NCV 2

MATHEMATICS

WORKBOOK

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CONTENTS

CHAPTER 1
Natural numbers, whole numbers and integers

In this chapter you will:

- Use the order of operations to do calculations
- Calculate factors of a number
- Classify numbers as either prime or composite
- Calculate prime factors
- Calculate the Highest Common Factor (HCF)
- Calculate multiples of a number
- Calculate the Lowest Common Multiple (LCM)
- Use laws of exponents to simplify expressions
- Calculate square roots and cube roots
- Distinguish between sets of natural numbers and whole numbers
- Add, subtract, multiply and divide using a calculator
- Find squares, cubes and higher powers on a calculator
- Find square roots, cube roots and higher roots on a calculator
- Estimate answers
- Add and subtract integers using a number line
- Add and subtract integers without using a number line
- Multiply and divide integers
- Distinguish between integers, whole numbers and natural numbers.

This chapter covers material from Topic 1: Numbers
SUBJECT OUTCOME 1.1:
Use Computational Tools and Strategies and Make Estimates and Approximations
Learning Outcome 1: Use a scientific calculator competently and efficiently
Learning Outcome 2: Execute algorithms appropriately in calculations

§ 1.1 ORDER OF OPERATIONS

- If a calculation has more than one operation, we do them in the following **fixed order**:
  1. ( ) Brackets
  2. **Exponents**
  3. × ÷ Multiplication and Division, working from left to right
  4. **+ –** Addition and Subtraction, working from left to right

- **BEMDAS** will help you to remember the order in which you must work. Unless you follow this order, you will get wrong answers.

EXAMPLES:	**SOLUTIONS**
Calculate the answers without using a calculator: 1) $5 + 2 - 7$	$5 + 2 - 7 = 7 - 7 = 0$ ….. *Work from left to right*
2) $24 \times 2 \div 8 \div 3$	$24 \times 2 \div 8 \div 3 = 48 \div 8 \div 3 = 6 \div 3 = 2$ ….. *Work from left to right*
3) $9 - 4 \times 2$	$9 - 4 \times 2 = 9 - (4 \times 2) = 9 - 8 = 1$ ….. *Multiply before subtracting*
4) $7 + 4 \times (3 + 2)$	$7 + 4 \times (3 + 2) = 7 + (4 \times 5) = 7 + 20 = 27$ …..*Simplify brackets, multiply, then add*
5) $2^2 + 4 \times 3$	$2^2 + 4 \times 3 = 4 + (4 \times 3) = 4 + 12 = 16$ ….. *Simplify the exponent, multiply, then add*
6) $8 - 2 \times 6 + 5$	$8 - 2 \times 6 + 5 = 8 - (2 \times 6) + 5 = 8 - 12 + 5 = 1$ ….. *Multiply, subtract, then add and subtract*
7) $6 + 3^2 \div 3$	$6 + 3^2 \div 3 = 6 + (9 \div 3) = 6 + 3 = 9$ …..*Simplify the exponent, divide, then add*
8) $(6 + 9) \div 3$	$(6 + 9) \div 3 = 15 \div 3 = 5$ ….. *Simplify brackets, then divide*
9) $2(3 + 1) - 4 \div 2$	$2(3 + 1) - 4 \div 2 = 2(4) - (4 \div 2) = 8 - 2 = 6$ ….. *Simplify brackets, divide, then subtract*

Exercise 1.1

Calculate the answers without using a calculator:

1) $9 - 8 + 1 =$ ……………………………...   **2)** $13 - 10 - 3 =$ …………………………….

3) $8 + 5 - 2 + 3 =$ ………………………….   **4)** $12 + 5 - 2 =$ ……………………………...

5) $12 \div 6 \div 2 =$ …………………………….   **6)** $4 \times 6 - 3 =$ ……………………………….

7) $15 - 3 \times 4 =$ …………………………….   **8)** $(15 - 3) \times 4 =$ …………………………….

9) $12 \div 4 + 2 =$ ……………………………   **10)** $12 \div (4 + 2) =$ …………………………….

11) $3 \times (2 + 1) - 3 \div 3 =$ ……………………………………………………………………….

12) $7 - 2 \times 3 + 4 \div 4 =$ …………………………………………………………………………

13) $19 - 4(5 - 3) \times 2 =$ …………………………………………………………………………

14) $9 \times 105 \times 28 \times (3 - 3) =$ ………………………………………………………………….

15) $23 - 15 + 4 - 12 + 9 - 3 =$ ………………………………………………………………….

16) $6 + 2 \times 8 - 8 - 5 =$ …………………………………………………………………………

§ 1.2 FACTORS

✧ **A factor of a number divides into that number exactly, without a remainder.**
3 is a factor of 12, since $12 \div 3 = 4$. 3 divides exactly into 12.
3 is not a factor of 16, since $16 \div 3 = 5$ remainder 1.
There are other factors of 12. To find them, divide by 1, 2, 3, ….. in this order. Those that divide into 12 exactly are factors.

Note:

- F stands for factor and F_{12} means "the set of factors of 12".

EXAMPLE	SOLUTION
1) Is 7 a factor of 42? Why?	7 **is** a factor of 42 since $42 \div 7 = 6$
2) Is 7 a factor of 41? Why?	7 is **not** a factor of 41, since $41 \div 7 = 5$ remainder 6
3) List all the factors of 12.	$12 \div 1 = 12$, $12 \div 2 = 6$, $12 \div 3 = 4$, $12 \div 4 = 3$, $12 \div 6 = 2$, $12 \div 12 = 1$. So $F_{12} = \{1, 2, 3, 4, 6, 12\}$. There are 6 factors of 12. 1 is the least factor and 12 is the greatest factor.
4) List all the factors of 30.	$F_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Note:

- The least factor of any number is always 1, since every number can be divided by 1.
- The greatest factor of any number is the number itself.

Exercise 1.2

1) Is 6 a factor of 54? Why? …………………………………………………………………………

2) Is 6 a factor of 46? Why? …………………………………………………………………………

3) a) List all the factors of 18. $F_{18} = \{$……………………………………………………………

   b) How many factors does 18 have? ……………………………………………………………

   c) What is the least factor of 18? ………………………………………………………………

   d) What is the greatest factor of 18? ……………………………………………………………

4) List all the factors of 24. $F_{24} = \{$ ……………………………………………………………

5) List all the factors of 36. $F_{36} = \{$ ……………………………………………………………

6) List all the factors of 45. …………………………………………………………………………

7) What are the greatest and the least factors of 179?

   Greatest factor = ……......……………………………………………

   Least factor = ……………………………………………………………

§ 1.3 PRIME AND COMPOSITE NUMBERS

✧ **A prime number has only 2 factors, itself and 1.**
✧ **A composite number has more than 2 factors.**

NOTE:
- **7 is prime**, since its only factors are 1 and 7. It has **only 2 factors**.
- **6 is not prime**, since its factors are 1, 2, 3, and 6. **6 is a composite number** since it has **more than 2 factors**.
- **1 is not prime**. The only factor of 1 is 1, it does not have 2 factors. So **1 is neither prime nor composite**.

EXAMPLE	SOLUTION
1) Is 1 740 prime or composite? Give a reason for your answer.	1, 10, 174 and 1 740 are some of the factors of 1 740. It is composite since it has more than 2 factors.
2) Is 365 prime? Why?	No, 365 is NOT prime. 1, 5, and 365 are factors of 365, so it has more than 2 factors.

NOTE:
- Numbers that end in 0 have a factor of 10
- Numbers that end in 5 have a factor of 5
- The set of even numbers = {2; 4; 6; 8; 10; ………………..}.
  In other words, numbers that end in an even number have a factor of 2.

Exercise 1.3

1) Are the following numbers prime or composite? Give a reason for your answer.

   a) 8 …………………………………………………………………………………….............

   b) 11 ………………………………………………………………………………………………

   c) 14 ………………………………………………………………………………………………

   d) 19 ………………………………………………………………………………………………

   e) 1 …………………………………………………………………………………….............

   f) 23 ………………………………………………………………………………………………

   g) 445 ……………………………………………………………………………….................

   h) 986 ……………………………………………………………………………….................

2) List all the prime numbers up to and including 20 . ……………………………………………..

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

3) List all the composite numbers up to and including 20 …………………………………………….

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

§ 1.4 PRIME FACTORS

✧ A **prime factor** is a factor that is a prime number.

EXAMPLE	SOLUTION
1) We said that $F_{12} = \{1, 2, 3, 4, 6, 12\}$ List the **prime factors** of 12	The prime factors are 2 and 3
2) Which **factors** of 12 are **composite**?	The composite factors are 4; 6; 12
3) Why is 1 not a prime factor of 12?	1 has only one factor, so it is not prime.
4) Factorise 36 into prime factors	See below

Prime factors	Other factors
2	**36**
2	18
3	9
3	3
	1

After dividing, we can say
$36 = 2\times2\times3\times3$
$= 2^2\times3^2$

Note:
We do not include 1 because it is not a prime number

Exercise 1.4

1) a) List all the factors of 15: $F_{15} = \{ \quad ; \quad ; \quad ; \quad \}$

b) List the prime factors of 15 ……………………………………………………………………

c) Which factors of 15 are composite? ……………………………………………………………

4) **a)** Factorise 60, 72, and 90 into prime factors.

Prime factors	Other factors
	60

Prime factors	Other factors
	72

Prime factors	Other factors
	75

b) List the prime factors of 60 ……………………………………………………………………

c) List the prime factors of 72 ……………………………………………………………………

d) List the prime factors of 75 ……………………………………………………………………

e) Is 1 a factor of 60? Give a reason for your answer

……………………………………………………………………………………………………

f) Is 1 a **prime** factor of 60? Give a reason for your answer

……………………………………………………………………………………………………

§ 1.5 HIGHEST COMMON FACTOR

✧ **HCF** stands for **Highest Common Factor. It is the greatest factor common to two or more numbers.**

<table>
<tr><th>EXAMPLE</th><th>SOLUTION</th></tr>
<tr><td>1) Find the HCF of 36 and 24 by listing all their factors.</td><td>$F_{24} = \{1, 2, 3, 4, 6, 8, \mathbf{12}, 24\}$
$F_{36} = \{1, 2, 3, 4, 6, 9, \mathbf{12}, 18, 36\}$
The HCF of 36 and 24 = 12, since this is the highest factor common to both.</td></tr>
<tr><td>2) Find the HCF of 36 and 24 by breaking each number into its prime factors.</td><td>
<table>
<tr><th>Prime factors</th><th>Other factors</th></tr>
<tr><td>2</td><td>24</td></tr>
<tr><td>2</td><td>12</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>3</td></tr>
<tr><td></td><td>1</td></tr>
</table>
<table>
<tr><th>Prime factors</th><th>Other factors</th></tr>
<tr><td>2</td><td>36</td></tr>
<tr><td>2</td><td>18</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>3</td><td>3</td></tr>
<tr><td></td><td>1</td></tr>
</table>
Look for the factors that are common to both numbers
$24 = \underline{\mathbf{2}} \times \underline{\mathbf{2}} \times 2 \times \underline{\mathbf{3}}$
$36 = \underline{\mathbf{2}} \times \underline{\mathbf{2}} \times \underline{\mathbf{3}} \times 3$
The HCF of 36 and 24 = $\underline{\mathbf{2}} \times \underline{\mathbf{2}} \times \underline{\mathbf{3}} = 12$.</td></tr>
</table>

Exercise 1.5

1) Find the HCF of 120 and 90 by breaking each number into its prime factors.

