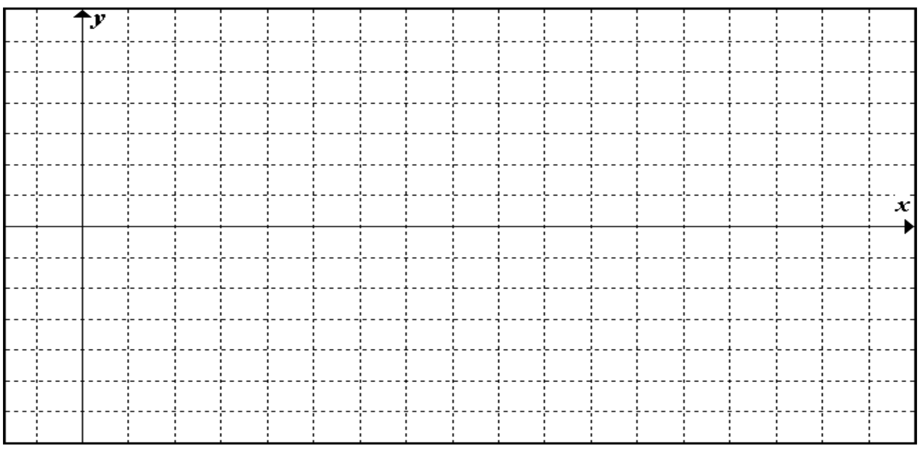
Note:

- Graphs have period 360°. The value of a does not affect the period of $y = a \sin x$.
- $a > 0$ **and** $y > 0$, the graph lies in quadrants 1 and 2
- $a > 0$ **and** $y < 0$, the graph lies in quadrants 3 and 4
- $a < 0$ **and** $y < 0$, the graph lies in quadrants 1 and 2
- $a < 0$ **and** $y > 0$, the graph lies in quadrants 3 and 4
- The positive numerical value of a gives the amplitude of the graph of $y = a \sin x$.

Exercise 9.11

Complete the table, draw and label the graphs of $y = 3\sin x$ and $y = -\sin x$ for $0 \leq x \leq 360°$.

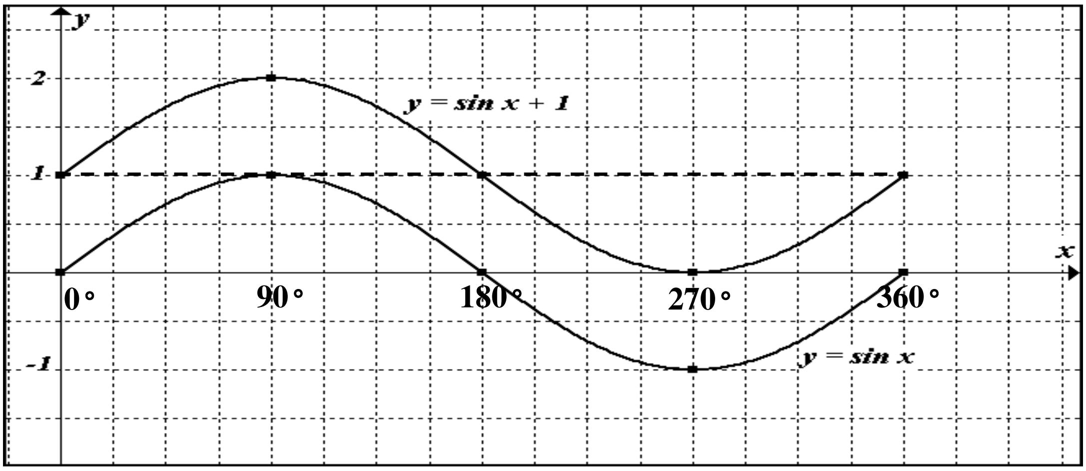
x	0°	30°	90°	150°	180°	210°	270°	330°	360°
$y = 3 \sin x$									
$y = -\sin x$									