Prime factors	Other factors
	120

Prime factors	Other factors
	90

120 = ……………………………………………………………………….

90 = …………………………………………………………………………

HCF = ……………………………………………………………………….

2) Find the HCF of 125 and 75 by listing all their factors.

F_{125} = { ; ; ; }

F_{75} = { ; ; ; ; ; }

HCF = ……………………………………………………………………….

3) Find the HCF of 180 and 150 by listing all their factors.

F_{180} = {……………………………………………………………………….

F_{150} = {……………………………………………………………………….

HCF = ……………………………………………………………………....

§ 1.6 MULTIPLES

- **A multiple of a whole number is found by multiplying it by any whole number.**
  Examples:
  The multiples of 5 are $5 \times 1, 5 \times 2, 5 \times 3, 5 \times 4, 5 \times 5$, …… = 5, 10, 15, 20, 25, ………..
  The multiples of 1 are 1, 2, 3, 4, 5, …………
- Note that M stands for multiple and M_{24} means "the set of multiples of 24"
- We cannot list all the multiples of a number since there are an infinite number of them, so we use dots to show that they are ongoing

EXAMPLE	SOLUTION
1) **a)** List the factors of 12 **b)** List the multiples of 12	**a)** $F_{12} = \{1; 2; 3; 4; 6; 12\}$ **b)** Multiples of 12 are 12×1; 12×2; 12×3; …. $M_{12} = \{12; 24; 36; 48; 60;$ …………..$\}$
2) **a)** Write down the factors of 18 **b)** Write down the multiples of 18.	**a)** $F_{18} = \{1; 2; 3; 6; 9; 18\}$ **b)** The multiples of 18 are $18\times1 = 18$, $18\times2 = 36$, $18\times3 = 54$ etc. So $M_{18} = \{18; 36; 54;$ ……$\}$
3) Which of the numbers 11, 22, 41, 77 are multiples of 11?	$11\times1 = 11$, $11\times2 = 22$ and $11\times7 = 77$, so <u>11, 22 and 77</u> are multiples of 11

Note:
- factors of a number are the number itself and smaller
- multiples of a number are the number itself and greater

Exercise 1.6

1) a) List the factors of 15 ………………………………………………………………………..
   b) List the multiples of 15 ……………………………………………………………………..
   c) Which number is both a factor and a multiple of 15? …………………………………………

2) a) List the multiples of 20 ……………………………………………………………………..
   b) List the factors of 20 ………………………………………………………………………..
   c) Which multiple of 20 is also a factor of 20? …………………………………………………..

3) Which of the following numbers are multiples of 9: 9, 18, 36, 109? Why?
   ……………………………………………………………………………………………………
   ……………………………………………………………………………………………………

4) Write down the first twelve multiples of 8
   ……………………………………………………………………………………………………
   ……………………………………………………………………………………………………

§ 1.7 LCM

✧ **LCM** stands for **Lowest Common Multiple. It is the lowest number into which two or more numbers divide.**

<table>
<tr><th>EXAMPLE</th><th>SOLUTION</th></tr>
<tr><td>1) Find the Lowest Common Multiple (LCM) of 36 and 24 by listing the multiples of each.</td><td>$M_{36} = \{36, \underline{\mathbf{72}}, 108, \dots\}$
$M_{24} = \{24, 48, \underline{\mathbf{72}}, \dots\}$

The LCM = 72. This is the lowest multiple which is common to both 36 and 24.</td></tr>
<tr><td>2) Use prime factors to find the HCF and LCM of 504 and 180.</td><td>
<table>
<tr><th>Prime factors</th><th>Other factors</th></tr>
<tr><td>2</td><td>504</td></tr>
<tr><td>2</td><td>252</td></tr>
<tr><td>2</td><td>126</td></tr>
<tr><td>3</td><td>63</td></tr>
<tr><td>3</td><td>21</td></tr>
<tr><td>7</td><td>7</td></tr>
<tr><td></td><td>1</td></tr>
</table>
<table>
<tr><th>Prime factors</th><th>Other factors</th></tr>
<tr><td>2</td><td>180</td></tr>
<tr><td>2</td><td>90</td></tr>
<tr><td>3</td><td>45</td></tr>
<tr><td>3</td><td>15</td></tr>
<tr><td>5</td><td>5</td></tr>
<tr><td></td><td>1</td></tr>
</table>
$504 = \underline{2}\times\underline{2}\times 2\times\underline{3}\times\underline{3}\times 7$
$180 = \underline{2}\times\underline{2}\times\underline{3}\times\underline{3}\times 5$
HCF = $2\times 2\times 3\times 3 = 36$

To find the LCM, multiply the HCF by those factors that are not common.
LCM = $36\times 2\times 5\times 7 = 2\,520$
</td></tr>
</table>

<u>Exercise 1.7</u>

1) Find the LCM of 15 and 20 by listing the multiples of each.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2) Find the HCF of 105 and 140 by listing the factors of each.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………..

3) Find the LCM of 105 and 140 by listing the multiples of each.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

§ 1.8 EXPONENTS, SQUARES AND CUBES

- In the expression 3^7, 3 is called the **base** and 7 is called the **exponent, index** or **power.**
- **The exponent gives the number of times the base is multiplied by itself.**

  3^7 expanded means $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
- 3^7 is said to be in **index form** or **exponent form** or written as a **power of the base**.

EXAMPLE	SOLUTION
1) Expand the following:	
a) 2^4	$2^4 = 2 \times 2 \times 2 \times 2$
b) x^3	$x^3 = x \times x \times x$
c) $(a+b)^2$	$(a+b)^2 = (a+b)(a+b)$
2) Write the following in exponential form:	
a) $6 \times 6 \times 6$	$6 \times 6 \times 6 = 6^3$
b) $x \times x$	$x \times x = x^2$
c) $p \times p \times p \times p \times p$	$p \times p \times p \times p \times p = p^5$
d) $(d+g)(d+g)$	$(d+g)(d+g) = (d+g)^2$

Exercise 1.8

1) Expand the following:

   a) 5^4 = …………………… b) x^3 = …………………… c) $(x+y)^2$ = ……………………

2) Write the following in index form:

   a) $8 \times 8 \times 8 \times 8$ = ………... b) $b \times b \times b$ = ………… c) $(p+q)(p+q)(p+q)$ = ……………

- The **square** of any number is the number multiplied by itself.
- The **cube** of any number is the number multiplied by itself and by itself again.

EXAMPLE	SOLUTION
Evaluate:	
1) 7^2	$7^2 = 7 \times 7 = 49$
2) 2^3	$2^3 = 2 \times 2 \times 2 = 8$
3) $(4+2)^2$	$(4+2)^2 = 6^2 = 36$
4) $4^2 + 2^2$	$4^2 + 2^2 = (4 \times 4) + (2 \times 2) = 16 + 4 = 20$

Note:
- 7^2 is read "seven **squared**" and 2^3 is read "two **cubed**"
- $49 = 7^2$ is called a "**perfect square**" and $8 = 2^3$ is called a "**perfect cube**".

3) Calculate:

   a) $5^2 + 1^2$ = ……………..…… b) $(5+1)^2$ = ………………..… c) $1^4 + 4^1$ = ……………..…

   d) $(4-3)^2$ = ……………… e) $4^3 - 3^2$ = ………………… f) $2^5 - 2$ = …………………

   g) $(1+2)^3$ = ……………..….. h) $(2+5-3)^2$ = ...

4) a) 36 is the square of ……..... b) 64 is the cube of ……… and the square of …….........

   c) Twelve squared = ……..... d) Ten squared = ………….. e) Five cubed = ……......…

   f) …..….. is the square of 11 g) The square of 9 = ... h) Twenty squared = ...

§ 1.9 SQUARE ROOTS AND CUBE ROOTS

Note:

- $\sqrt{\ }$ is the symbol for **square root** and means $\sqrt[2]{\ }$ where the 2 is assumed
- $\sqrt[3]{\ }$ is the symbol for **cube root**
- $\sqrt{16}$ is read "the square root of 16"
- $\sqrt[3]{27}$ is read "the cube root of 27"

- The **square root** of a number multiplied by itself gives the number.
  $\sqrt{3} \times \sqrt{3} = \sqrt{3\times3} = 3$
  Take **one** from **every pair** of identical numbers under the square root sign as the answer.
  So $\sqrt{3\times5\times3\times5} = \sqrt{3\times3} \times \sqrt{5\times5} = 3 \times 5 = 15$

- The **cube root** of a number multiplied by itself and by itself again gives the number.
  $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = \sqrt[3]{2\times2\times2} = 2$
  Take **one** from every **three** identical numbers under the cube root sign as the answer.
  So $\sqrt[3]{2\times3\times3\times2\times3\times2} = \sqrt[3]{2\times2\times2} \times \sqrt[3]{3\times3\times3} = 2 \times 3 = 6$

EXAMPLE		SOLUTION
Evaluate		
1)	$\sqrt{16}$	$\sqrt{16} = \sqrt{4\times4} = 4$
2)	$\sqrt[3]{27}$	$\sqrt[3]{27} = \sqrt[3]{3\times3\times3} = 3$
3)	$\sqrt{0}$	$\sqrt{0} = \sqrt{0\times0} = 0$
4)	$\sqrt[3]{1}$	$\sqrt[3]{1} = \sqrt[3]{1\times1\times1} = 1$
5)	$\sqrt{9+16}$	$\sqrt{9+16} = \sqrt{25} = \sqrt{5\times5} = 5$
6)	$\sqrt{78\times78}$	$\sqrt{78\times78} = 78$
7)	$\sqrt[3]{59\times59\times59}$	$\sqrt[3]{59\times59\times59} = 59$
8)	$\sqrt{9} + \sqrt{16}$	$\sqrt{9} + \sqrt{16} = \sqrt{3\times3} + \sqrt{4\times4} = 3 + 4 = 7$
9)	$\sqrt{1\frac{7}{9}}$	$\sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{\sqrt{4\times4}}{\sqrt{3\times3}} = \frac{4}{3}$
10)	$\sqrt{8\times18}$	$\sqrt{8\times18} = \sqrt{(2\times2\times2)\times(2\times3\times3)} = 2 \times 2 \times 3 = 12$

Exercise 1.9

Calculate

1) $\sqrt{81}$ = ……….
2) $\sqrt{144}$ = …..…..
3) $\sqrt[3]{27}$ = ……....
4) $\sqrt[3]{8}$ = …..…..
5) $\sqrt{16} - \sqrt{4}$ = ………………..……
6) $\sqrt{169-25}$ = ..…………
7) $\sqrt{169} - \sqrt{16}$ = ……………………
8) $\sqrt[3]{1} + \sqrt{81}$ = ……..…………………
9) $\sqrt{25-16}$ = ……………………..
10) $\sqrt{36} + \sqrt[3]{125}$ = ………………………
11) $\sqrt{\frac{16}{25}}$ = …………..
12) $\frac{\sqrt{100}}{\sqrt[3]{64}}$ = …………………..
13) $\sqrt[3]{\frac{27}{125}}$ = ……………
14) $\sqrt{5\frac{4}{9}}$ = ………………………….…..
15) $\sqrt[3]{19\times19\times19}$ = ……………
16) $\sqrt[3]{9\times4\times6}$ = …………………………………

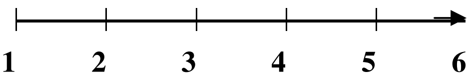
§ 1.10 THE SET OF NATURAL NUMBERS AND WHOLE NUMBERS

- Numbers belong in number sets which have names. Some numbers belong to more than one number set.
- So far, all the numbers we have worked with have been either natural numbers or whole numbers. Here are the first two sets. Remember, the dots in the set brackets mean that the numbers continue in the same way without end.

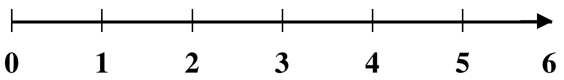
  The set of **Natural numbers** (also called counting numbers): ***N* = {1; 2; 3; 4; …………..}**

  The set of **Whole numbers**: ***W* or N_0 = {0; 1; 2; 3; …………….}**

- These numbers can be shown on a number line. The arrow on the end means that the numbers continue in the same way without end.



***N* = {1; 2; 3; 4; …………….}**



***W* = {0; 1; 2; 3; ……….}**

EXAMPLE:
1) 0 is a Whole number (*W*)
2) 1 is a Natural numbers (*N*) and a Whole numbers (*W*)
3) 245 is a Natural numbers (*N*) and a Whole numbers (*W*)

Exercise 1.10

1) Write down the letter that stands for:
   a) the set of whole numbers ………………………………………………………………
   b) the set of natural numbers ……………………………………………………………...
2) What is the smallest natural number? ………………………………………………………….
3) What is the greatest natural number? …………………………….…………………………….
4) What is the smallest whole number? …………………………….……………………………..
5) Write down a number that is a whole number but not a natural number. ..……………….…
6) Are the following numbers whole numbers and/or natural numbers?
   a) 4 ………………………………………………………………………………………
   b) 56 ……………………………………………………………………………………..
   c) 0 ……………………………………………………………………………………….
   d) 249 …………………………………………………………………………………….

§ 1.11 ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION USING A CALCULATOR

- Remember **BIMDAS**, the correct order of operations. Your **scientific calculator** follows this order automatically.

EXAMPLE	**SOLUTION**		
Calculate	**Key sequence**	**Display**	**Answer**
1) $\frac{8-2}{3}$	8 $\boxed{-}$ 2 $\boxed{=}$ $\boxed{\div}$ 3 $\boxed{=}$	2	$\frac{8-2}{3} = 2$
2) $(8-2) \div 3$	$\boxed{(}$ 8 $\boxed{-}$ 2 $\boxed{)}$ $\boxed{\div}$ 3 $\boxed{=}$	2	$(8-2) \div 3 = 2$
3) $8-2\div3$	8 $\boxed{-}$ 2 $\boxed{\div}$ 3 $\boxed{=}$	7.33…	$8-2\div3 = 8 - \frac{2}{3} = 7{,}33\ldots$
4) $8 - \frac{2}{3}$			

Note:
- For **1)**, you have to enter $\boxed{=}$ after 8 – 2 before dividing by 4. This is because the entire numerator is divided by 3.
- For **2)**, you use brackets instead of $\boxed{=}$. Questions **1)** and **2)** are identical.
- For **3)** and **4)** the answer is not 2. The calculator first divides 2 by 3 before subtracting the answer from 8.

Exercise 1.11

Use your calculator to calculate the following:

1) $100 - 84 \div 2 =$ ……….

2) $(100 - 84) \div 2 =$ ……….

3) $310 \times 54 - 22 =$ ……….

4) $310 \times (54 - 22) =$ ……….

5) $840 \div 21 + 3 =$ ……….

6) $840 \div (21 + 3) =$ ……….

7) $321 + 12 \div 3 =$ ……….

8) $(321 + 12) \div 3 =$ ……….

9) $102 - 34 \times 5 + 96 =$ ……….

10) $102 - 34 \times (5 + 96) =$ ……….

11) $(102 - 34) \times 5 + 96 =$ ……….

12) $(102 - 34) \times (5 + 96) =$ ……….

13) $126 + 18 \div 9 =$ ……….

14) $(126 + 18) \div 9 =$ ……….

15) $145 \times 23 \div 5 =$ ……….

16) $114 \div 19 \div 2 =$ ……….

17) $459 \div 3 \times 9 =$ ……….

18) $459 \div (3 \times 9) =$ ……….

19) $154 - 6 \div 2 =$ ……….

20) $(154 - 6) \div 2 =$ ……….

21) $119 \div 17 - 7 =$ ……….

22) $119 \div (17 - 7) =$ ……….

§ 1.12 SQUARES, CUBES AND HIGHER POWERS ON A CALCULATOR

On a CASIO *fx*-82ES scientific calculator,

- $\boxed{x^2}$ is used for squaring a number
- $\boxed{x^3}$ is used for cubing a number
- $\boxed{x^{\square}}$ is used for raising a base to any exponent

Note:

- On some calculators the key $\boxed{x^{\square}}$ is marked $\boxed{\wedge}$ or $\boxed{x^y}$ or $\boxed{y^x}$

EXAMPLE		**SOLUTION**	
Use your calculator to calculate:		**Key Sequence**	**Answer**
1)	198^2	198 $\boxed{x^2}$ $\boxed{=}$	$198^2 = 39\ 204$
2)	18^3	18 $\boxed{x^3}$ $\boxed{=}$	$18^3 = 5\ 832$
3)	14^5	14 $\boxed{x^{\square}}$ 5 $\boxed{=}$	$14^5 = 537\ 824$
4)	25×16^3	25 $\boxed{\times}$ 16 $\boxed{x^3}$ $\boxed{=}$	$25 \times 16^3 = 102\ 400$
5)	$2\ 379^2 - 45^3$	2 379 $\boxed{x^2}$ $\boxed{-}$ 45 $\boxed{x^3}$ $\boxed{=}$	$123^2 - 45^3 = 5\ 568\ 516$
6)	$(23 + 19)^3$	$\boxed{(}$ 23 $\boxed{+}$ 19 $\boxed{)}$ $\boxed{x^3}$ $\boxed{=}$	$(23 + 19)^3 = 74\ 088$
7)	$(17^2 - 13^2)^4$	$\boxed{(}$ 17 $\boxed{x^2}$ $\boxed{-}$ 13 $\boxed{x^2}$ $\boxed{)}$ $\boxed{x^{\square}}$ 4 $\boxed{=}$	$(17^2 - 13^2)^4 = 207\ 360\ 000$

Note:

- On some calculators you may not have to enter the $\boxed{=}$ key for squares and cubes.

Exercise 1.12

Use your calculator to evaluate:

1) $54^2 =$ ……….

2) $43^3 =$ ……….

3) $18^4 =$ ……….

4) $15^3 - 14^3 =$ ……….

5) $17^2 - 16^2 + 5^2 =$ ……….

6) $(22^2 - 19^2)^3 =$ ……….

7) $18^2 - 8^2 =$ ……….

8) $(18 - 8)^2 =$ ……….

9) $(43 - 19)^4 =$ ……….

10) $43^4 - 19^4 =$ ……….

11) $15^3 - 14^2 + 16^2 =$ ……….

12) $14^4 - (13^2 + 12^3) =$ ……….

13) $123^2 - 13^3 + 12^4 =$ ……….

14) $(25^2 - 20^2 - 5^2)^2 =$ ……….

15) $(23^3 + 12^4)^2 =$ ……….

16) $(56 - 23 + 45)^4 =$ ……….

17) $12 \times 15^2 \times 13^2 =$ ……….

18) $(32 + 47)^3 =$ ……….

§ 1.13 SQUARE ROOTS, CUBE ROOTS AND HIGHER ROOTS ON A CALCULATOR

On your CASIO *fx*-82ES scientific calculator, the key:

- $\boxed{\sqrt{\square}}$ is used for finding the square root
- $\boxed{\sqrt[3]{\square}}$ is used for finding the cube root
- $\boxed{\sqrt[\square]{\square}}$ is used for finding other roots greater than 3

Note:

- On some calculators the key $\boxed{\sqrt[\square]{\square}}$ is marked $\boxed{x^{\frac{1}{y}}}$ or $\boxed{y^{\frac{1}{x}}}$ or $\boxed{\sqrt[x]{\ }}$ or $\sqrt[x]{y}$

EXAMPLES		**SOLUTION**	
Use your calculator to calculate:		**Key sequence**	**Answer**
1)	$\sqrt{841}$	$\boxed{\sqrt{\square}}$ 841 $\boxed{=}$	$\sqrt{841} = 29$
2)	$\sqrt[5]{1\,889\,568}$	$\boxed{\sqrt[\square]{\square}}$ 5 ▶ 1 889 568 $\boxed{=}$	$\sqrt[5]{1\,889\,568} = 18$
3)	$\sqrt{9+16}$	$\boxed{\sqrt{\square}}$ $\boxed{(}$ 9 $\boxed{+}$ 16 $\boxed{)}$ $\boxed{=}$	$\sqrt{9+16} = 5$
4)	$\sqrt[3]{9\,261} - \sqrt{361}$	$\boxed{\sqrt[3]{\square}}$ 9 261 ▶ $\boxed{-}$ $\boxed{\sqrt{\square}}$ 361 $\boxed{=}$	$\sqrt[3]{9\,261} - \sqrt{361} = 2$
5)	$\sqrt[4]{2\,825\,761}$	$\boxed{\sqrt[\square]{\square}}$ 4 ▶ 2 825 761 $\boxed{=}$	$\sqrt[4]{2\,825\,761} = 41$

Note:

- In 3) the brackets around the '9 + 16' are essential to get the right answer
- If you forget the brackets in **3**), your calculator will calculate $\sqrt{9} + 16 = 19$

Exercise 1.13

Use your calculator to calculate:

1) $\sqrt[5]{537\,824}$ = ……….

2) 14^5 = ……….

3) $\sqrt[3]{12\,167}$ = ……….

4) 23^3 = ……….

5) $\sqrt{14\,400}$ = ……….

6) $\sqrt{729} + \sqrt[3]{3\,375}$ = ……….

7) $\sqrt[7]{62\,748\,517}$ = ……….

8) $\sqrt{961} - \sqrt[4]{28\,561}$ = ……….