§ 9.12 THE GRAPH OF $y = \sin x + q$

EXAMPLE
Complete the table and on the same set of axes draw the graphs of $y = \sin x$ and $y = \sin x + 1$, where $0 \leq x \leq 360°$.
SOLUTION

x	$0°$	$90°$	$180°$	$270°$	$360°$
$y = \sin x$	0	1	0	-1	0
$y = \sin x + 1$	1	2	1	0	1



Note:

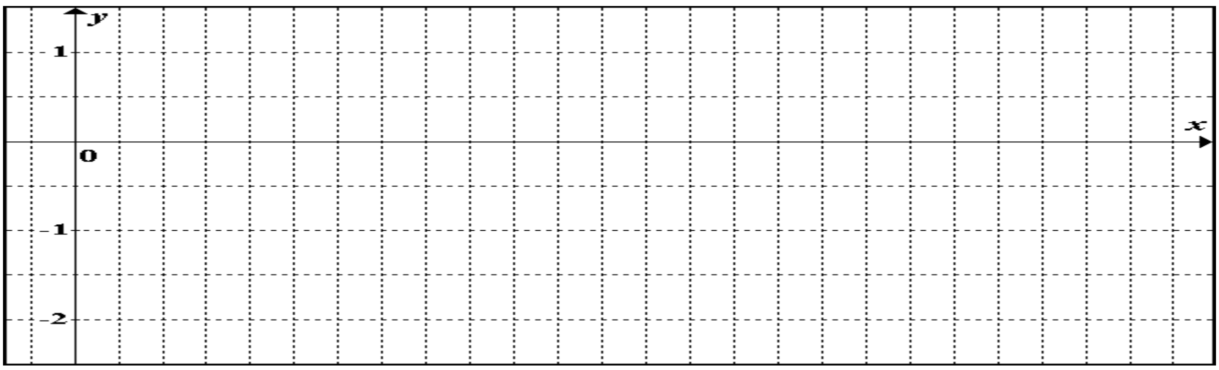
For $y = \sin x + 1$:

- The period of this graph is $360°$, so q does not affect the period.
- The graph rotates about the line $y = 1$ (dotted line). The amplitude is still 1.
- The effect of $q = 1$ is to raise the graph vertically through 1 unit.

Exercise 9.12

Complete the following table and on the same axes draw the graphs of $y = \sin x$ and $y = \sin x - 1$, where $0 \leq x \leq 360°$.

x	$0°$	$30°$	$90°$	$150°$	$180°$	$210°$	$270°$	$330°$	$360°$
$y = \sin x$									
$y = \sin x - 1$									



§ 9.13 THE GRAPH OF $y = a \sin x + q$

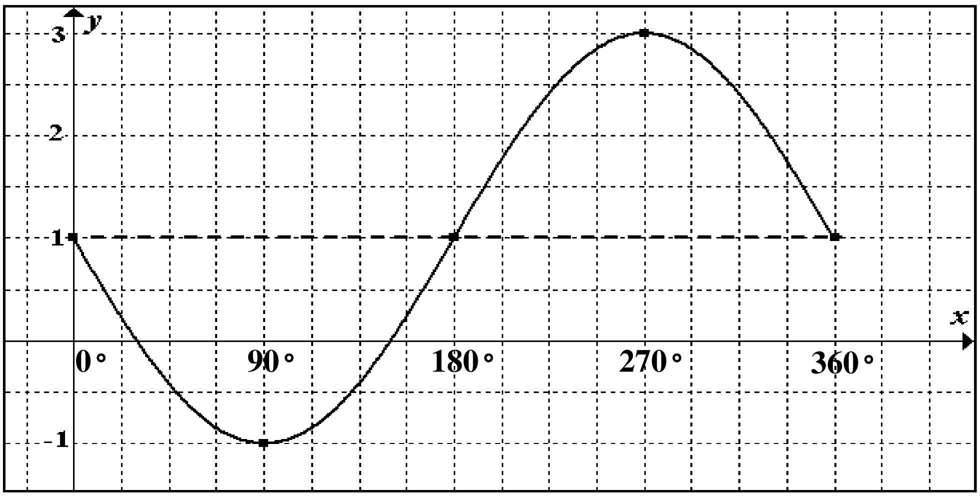
EXAMPLE

Complete the table and then draw the graph of $y = -2 \sin x + 1$, where $0 \leq x \leq 360°$.