9) $\sqrt{25^2 - 24^2}$ = ……….

10) $32^3 - \sqrt{361} - \sqrt[3]{1\,331}$ = ……….

11) $(\sqrt[4]{130\,321} + \sqrt{289})^2$ = ……….

12) $(\sqrt{256} - \sqrt{225})^2$ = ……….

13) $\sqrt[3]{35\,937} - \sqrt{256}$ = ……….

14) $\sqrt[4]{130\,000 + 321}$ = ……….

15) $\sqrt{100+100+100+100}$ = ……………….

16) $\sqrt[3]{19 \times 19 \times 19}$ = ……………………

§ 1.14 ESTIMATING AN ANSWER

- **Rounding off or correcting to 1 significant figure:**
  As you read a number from **left to right**, the first figure you come to that is **not zero** is called the **first significant figure**.

Numbers greater than 1			
1st significant figure	**3**25	**1** 456	**2**,789
The same is true for numbers less than 1			
1st significant figure	0,0**3**45	0,00**9**2	0,000 **5**49

- An easy way to approximate a number is to round it off to the first significant figure.

Procedure for rounding to 1 significant figure:

i) Underline the first significant figure
ii) Look at the number to the right:
- If it is **less than 5**, the first significant figure remains the same
- If it is **5 or more**, the first significant figure is increased by 1

iii) If the number is more than 1, fill in zeros to keep the place value correct

EXAMPLE	SOLUTION
1) Round the following numbers off to 1 significant figure:	
a) 634 810	6**3**4 810 ≈ 600 000
b) 38 126	3**8** 126 ≈ 40 000
c) 0,0346	0,03**4**6 ≈ 0,03
d) 0,006 825 13	0,006 **8**25 13 ≈ 0,007
2) a) Calculate 12 012 × 45 ÷ 462 using a calculator	Calculator: 12 012 × 45 ÷ 462 = 1 170
b) As a check, estimate the answer to **a)** by rounding each number off to 1 significant figure	Estimate: 10 000 × 50 ÷ 500 = 500 000 ÷ 500 = 1 000 The answers to **a)** and **b)** are nearly the same, so the answer to **a)** is probably correct

Exercise 1.14

1) Round each of the following numbers to 1 significant figure:

a) 5 499 ≈ …………………………………………………

b) 491 ≈ ……………………………………………………

c) 35 ≈ ……………………………………………………..

d) 0,005 53 ≈ ……………………………………………….

2) a) Use a calculator to calculate: 924 × 28 ÷ 87 ………………………………………………..

b) Check your answer, by rounding each number to 1 significant figure

…………………………………………………………………………………………………

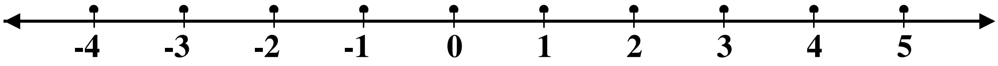
3) Round each number to 1 significant figure, to estimate the answer to 5059 × 99 × 17

……………………………………………………………………………………………………...

§ 1.15 ADDITION AND SUBTRACTION OF INTEGERS USING A NUMBER LINE

✧ The set of integers is the set of numbers {….; –5; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …….}

We can show these integers on a number line like this:



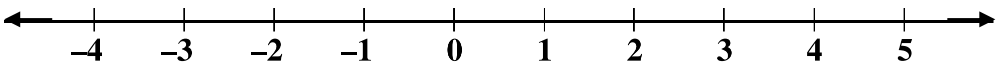
✧ The set of integers consists of the set of whole numbers as well as the negatives of the whole numbers.

✧ When we **add integers**, we move a number of digits to the **right**.
When we **subtract integers** , we move a number of digits to the **left**.

Note:

- If there is no sign in front of a digit, it is assumed to be +
- The sign before an integer belongs to that integer

Use the number line for the following:



EXAMPLE	SOLUTION
1. Add 2 and 3	**Start at 2** and **move 3 digits to the right**. Your answer is 2 + 3 = 5.
2. Add –3 and 4	**Start at –3** and **move 4 digits to the right**. Your answer is –3 + 4 = 1.
3. Add –1 and –3	A tricky one! **Start at –1** and **move –3 digits to the right, which is the same as moving 3 digits to the left**. Your answer is –1 + (–3) = –4.
4. Subtract 4 from 3	**Start at 3** and **move 4 digits to the left**. Your answer is 3 – 4 = –1.
5. Subtract –3 from –2	Another tricky one! **Start at –2** and **move –3 to the left, which is the same as moving +3 to the right.** Your answer is **–**2 – (–3) = –2 + 3 = 1.

Exercise 1.15

1) Use the number line above to help you to calculate the answers to each question.

a) –2 + 7 = …… **b)** 5 – 9 = ……… **c)** 1 – (–2) = ……… **d)** 1 – 5 = ………

e) –3 – (–2) = ……... **f)** –1 – 3 = …….. **g)** –2 + (–1) = …….. **h)** –2 – (–3) = ……..

2) Use the centimetre marks along a full length ruler if the number line above is too small.

a) –4 + 2 = …….. **b)** –1 – 2 = …….. **c)** –3 + (–1) = …….. **d)** –5 – (–2) = ……..

e) 2 – 3 = …….. **f)** 2 – (–2) = …….. **g)** –2 – 1 = …….. **h)** –7 + 12 = ……..

i) –3 – 6 = …….. **j)** –1 + (–2) = …….. **k)** 4 + (–4) = …….. **l)** –11 – 1 = ……..

m) 4 – 6 = …….. **n)** –3 + 5 = …….. **o)** 1 – (–2) = …….. **p) –**10 – (–1) = …….

§ 1.16 ADDITION AND SUBTRACTION OF INTEGERS WITHOUT USING A NUMBER LINE

As a short cut, remember:

- $+ \boxtimes + = +$
- $+ \boxtimes - = -$
- $- \boxtimes + = -$
- $- \boxtimes - = +$

When multiplying:
- if the signs are the **same**, the answer is **+**
- if the signs are **different**, the answer is **–**

EXAMPLE	**SOLUTION**
Evaluate 1) +5 + (+4)	 +5 + (+4) = 5 + 4 = 9
2) –3 + (–5)	–3 + (–5) = – 3 – 8 = – 8
3) –8 – (+3) = –11	–8 – (+3) = – 8 – 3 = –11
4) –2 – (–4)	–2 – (–4) = –2 + 4 = 2
5) –2 – (+6) + (+1) + (–3) – (–5)	–2 – (+6) + (+1) + (–3) – (–5) = –2 – 6 + 1 – 3 + 5 = – 5
6) 12 + (–6) – (–3) – (+2) – (–1)	12 + (–6) – (–3) – (+2) – (–1) = 12 – 6 + 3 – 2 + 1 = 8
7) –15 + (–18) + (+6) – (–2) + (–5) – (+7)	–15 + (–18) + (+6) – (–2) + (–5) – (+7) = –15 – 18 + 6 + 2 – 5 – 7 = – 37
8) –11 – (–6) + (–3) + 4 – 2	–11 – (–6) + (–3) + 4 – 2 = – 11 + 6 – 3 + 4 – 2 = – 6

Exercise 1.16

Evaluate without the use of a calculator.

1) – (+4) = ……………………………………
2) – (–5) = ……………………………………
3) –7 – (+15) = ………………………………
4) 13 – (–9) = ………………………………
5) +14 + (+6) = ………………………………
6) –9 + (–5) = ………………………………
7) –9 + 0 = ……………………………………
8) 19 – (–7) = ………………………………
9) –13 + (–5) = ………………………………
10) –13 – 0 = …………………………………
11) 23 – (–4) = ………………………………...
12) –25 – (+12) = ……………………………
13) 0 – (+13) = ………………………………...
14) 0 – (–23) = ………………………………
15) –3 – 4 + 2 + (–7) + 6 = ……………………………………………………………………..
16) 14 – 17 + 4 + 3 = ……………………………………………………………………………
17) –4 – (–3 + –5) = …………………………………………………………………………….
18) 11 + 9 + (–6) – 2 = ………………………………………………………………………….
19) 24 – 7 – 5 – (+3) + 1 = ……………………………………………………………………..
20) –19 +(–21) – (–32) + 5 = …………………………………………………………………..
21) –23 – 12 – 34 – 15 = ……………………………………………………………………….

§ 1.17 MULTIPLICATION AND DIVISION OF INTEGERS

Remember:

- **+ $\boxed{\times \div}$ + = +**
- **+ $\boxed{\times \div}$ – = –**
- **– $\boxed{\times \div}$ + = –**
- **– $\boxed{\times \div}$ – = +**

When dividing, as for multiplying:

- if the signs are the **same**, the answer is **+**
- if the signs are **different**, the answer is **–**

EXAMPLE	SOLUTION		EXAMPLE	SOLUTION
1) Evaluate a) 7 × 11	7 × 11 = 77		2) Evaluate a) 24 ÷ –3	24 ÷ –3 = –8
b) 5 × –3	5 × -3 = –15		b) –15 ÷ 3	–15 ÷ 3 = –5
c) –4 × 7	–4 × 7 = –28		c) –49 ÷ –7	–49 ÷ –7 = 7
d) –6 × –2	–6 × –2 = 12		d) –2 × 5 × –6 × 4	–2 × 5 × –6 × 4 = 240
e) 18 ÷ 6	18 ÷ 6 = 3		e) (–2 + –3)(9 – 11)	(–2 + –3)(9 – 11) = –5 × –2 = 10

Note:
- ✧ An integer multiplied by zero gives an answer of zero, for example –6 × 0 = –6
- ✧ Zero divided by any integer other than zero, is zero, for example 0 ÷ 9 = 0
- ✧ Multiplying an integer by 1 does not change the integer, for example –8 × 1 = –8
- ✧ Division by zero is undefined, for example 3 ÷ 0 = undefined

Exercise 1.17

Calculate:

1) 0 × 24 = ………………
2) –6 × 1 = ……………
3) 1 × –6 = …………......

4) –2 × 7 = ………………
5) 5 × –4 = ……………
6) –4 × –1 = ……………

7) –5 × 0 = ………………
8) 15 × 1 = ……………
9) 0 ÷ 5 = ……………..

10) –24 × –10 = …………..
11) –9 ÷ 0 = ……………
12) –1 × 4 = ……………

13) –12 ÷ –4 = ……..
14) –8 ÷ –2 = …………...
15) –320 ÷ 10 = …………

16) –2 × –3 × –1 × 5
= ………………………
17) –3 × 4 × –2 × 5
= ……………………
18) 4 × –5 × 7 × – 2
= ……………………

19) (–2 + –1)(–4 +–5)
= ………………………
20) 28 ÷ –7 × –2
= ……………………
21) –4(–1 – 6)
= ……………………

22) 56 ÷ –8 –4
= ………………………
23) –24 ÷ (7 – 4)
= ……………………
24) 12 ÷ –4 + 2
= ……………………

25) (–5 × 2) × 3 × –2
= ………………………
26) 7 × (–3 – 2)
= ……………………..
27) 24 ÷ –3 × 4 ÷ –2
= ……………………

§ 1.18 THE SET OF INTEGERS

So far we have worked with three sets of numbers.

- The set of **integers**: Z = {……; -3; -2; -1; 0; 1; 2; 3; ………}
- The set of **whole numbers:** W = {0; 1; 2; 3; …………….}
- The set of **natural numbers:** N = {1; 2; 3; 4; ……………..}

Exercise 1.18

1) Answer the following:
   a) In which set/s of numbers do negative numbers occur? ………………………………
   b) Is there a smallest integer? ……………………………………………………….....
   c) Is there a greatest integer? ………………………………………………………………
2) In which set/s of number do you find each of the following numbers?
   a) 3 …………………………………………………………………
   b) 1 …………………………………………………………………
   c) –9 ………………………………………………………………..
   d) 0 …………………………………………………………………
   e) 27 ………………………………………………………………..
   f) 14 ………………………………………………………………..
   g) –41 ………………………………………………………………
   h) 102 ………………………………………………………………
   i) –15 ………………………………………………………………
   j) 200 ………………………………………………………………
3) Answer True or False:
   a) The set of natural numbers includes zero ………………………………………………
   b) There are negative numbers in N_0 ……………………………………………………
   c) Zero is an integer but not a natural number ……………………………………………
   d) Zero is a whole number but not an integer …………………………………………….
   e) All integers are whole numbers ………………………………………………………..
   f) All natural numbers are whole numbers ……………………………………………….
   g) All natural numbers are integers ……………………………………………………….
   h) All whole numbers are integers ………………………………………………………..

CHAPTER 2
Fractions and decimals

In this chapter you will:

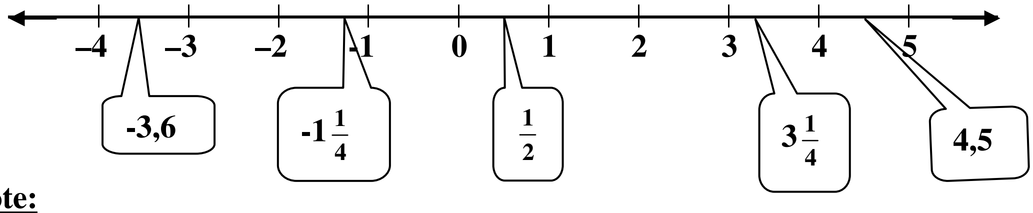
- Define fractions
- Find equivalent fractions
- Compare fractions
- Convert between mixed numbers and improper fractions
- Add and subtract fractions with the same denominators
- Add and subtract fractions with different denominators
- Multiply fraction
- Find the reciprocal of a number
- Divide fractions
- Convert fractions to decimals
- Compare the sizes of decimals
- Multiply decimals
- Divide decimals
- Round off to the nearest whole number and to 1 or 2 decimal places
- Round off and estimate answers
- Work with fractions and decimals on the scientific calculator
- Add and subtract fractions on a scientific calculator
- Use a scientific calculator to convert from recurring decimals to fractions

This chapter covers material from Topic 1: Numbers
SUBJECT OUTCOME 1.1:
Use Computational Tools and Strategies and Make Estimates and Approximations
Learning Outcome 1: Use a scientific calculator competently and efficiently
Learning Outcome 2: Execute algorithms appropriately in calculations

SUBJECT OUTCOME 1.2:
Demonstrate understanding of numbers and relationships among numbers and number systems and represent numbers in different ways
Learning Outcome 1: Identify rational numbers and convert between terminating or recurring decimals like $\frac{a}{b}$; $a; b \in Z; b \neq 0$

§ 2.1 FRACTIONS

- Examples of fractions and decimals are $3\frac{1}{4}$, $\frac{1}{2}$, $-1\frac{1}{4}$, -3,6 and 4,5.
- The position of these numbers can be shown on a **number line**:

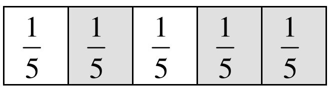


Note:
- the arrows on the ends of the number line show that it extends in both directions
- there is an infinite number of positive and negative numbers along the number line

This new set of numbers is called **the set of rational numbers**.

- Between any two integers there are an infinite number of rational numbers, so it is impossible to list them all.

Note:
- In the fraction $\frac{2}{3}$, 2 is called the **numerator** and 3 is called the **denominator**.
- $\frac{4}{5}$ is a **proper fraction,** since the numerator is less than the denominator
- $\frac{12}{5}$ is an **improper fraction**, since the numerator is greater than the denominator
- $2\frac{3}{4}$ is a **mixed number**, since it is made up of a whole number and a proper fraction

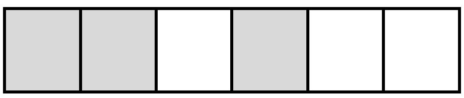


- The rectangle has 5 equal parts.
- **Each piece** is 1 part of the 5 equal parts.
- **Each piece** is $\frac{1}{5}$ of the rectangle: $5 \times \frac{1}{5} = 1$
- The shaded area is 3 parts of the 5 equal parts, so it is written as $\frac{3}{5}$ of the rectangle.

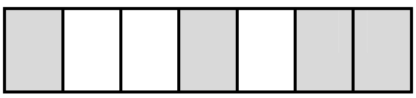
Exercise 2.1

For each of the figures below, answer the following questions:

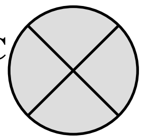
A



B



C



	A	B	C
1) How many equal parts does the figure have?			
2) How many of these parts are shaded?			
3) What fraction is the shaded area of the whole figure?			

§ 2.2 EQUIVALENT FRACTIONS

✧ We call $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$ and $\frac{4}{12}$ equivalent fractions. This is because $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$.

✧ We can find equivalent fractions by:

- multiplying the numerator and the denominator by the same number
- dividing the numerator and the denominator by the same number

Examples:

1) $\frac{2}{3} = \frac{4}{6}$, by multiplying both the numerator and the denominator by 2
2) $\frac{2}{3} = \frac{6}{9}$, by multiplying both the numerator and denominator by 3

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

3) $\frac{12}{24} = \frac{6}{12}$, dividing both the numerator and the denominator by 2
4) $\frac{6}{12} = \frac{3}{6}$, dividing both the numerator and the denominator by 2
5) $\frac{3}{6} = \frac{1}{2}$, dividing both the numerator and the denominator by 3

$$\frac{12}{24} = \frac{6}{12} = \frac{3}{6} =$$

Exercise 2.2

1) Fill in the missing numbers to make the fractions equivalent:

a) $\frac{1}{2} = \frac{\_}{8}$ b) $\frac{2}{3} = \frac{\_}{9}$ c) $\frac{2}{5} = \frac{10}{\_}$ d) $\frac{3}{4} = \frac{\_}{8}$

e) $\frac{4}{8} = \frac{\_}{2}$ f) $\frac{12}{18} = \frac{2}{\_}$ g) $\frac{3}{4} = \frac{9}{\_}$ h) $\frac{2}{5} = \frac{6}{\_}$

i) $\frac{3}{7} = \frac{21}{\_}$ j) $\frac{\_}{4} = \frac{12}{16}$ k) $\frac{3}{5} = \frac{\_}{45}$ l) $\frac{5}{8} = \frac{\_}{16}$

m) $\frac{10}{15} = \frac{\_}{3}$ n) $\frac{36}{48} = \frac{3}{\_}$ o) $\frac{\_}{81} = \frac{5}{9}$ p) $\frac{4}{\_} = \frac{2}{9}$

q) $\frac{21}{\_} = \frac{7}{8}$ r) $\frac{24}{\_} = \frac{6}{7}$

2) Write down the first five equivalent fractions starting with the number given:

a) $\frac{1}{5} = \quad = \quad = \quad = \quad$ b) $\frac{4}{7} = \quad = \quad = \quad = \quad$

c) $\frac{2}{3} = \quad = \quad = \quad = \quad$ d) $\frac{4}{9} = \quad = \quad = \quad = \quad$

e) $\frac{5}{8} = \quad = \quad = \quad = \quad$ f) $\frac{11}{12} = \quad = \quad = \quad = \quad$

g) $\frac{3}{4} = \quad = \quad = \quad = \quad$ h) $\frac{2}{5} = \quad = \quad = \quad = \quad$

§ 2.3 COMPARING FRACTIONS

1) <u>When the denominators are the same</u>:

Is $\frac{4}{5}$ greater than or less than $\frac{3}{5}$?

$\frac{4}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

From the diagrams we can see that $\frac{4}{5}$ is **greater** than $\frac{3}{5}$. We write $\frac{4}{5} > \frac{3}{5}$.

2) <u>When numerators are the same</u>:

Which is the smaller, $\frac{5}{8}$ or $\frac{5}{6}$?

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{8}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{6}$

From the diagram we can see that $\frac{5}{8}$ is less than $\frac{5}{6}$. So $\frac{5}{8} < \frac{5}{6}$.

3) <u>When numerators and denominators are different</u>:

Compare $\frac{4}{5}$ with $\frac{5}{6}$. Which is greater?

Choose the lowest multiple (LCM) of 5 and 6, which is 30.
Then write each fraction as an equivalent fraction having a denominator of 30.

$\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$ $\quad$ $\frac{5}{6} = \frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$ $\quad$ But $\frac{25}{30} > \frac{24}{30}$, so $\frac{5}{6} > \frac{4}{5}$.

<u>Exercise 2.3</u>

1) Which fraction is greater? Show all calculations.

a) $\frac{4}{7}$ or $\frac{6}{7}$ ……………………………………………………………………………….....

b) $\frac{5}{7}$ or $\frac{5}{9}$ ………………………………………………………………………………

c) $\frac{7}{9}$ or $\frac{20}{27}$ …………………………………………………………………………..……

d) $\frac{2}{3}$ or $\frac{5}{8}$ …………………………………………………………………………..……...

………………………………………………………………………………

2) Which fraction is smallest, $\frac{4}{5}$ or $\frac{3}{4}$ or $\frac{5}{6}$? Show all calculations.

…………………………………………………………………………………….……

……………………..……………………………………………………………………...

§ 2.4 CONVERTING FRACTIONS

1. Write fractions in simplest form:

A fraction is in **simplest form** when its numerator and denominator are as small as possible. They cannot be divided by any further number and still remain whole numbers.