SOLUTION

x	0°	90°	180°	270°	360°
$y = -2 \sin x + 1$	1	–1	1	3	1

$y = -2 \sin x + 1$



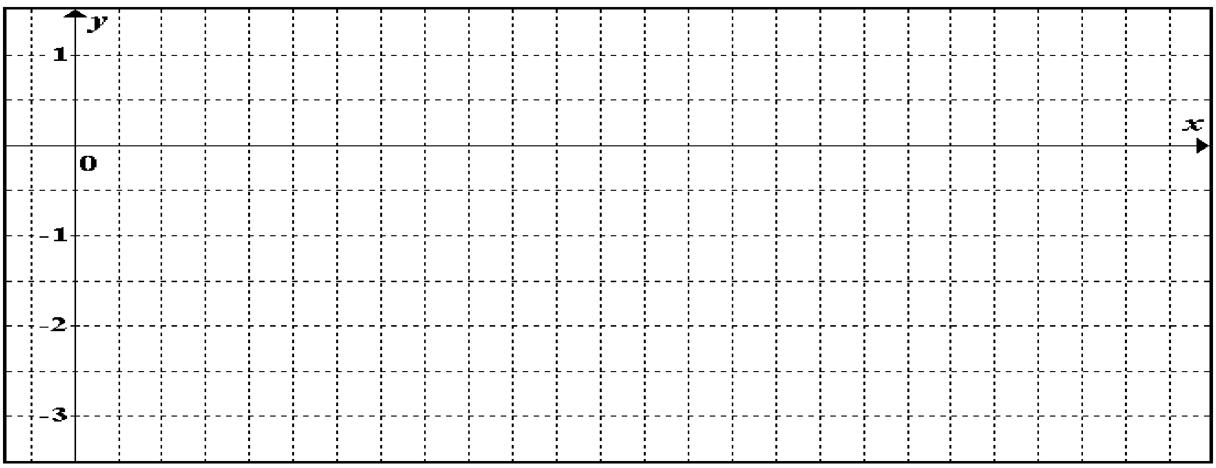
Note:

- The period is 360°
- The graph of $y = -2 \sin x + 1$ rotates about the line $y = 1$ (dotted line)
- The amplitude is 2

Exercise 9.13

Complete the table and draw the graph of $y = 2 \sin x - 1$, $0 \leq x \leq 360°$.

x	0°	30°	90°	150°	180°	210°	270°	330°	360°
$y = \sin x$									
$y = 2 \sin x$									
$y = 2\sin x - 1$									