Examples:

Write **a)** $\frac{24}{30}$, **b)** $\frac{15}{45}$ in simplest form (simplify)

Solution: **a)** $\frac{24}{30} = \frac{24 \div 6}{30 \div 6} = \frac{4}{5}$ **b)** $\frac{15}{45} = \frac{15 \div 5}{45 \div 5} = \frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$

2. Convert improper fractions to mixed numbers:

Examples:

1) $\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3$ **2)** $\frac{7}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = 1 + 1 + \frac{1}{3} = 2\frac{1}{3}$

3) $\frac{9}{5} = \frac{5}{5} + \frac{4}{5} = 1\frac{4}{5}$ As a shortcut: $\frac{9}{5}$ = 1 remainder 4 = $1\frac{4}{5}$

3. Convert mixed numbers into improper fractions:

Examples:

1) $1\frac{1}{7} = 1 + \frac{1}{7} = \frac{7}{7} + \frac{1}{7} = \frac{8}{7}$ **2)** $2\frac{3}{4} = 1 + 1 + \frac{3}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$

As a shortcut: $2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4}$

Exercise 2.4

1) Write each fraction in simplest form / simplify:

a) $\frac{4}{10}$ = …………………….. **b)** $\frac{14}{28}$ = …………………… **c)** $\frac{25}{40}$ = ………………

d) $\frac{12}{27}$ = …………………….. **e)** $\frac{16}{28}$ = …………………… **f)** $\frac{33}{55}$ = ………………

2) Convert these improper fractions to mixed numbers in simplest form:

a) $\frac{12}{5}$ = ………………………………………… **b)** $\frac{48}{7}$ = ……………………………………

c) $\frac{64}{9}$ = ………………………………………… **d)** $\frac{38}{4}$ = ……………………………………

e) $\frac{75}{9}$ = ………………………………………… **f)** $\frac{44}{8}$ = ……………………………………

3) Convert these mixed numbers to improper fractions:

a) $1\frac{1}{5}$ = ………………………………………… **b)** $2\frac{3}{4}$ = ……………………………………

c) $3\frac{5}{8}$ = ………………………………………… **d)** $4\frac{2}{7}$ = ……………………………………

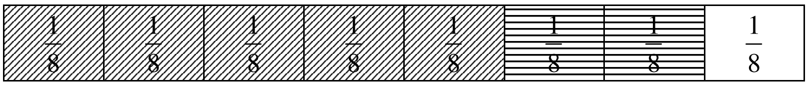
§ 2.5 ADDING AND SUBTRACTING FRACTIONS WITH THE SAME DENOMINATORS

To add or subtract fractions:

- Make sure their denominators are the same – use equivalent fractions if they are not
- Add or subtract the numerators
- Simplify the answer

EXAMPLES

1)



$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

2) $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

3) $2 - \frac{1}{8} = 2(\frac{8}{8}) - \frac{1}{8} = \frac{16}{8} - \frac{1}{8} = \frac{15}{8} = 1\frac{7}{8}$

Note: $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5}$ etc.

Exercise 2.5

Calculate and simplify your answers:

1) $\frac{3}{7} + \frac{2}{7} =$ ……………………………………………………………………

2) $\frac{1}{5} + \frac{3}{5} =$ ……………………………………………………………………

3) $\frac{7}{9} - \frac{4}{9} =$ ……………………………………………………………………

4) $\frac{5}{6} - \frac{2}{6} =$ ……………………………………………………………………

5) $1 - \frac{1}{5} =$ ……………………………………………………………………

6) $1 - \frac{3}{4} =$ ……………………………………………………………………

7) $\frac{1}{8} + \frac{3}{8} + \frac{2}{8} =$ ……………………………………………………………………

8) $1 + \frac{2}{9} - \frac{1}{9} =$ ……………………………………………………………………

9) $1 - \frac{2}{7} - \frac{1}{7} =$ ……………………………………………………………………

10) $\frac{14}{15} - \frac{11}{15} + \frac{2}{15} =$ ……………………………………………………………………

11) $2 - \frac{4}{5} + \frac{1}{5} =$……………………………………………………………………

12) $1\frac{3}{8} + \frac{1}{8} =$ ……………………………………………………………………

13) $\frac{7}{20} - \frac{3}{20} + \frac{1}{20} =$ ……………………………………………………………………

14) $3 - \frac{4}{5} =$ ……………………………………………………………………

§ 2.6 ADDING AND SUBTRACTING FRACTIONS WITH DIFFERENT DENOMINATORS

✧ To add or subtract fractions like $\frac{1}{2}$ and $\frac{1}{3}$ they must each be written as equivalent fractions with the same denominator.

EXAMPLES	
Calculate: 1) $\frac{1}{2}+\frac{1}{3}$ **2)** $\frac{5}{12}-\frac{1}{3}$ **3)** $\frac{3}{4}+\frac{2}{5}$ **4)** $3\frac{1}{8}-1\frac{3}{4}$	
SOLUTION	
1) The LCM of 2 and 3 is 6. $\frac{1}{2}=\frac{1\times3}{2\times3}=\frac{3}{6}$ and $\frac{1}{3}=\frac{1\times2}{3\times2}=\frac{2}{6}$ So $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$	**2)** The LCM of 12 and 3 is 12 $\frac{5}{12}-\frac{1}{3}=\frac{5}{12}-\frac{1\times4}{3\times4}=\frac{5}{12}-\frac{4}{12}=\frac{1}{12}$
3) $\frac{3}{4}+\frac{2}{5}$ The LCM of 4 and 5 is 20. $=\frac{3\times5}{4\times5}+\frac{2\times4}{5\times4}$ $=\frac{15}{20}+\frac{8}{20}$ $=\frac{23}{20}=1\frac{3}{20}$	**4)** $3\frac{1}{8}-1\frac{3}{4}$ The LCM of 8 and 4 is 8 $=\frac{25}{8}-\frac{7}{4}$ $=\frac{25}{8}-\frac{7\times2}{4\times2}$ $=\frac{25}{8}-\frac{14}{8}=\frac{11}{8}=1\frac{3}{8}$

<u>Exercise 2.6</u>

Calculate without a calculator. Write your answers as mixed numbers where possible.

1) $\frac{1}{2}+\frac{1}{8}=$ ……………………………………………………………………………………

2) $\frac{3}{4}+\frac{5}{8}=$ ……………………………………………………………………………………

3) $\frac{4}{5}-\frac{1}{3}=$ ……………………………………………………………………………………..

4) $\frac{5}{12}+\frac{2}{3}+\frac{3}{8}=$ ……………………………………………………………………………

5) $1\frac{5}{6}-\frac{2}{3}=$ ……………………………………………………………………………….

6) $3\frac{5}{8}-1\frac{2}{3}=$ ………………………………………………………………………………

7) $1\frac{1}{3}-\frac{5}{6}+\frac{1}{6}=$ ……………………………………………………………………………

8) $2\frac{1}{2}-1\frac{3}{4}=$ ………………………………………………………………………………

9) $2\frac{7}{8}-1\frac{5}{6}=$ ………………………………………………………………………………

10) $4\frac{7}{9}-2\frac{2}{3}=$ ……………………………………………………………………..........

§ 2.7 MULTIPLICATION OF FRACTIONS

EXAMPLE	
Find the following products: **1)** $\frac{2}{5} \times \frac{4}{9}$ **2)** $\frac{3}{9} \times \frac{3}{14}$ **3)** $2\frac{3}{5} \times 1\frac{2}{13}$ **4)** $3\frac{3}{7} \times \frac{1}{2}$	
SOLUTION	
1) $\frac{2}{5} \times \frac{4}{9} = \frac{2\times 4}{5\times 9} = \frac{8}{45}$	**2)** $\frac{3}{9} \times \frac{3}{14} = \frac{3\times 3}{9\times 14} = \frac{1}{4}$ 3 is a factor of 3 and 9, so cancel
3) $2\frac{3}{5} \times 1\frac{2}{13} = \frac{13}{5} \times \frac{15}{13}$ $= \frac{13\times 15}{5\times 13}$ cancel numerator and denominator by 13 and 5 $= 3$	
4) $3\frac{3}{7} \times \frac{1}{2} = \frac{24}{7} \times \frac{1}{2} = \frac{24\times 1}{7\times 2}$ 2 is a factor of 24 and 2, so you can cancel by 2 $= \frac{12}{7} = 1\frac{5}{7}$ The answer is given as a mixed number	

Exercise 2.7

Find the following products. Whenever possible, write your answers as mixed numbers.

1) $\frac{9}{10} \times \frac{3}{2} =$ ……………………………………………………………………………

2) $\frac{2}{7} \times \frac{3}{5} =$ ……………………………………………………………………………...

3) $\frac{5}{6} \times \frac{9}{10} =$ ……………………………………………………………………………

4) $\frac{13}{16} \times \frac{4}{5} =$ ……………………………………………………………………………

5) $\frac{2}{5} \times 3\frac{3}{4} =$ ……………………………………………………………………………

6) $3\frac{1}{4} \times \frac{2}{9} =$ ……………………………………………………………………………

7) $2\frac{1}{19} \times 12\frac{2}{3} =$ ……………………………………………………………………………

8) $1\frac{3}{5} \times 2\frac{1}{2} =$ ……………………………………………………………………………

9) $\frac{3}{4} \times \frac{5}{9} \times \frac{8}{25} =$ ……………………………………………………………………………

10) $\frac{3}{4} \times 2\frac{2}{3} \times 3\frac{1}{4} =$ …………………………………………………………………………..

11) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} =$ ……………………………………………………………………………

12) $2\frac{8}{9} \times 1\frac{2}{13} =$ …………………………………………………………………………..

§ 2.8 THE RECIPROCAL OF A NUMBER

When we multiply any number by its reciprocal, we get 1 as an answer.

- The reciprocal of 2 is $\frac{1}{2}$, because $2 \times \frac{1}{2} = 1$.
- The reciprocal of $\frac{1}{6}$ is 6, because $\frac{1}{6} \times 6 = 1$
- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \times \frac{3}{2} = 1$

EXAMPLES	SOLUTIONS
What is the reciprocal of 1) 3?	The reciprocal of 3 is $\underline{\frac{1}{3}}$, since $3 \times \frac{1}{3} = 1$
2) $\frac{3}{4}$?	The reciprocal of $\frac{3}{4}$ is $\underline{\frac{4}{3}}$, since $\frac{3}{4} \times \frac{4}{3} = 1$
3) -5	The reciprocal of -5 is $\underline{-\frac{1}{5}}$, since $-5 \times -\frac{1}{5} = 1$
4) $-\frac{1}{8}$?	The reciprocal of $-\frac{1}{8}$ is $\underline{-8}$, since $-\frac{1}{8} \times -8 = 1$
5) $2\frac{3}{8}$?	The reciprocal of $2\frac{3}{8}$ is $\underline{\frac{8}{19}}$, since $2\frac{3}{8} = \frac{19}{8}$ and $\frac{19}{8} \times \frac{8}{19} = 1$

Exercise 2.8

1) The reciprocal of 6 is ……….. since ……………………………………………………

2) The reciprocal of $\frac{2}{3}$ is ……….. since …………………………………………………...

3) The reciprocal of $\frac{-4}{3}$ is ……….. since …………………………………………………..

4) The reciprocal of -2 is ……….. since ……………………………………………………

5) The reciprocal of $\frac{5}{9}$ is ……….. since ……………………………………………………

6) The reciprocal of $\frac{-6}{7}$ is ……….. since ……………………………………………………

7) The reciprocal of $3\frac{1}{2}$ is ……….. since ……………………………………………………

8) The reciprocal of -9 is ……….. since ……………………………………………………

9) The reciprocal of $2\frac{3}{4}$ is ……….. since …………………………………………………..