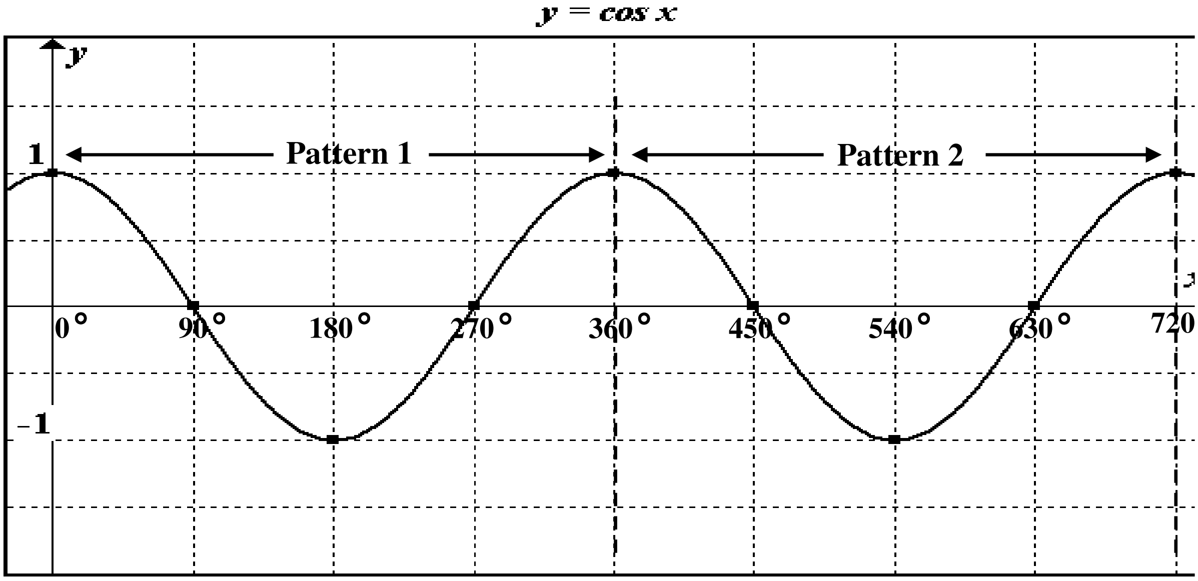
§ 9.14 THE GRAPH OF THE COS FUNCTION

EXAMPLE

Complete the table using a calculator. Use the table to draw the graph of $y = \cos x$

SOLUTION

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
$y = \cos x$	1	0	– 1	0	1	0	– 1	0	1

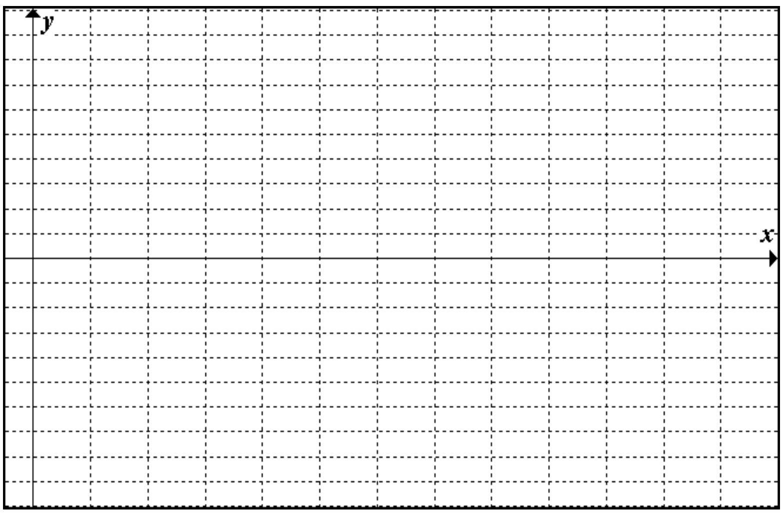


- One graph pattern is completed in 360°. $y = \cos x$ has **period = 360°**. This graph shows two complete patterns. Taking x values to the left and right of those above would show that this pattern is repeated endlessly, in both the negative and the positive directions.
- The distance between the y-axis and the maximum y value is 1, so the **amplitude = 1**.

<u>Exercise 9.14</u>

1) Use a calculator to complete the table. **2)** Draw the graph of $y = \cos x$ for $0° \leq x \leq 180°$.