§ 2.9 DIVISION OF FRACTIONS

1) If we are asked to calculate $3 \div \frac{1}{6}$, we are actually being asked "how many sixths are there in 3 wholes".

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

There are 18 sixths in 3 wholes, so $3 \div \frac{1}{6} = 18$ OR $3 \div \frac{1}{6} = \frac{3}{\frac{1}{6}} = \frac{3\times 6}{\frac{1}{6}\times 6} = \frac{18}{1} = 18$

2) $6 \div 2 = 3$ but $6 \times \frac{1}{2} = 3$, so division by 2 is the same as multiplying by $\frac{1}{2}$, its reciprocal. As a shortcut, when you divide by a fraction, you multiply by its reciprocal.

EXAMPLES	**SOLUTIONS**
Calculate 1) $\frac{2}{3} \div \frac{1}{3}$	$\frac{2}{3} \div \frac{1}{3} = \frac{2}{3}\times\frac{3}{1} = \frac{2}{\cancel{3}}\times\frac{\cancel{3}}{1} = 2$
2) $\frac{-6}{7} \div \frac{3}{14}$	$\frac{-6}{7} \div \frac{3}{14} = \frac{-6}{7}\times\frac{14}{3} = \frac{-{}^{2}\cancel{6}}{\cancel{7}}\times\frac{14}{\cancel{3}} = -4$
3) $\frac{9}{10} \div 18$	$\frac{9}{10} \div 18 = \frac{9}{10}\times\frac{1}{18} = \frac{1}{20}$
4) $2\frac{1}{3} \div 3\frac{1}{2}$	$2\frac{1}{3} \div 3\frac{1}{2} = \frac{7}{3} \div \frac{7}{2} = \frac{7}{3}\times\frac{2}{7} = \frac{2}{3}$

Exercise 2.9

Find the following quotients and simplify your answers:

1) $-\frac{5}{6} \div \frac{5}{12} =$ ……………………………………………………………………………

2) $\frac{2}{3} \div 6 =$ ……………………………………………………………………………

3) $45 \div 7\frac{1}{2} =$ ……………………………………………………………………………

4) $4\frac{3}{4} \div \frac{1}{4} =$ ……………………………………………………………………………

5) $-3 \div \frac{3}{4} =$ ……………………………………………………………………………..

6) $2\frac{5}{8} \div 3\frac{1}{2} =$ ……………………………………………………………………………

7) $4\frac{4}{5} \div 1\frac{1}{5} =$ ……………………………………………………………………………

8) $3\frac{4}{7} \div 1\frac{1}{14} =$ ……………………………………………………………………………..

9) $\frac{2}{7} \div \frac{5}{7} \div \frac{1}{10} =$ ……………………………………………………………………………

§ 2.10 CONVERTING FRACTIONS TO DECIMALS

- To convert a fraction to a decimal, divide the denominator into the numerator.

Note:

- $2 = 2{,}0 = 2{,}00 = 2{,}0000\ldots.$
- $3{,}5 = 3{,}50 = 3{,}500 = 3{,}5000 = 3{,}50000\ldots.$

We can write any number of zeros after the decimal comma.

EXAMPLES	SOLUTIONS
Convert the following fractions to decimals 1) $\frac{1}{2}$	$\frac{1}{2} = \frac{1{,}0}{2} = 0{,}5$
2) $1\frac{3}{4}$	$1\frac{3}{4} = \frac{7}{4} = \frac{7{,}000}{4} = 1{,}75$
3) $\frac{19}{1000}$	$\frac{19}{1000} = 0{,}019$

Exercise 2.10

1) Convert these fractions to decimals:

a) $2\frac{1}{2}$ = ……….. **b)** $3\frac{7}{10}$ = ……….. **c)** $\frac{7}{8}$ = ………….. **d)** $\frac{2}{100}$ = ……….

e) $\frac{7}{80}$ = ……….. **f)** $\frac{5}{8}$ = ………….. **g)** $\frac{10}{8}$ = ………… **h)** $\frac{45}{150}$ = ………...

Recurring decimal notation:

- Some answers will be a **recurring** or **repeating decimal**.
- In a recurring decimal, a digit or group of digits, which occur after the decimal comma, are repeated. The numbers which recur are written under a line, or under and between dots.

EXAMPLES	SOLUTIONS
Write as a decimal: 1) $\frac{1}{3}$	$\frac{1}{3} = 0{,}333333\ldots. = 0{,}\dot{3}$ or $1{,}\overline{3}$
2) $4\frac{23}{99}$	$4\frac{23}{99} = 4{,}23232323\ldots.. = 4{,}\dot{2}\dot{3}$ or $4{,}\overline{23}$
3) $9\frac{152}{999}$	$9\frac{152}{999} = 9{,}152\ 152\ 152\ \ldots.. = 9{,}\dot{1}5\dot{2}$
4) $3\frac{97}{666}$	$3\frac{97}{666} = 3{,}1\ 456\ 456\ 456\ldots. = 3{,}1\overline{456}$, since the 1 does not recur

2) Write as a recurring decimal:

a) 3,88888……… = ………………….. **b)** 12,45454545……….. = ……………….

c) 5,238238238238 …….. = …………… **d)** 3,417171717…… = …………………..

e) $1\frac{2}{3}$ = ……………………………….. **f)** $\frac{5}{6}$ = …………………………………

§ 2.11 COMPARING AND CONVERTING

Comparing the size of decimal numbers

1) **Compare the signs:**
   1,2 > – 6,7 since a positive number is always greater than a negative number.
   – 1,2 > – 3,8 since on the number line, the greater negative number is further from 0.
2) **Compare** the whole numbers:
   3,49 is greater than **2**,9 since 3 > 2.
3) **Compare the numbers in the first decimal place:**
   If the **integer parts are the same,** then we compare the numbers in the **first decimal place**.
   e.g. 4,**6**9 is greater than 4,**2**98 since in the first decimal place, 6 > 2.
4) **Compare the second decimal place:**
   If the **numbers in first decimal place are the same**, compare the numbers in the **second decimal place**.
   e.g. 2,3**6**9 9 is less than 2,3**9**1 97 since 6 < 9. We can continue in this way.
5) **Compare recurring numbers:**
   $1,\dot{3} > 1,3$ since 1,3333… is greater than 1,3

Exercise 2.11

1) Insert = or > or <:

a) 3,1659 …... 3,1700	**b)** 0,00879 …... 0,00891	**c)** 5,1903 …... 5,1903000
d) 1,001 …... 0,009	**e)** –3,6 …… 0,1	**f)** –2,78 …… –2,99
g) 2,5 …… $2,\dot{5}$	**h)** –0,019 …… 0	**i)** 11,009 …… 11,0019

Converting from a decimal to a fraction
We write one zero in the denominator for each decimal place

EXAMPLES		
Convert to a fraction and simplify: 1) 0,5 2) 0,34 3) 0,056 4) 3,79 5) 0,005		
SOLUTION		
1) $0,5 = \frac{5}{10}$	**2)** $0,34 = \frac{34}{100}$	**3)** $0,056 = \frac{56}{1\,000}$
4) $3,79 = 3\frac{79}{100} = \frac{379}{100}$	**5** $0,005 = \frac{5}{1\,000}$	

2) Convert to a fraction and simplify:

   a) 3,65 = ……………….. b) 0,1 = ………………..

   c) 7,5 = ………………... d) 2,04 = ………………..

   e) 0,045 = ………………..

§ 2.12 MULTIPLYING DECIMALS

EXAMPLES	SOLUTIONS
Calculate: 1) 1,2 × 0,3	$1,2 \times 0,3 = \frac{12}{10} \times \frac{3}{10} = \frac{36}{100} = 0,36$ **<u>Shortcut</u>:** Multiply the numbers ignoring the decimal comma. 3 × 12 = 36. There is 1 decimal place in each number, so there will be 1 + 1 = 2 decimal places in the answer.
2) 0,04 × 200	$0,04 \times 200 = \frac{4}{100} \times \frac{200}{1} = 8,00$ **<u>Shortcut</u>:** 4 × 200 = 800. There are 2 decimal places in the answer
3) 1,30 × 0,1	$1,30 \times 0,1 = \frac{13}{10} \times \frac{1}{10} = 0,130$ **<u>Shortcut</u>**: 130 × 1 = 130. There are 2 + 1 = 3 decimal places in the answer.
4) 0,0042 × 1,2	0,0042 × 1,2 = 0,00504 **<u>Shortcut:</u>** 42 × 12 = 504. There are 4 + 1 = 5 decimal places in the answer. Add zeros to the **left** of 504 in order to get the correct number of decimal places

<u>Exercise 2.12</u>

Simplify without using a calculator:

1) 0,25 × 4 = ……………………………………………………………………………..

2) 0,004 × 1,6 = ……………………………………………………………………….

3) 14,5 × 1,1 = ………………………………………………………………………...

4) 1,25 × 0,4 = ………………………………………………………………………...

5) 21,5 × 0,02 = ……………………………………………………………………….

6) 1,004 × 10 = ………………………………………………………………………..

7) 0,02 × 500 = ………………………………………………………………………..

8) 1,300 × 0,40 = ……………………………………………………………………...

9) 0,005 × 1,02 = ……………………………………………………………………...

10) 1,000 × 0,09 = ……………………………………………………………………...

11) 250 × 4,0 = …………………………………………………………………………

12) 2,5 × 0,002 = ……………………………………………………………………….

§ 2.13 DIVIDING DECIMALS

To divide decimals:
- Count the number of decimal places in the denominator.
- Make the **denominator a whole number** by multiplying both the numerator and the denominator by the same multiple of 10 as the number of decimal places in the denominator.

EXAMPLE	SOLUTION
Calculate 1) 4,2 ÷ 0,02	$4,2 \div 0,02 = \frac{4,2}{0,02} = \frac{4,2\times100}{0,02\times100} = \frac{420}{2} = 210.$ **Note:** There are two decimal places in the denominator, so multiply both numerator and denominator by 100
2) 32 ÷ 0,005	$32 \div 0,005 = \frac{32}{0,005} = \frac{32\times1\,000}{0,005\times1\,000} = \frac{32\,000}{5} = 64\,000$ **Note:** There are 3 decimal places in the denominator, so multiply both numerator and denominator by 1 000

Exercise 2.13

1) Calculate the following quotients **without using a calculator**:

a) 350 ÷ 100 = ………………………...
b) 37,4 ÷ 11 = …………………………
c) 12,65 ÷ 0,05 = ……………………...
d) 300,05 ÷ 0,005 = …………………...
e) 1,004 ÷ 0,02 = ……………………...
f) 284,6 ÷ 1,2 = ………………………
g) 3,411 ÷ 0,9 = ………………………
h) 45,550 ÷ 0,5 = ……………………...
i) 39,3 ÷ 0,3 = ………………………...
j) 1,44 ÷ 1,2 = ………………………...