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
$y = \cos x$									



§ 9.15 THE GRAPH OF $y = a \cos x + q$

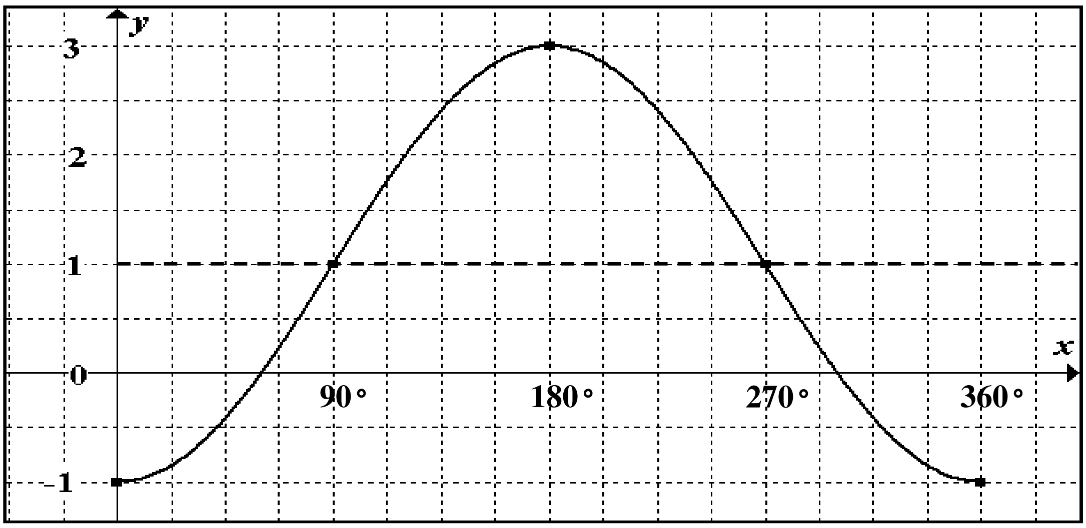
EXAMPLE

Complete the table and draw the graph of $y = -2 \cos x + 1$, $0° \leq x \leq 360°$.

SOLUTION

x	$0°$	$90°$	$180°$	$270°$	$360°$
$y = -2 \cos x + 1$	-1	1	3	1	-1

$y = -2 \cos x + 1$



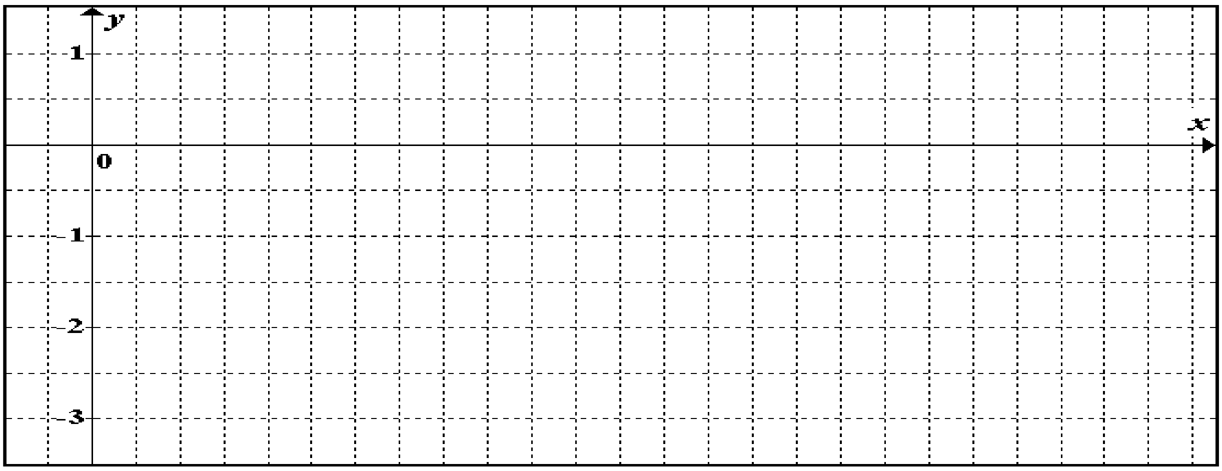
Note:

- The period is 360°
- The graph of $y = -2 \cos x + 1$ rotates about $y = 1$ (dotted line)
- The amplitude is 2

Exercise 9.15

Complete the table and draw the graphs of $y = 2 \cos x - 1$, $0 \leq x \leq 360°$.

x	$0°$	$60°$	$90°$	$120°$	$180°$	$240°$	$270°$	$300°$	$360°$
$y = \cos x$									
$y = 2\cos x - 1$									



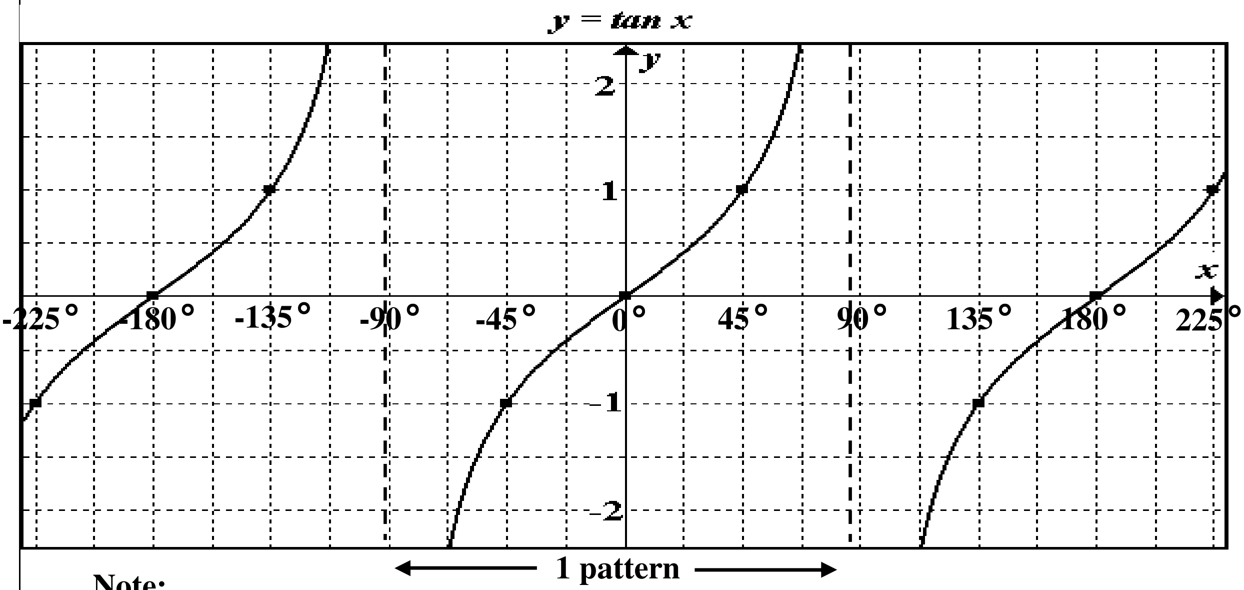
§ 9.16 THE GRAPH OF THE TAN FUNCTION

EXAMPLE

1) Complete the table using a calculator. **2)** Draw the graph of $y = \tan x$, $-225° \leq x \leq 225°$.

SOLUTION

x	$-225°$	$-180°$	$-135°$	$-45°$	$0°$	$45°$	$135°$	$180°$	$225°$
$y = \tan x$	-1	0	1	-1	0	1	-1	0	1



Note:

- The tan function has **no amplitude**
- The period of the graph is 180°, and $x = -90°$ and $x = 90°$ are asymptotes
- Points whose x-coordinates are multiples of 45° are added to show the shape clearly

Exercise 9.16

Complete the table, and draw and label graphs for $-90° \leq x \leq 90°$

x	$-90°$	$-45°$	$0°$	$45°$	$90°$
$y = \tan x$					
$y = \tan x + 1$					
$y = \tan x - 1$