2) Calculate the following products and quotients **without using a calculator**:

a) 600 ÷ 0,003 = ……………………………………………………………………
b) 0,002 × 60 = ……………………………………………………………………..
c) 120 × 0,4 = ………………………………………………………………………
d) 36 ÷ 1,2 = ………………………………………………………………………..
e) 0,2 × 0,3 × 200 = ……………………………………………………………..
f) 4 500 ÷ 0,5 ÷ 1 000 = ………………………………………………………...
g) 2,4 ÷ 0,02 × 5 = ……………………………………………………………
h) 900 ÷ 0,03 × 100 = …………………………………………………………...
i) 1,08 ÷ 9 ÷ 0,4 = ……………………………………………………………
j) 12,1 ÷ 0,11 × 0,002 = ……………………………………………………

§ 2.14 ROUNDING OFF TO THE NEAREST WHOLE NUMBER AND TO 1 OR 2 DECIMAL PLACES

1. Rounding off to the nearest whole number:
Underline the number in the **units** position. If the number to the **right** is **5 or greater**, round **up** to the next whole number. If it is **less than 5**, round **down** to the given whole number.

EXAMPLE	SOLUTION
Round the following numbers to the nearest whole number 1) 4,**2**76	 $\underline{4}$,**2**76 ≈ 4, since 2 < 5
2) 12,**5**3	1$\underline{2}$,**5**3 ≈ 13, since 5 ≥ 5
3) 0,199	$\underline{0}$,**1**99 ≈ 0, since 1 < 5

Exercise 2.14

1) Round off to the nearest whole number:

a) 3,29 ≈ ………. **b)** 21,51 ≈ ……... **c)** 0,482 ≈ …….. **d)** 0,59 ≈ ……..

e) 109,51 ≈ ……. **f)** 120,49 ≈ …… **g)** 2 000,82 ≈ … **h)** 199,67 ≈ …..

2. Rounding off or correcting to 1 decimal place:
Underline the number in the first decimal place. If the number to the **right** is **5 or greater**, round **up** the number in the first decimal place. If it is **less than 5**, round **down**.

Note:
To round off or correct to **any** number of **decimal places**, look at the next **decimal place** to see whether it is 5 or greater. If so, round **up**. If not, round **down**.

EXAMPLE	SOLUTION
Round the following numbers to 1 decimal place: 1) 3,551	 3,$\underline{5}$**5**1 ≈ 3,6 since 5 ≥ 5
2) 11,949	11,$\underline{9}$**4**9 ≈ 11,9 since 4 < 5
3) 3,991	3,$\underline{9}$**9**1 ≈ 4,0 since 9 ≥ 5

2) Round off to 1 decimal place:

a) 23,451 = …….. **b)** 36,623 = …….. **c)** 0,025 = …….. **d)** 329,95 = ……..

e) 145,459 = …… **f)** 6,019 = …….. **g)** 0,009 = …….. **h)** 26,951 = ……..

3) Round off to 2 decimal places:

a) 3,5608 = …….. **b)** 12,915 = …….. **c)** 99,995 = …… **d)** 0,0051 = ……..

§ 2.15 ROUNDING OFF AND ESTIMATING ANSWERS

EXAMPLE	SOLUTION
1) Use a calculator to calculate 6,179 × 180,5 correct to 2 decimal places.	6,179 × 180,5 = 1 115,3095… = 1 115,31 correct to 2 decimal places.
2) Check your answer by rounding each number to the nearest whole number.	6,179 × 180,5 ≈ 6 × 181 = 1 086 correct to the nearest whole number.
3) Check your answer by rounding each number to 1 significant figure	6,179 × 180,5 = 6 × 200 = 1 200

Exercise 2.15

1) a) Calculate $\frac{1,26\times 25,3}{0,995\,6\times 10,729}$ using a calculator correct to the nearest whole number.

b) Estimate the answer by rounding each number to one significant figure.

c) Does your answer seem to be correct?

2) a) Use a calculator to calculate 207,8 × 15,995 correct to 1 decimal place.

b) Check this answer by rounding each number to one significant figure.

c) Estimate the answer by rounding each number to the nearest whole number

d) Do these estimates show that your answer is reasonably correct?

§ 2.16 FRACTIONS AND DECIMALS ON THE SCIENTIFIC CALCULATOR

Fraction keys on the calculator enable you to:

- Simplify fractions (i.e. write them in their simplest form)
- Convert between mixed numbers, improper fractions and decimals
- Do calculations involving fractions

The **CASIO *fx*-82ES PLUS Scientific Calculator:**

- has fraction keys marked $\boxed{\frac{\blacksquare}{\square}}$ and $\boxed{\blacksquare\frac{\square}{\square}}$.
- $\boxed{\blacksquare\frac{\square}{\square}}$ is shown above the $\boxed{\frac{\blacksquare}{\square}}$ key and is used by pressing $\boxed{\text{shift}}$ and then $\boxed{\frac{\blacksquare}{\square}}$.
- The arrow keys move the cursor around the blocks on the display.

<u>Note:</u>

- First get the calculator in the correct mode by entering the following keys:
  MODE: Select 1: COMP **SETUP:** Select 1: Mth IO
- Repeated entering of $\boxed{S \Leftrightarrow D}$ will enable you to go back and forth between a decimal and an improper fraction, i.e. you **toggle** from one to the other.

EXAMPLE	SOLUTION	
1) Use your calculator to:	**KEY SEQUENCE**	**ANSWER**
a) Simplify $\frac{18}{27}$	$\boxed{\frac{\blacksquare}{\square}}$ 18 $\downarrow$ 27 $\boxed{=}$	$\frac{2}{3}$
b) Write $\frac{18}{27}$ as a decimal	$\boxed{S \Leftrightarrow D}$	0,6666
2) Use your calculator to write $2\frac{5}{9}$		
a) as an improper fraction	$\boxed{\blacksquare\frac{\square}{\square}}$ 2 $\triangleright$ 5 $\downarrow$ 9 $\boxed{=}$	$\frac{23}{9}$
b) as a decimal	$\boxed{S \Leftrightarrow D}$	2,5555..
3) Use your calculator to:		
a) Simplify $\frac{36}{15}$	$\boxed{\frac{\blacksquare}{\square}}$ 36 $\downarrow$ 15 $\boxed{=}$	$\frac{12}{5}$
b) Write $\frac{36}{15}$ as a mixed number	$\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$2\frac{2}{5}$
c) Write $\frac{36}{15}$ as a decimal	$\boxed{S \Leftrightarrow D}$	2,4

<u>Exercise 2.16</u>

Use your calculator to:

1) Simplify $\frac{24}{18}$ ……………………………… 2) Write $\frac{24}{18}$ as a decimal …

3) Write $\frac{24}{18}$ as a mixed number ………….. 4) Write 2,54 as an improper fraction …….

5) Write 2,54 as a mixed number …………………………………………………..

§ 2.17 ADDING AND SUBTRACTING FRACTIONS ON A SCIENTIFIC CALCULATOR

EXAMPLES

1) Calculate $\frac{4}{5} + \frac{2}{3}$, and write the answer as **a)** an improper fraction, **b)** a decimal and **c)** a proper fraction.

Key Sequence	(a)	K S	(b)	K S	(c)
$\frac{\blacksquare}{\square}$ 4 (↓) 5 (▶) $\boxed{+}$ $\frac{\blacksquare}{\square}$ 2 (↓) 3 $\boxed{=}$	$\frac{22}{15}$	$\boxed{S \Leftrightarrow D}$	1,466...	$\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$1\frac{7}{15}$

2) Calculate $1\frac{5}{7} + \frac{3}{4}$. Write your answer as a proper fraction.

Key sequence	Display	K S	Display
$\blacksquare\frac{\square}{\square}$ 1 (▶) 5 (↓) 7 (▶) $\boxed{+}$ $\frac{\blacksquare}{\square}$ 3 (↓) 4 $\boxed{=}$	$\frac{69}{28}$	$\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$2\frac{13}{28}$

3) Calculate $\dfrac{3}{\frac{2}{3} - \frac{1}{4}}$ as a proper fraction

Key sequence	Display	K S	Display
$\frac{\blacksquare}{\square}$ 3 (↓) $\frac{\blacksquare}{\square}$ 2 (↓) 3 (▶) $\boxed{-}$ $\frac{\blacksquare}{\square}$ 1 (↓) 4 (▶) $\boxed{=}$	$\frac{36}{5}$	$\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$7\frac{1}{5}$

Exercise 2.17

1) Use a calculator to write $\frac{12}{13}$ as a decimal (correct to 1 decimal place) ....…………

2) Use a calculator to write $\frac{23}{14}$ as

   a) a mixed number ………………………………….

   b) a decimal (correct to 1 decimal place) ………………………………………….

3) Use a calculator to write each answer as

   i) a decimal (correct to 2 decimal places) and ii) a mixed number.

   a) $\frac{3}{4} + \frac{4}{7}$ = ………………………………………………………………………………

   b) $7\frac{11}{12} - 5\frac{7}{8}$ = ………………………………………………………………………………

   c) $1\frac{6}{7} + 4\frac{5}{8}$ = ………………………………………………………………………………

   d) $3\frac{5}{8} - 2\frac{1}{4}$ = ………………………………………………………………………………

§ 2.18 USING A SCIENTIFIC CALCULATOR TO CONVERT FROM RECURRING DECIMALS TO FRACTIONS

EXAMPLE	SOLUTION	
Use your calculator to:	**KEY SEQUENCE**	**ANSWER**
1) Prove that $3,\dot{5} = 3\frac{5}{9}$	3,55555…… (until the display is full) $\boxed{=}$ $\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$3\frac{5}{9}$
2) Convert $1,\dot{1}4\dot{5}$ to a mixed number	1,1145145145 …… $\boxed{=}$ $\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$1\frac{145}{999}$
3) Convert $2,3\overline{45}$ to a mixed number	2,34545454545… $\boxed{=}$ $\boxed{\text{shift}}$ $\boxed{S \Leftrightarrow D}$	$2\frac{19}{55}$

Note:
- In all three examples it is necessary to fill the screen with the recurring decimals
- In example 3, the 3 in the first decimal place does not recur

Exercise 2.18

Use a calculator to write each recurring decimal as a fraction. Where possible, write as a mixed number.

1) $0,\dot{3}$ = ………………………………………
2) $2,\overline{8}$ = ………………………………………
3) $3,\dot{5}$ = ……………………………………….
4) $12,\overline{46}$ = …………………………………….
5) 1,444 ….. = ………………………………..
6) $0,\dot{3}\dot{5}$ = ……………………………………..
7) $5,\dot{1}5\dot{3}$ = …………………………………….
8) $1,2\dot{4}$ = ……………………………………...

CHAPTER 3
Rational numbers and real numbers

In this chapter you will:

- Find ratios of two or more quantities
- Find equivalent ratios
- Divide a quantity in a given ratio
- Calculate rates
- Work with ratios in direct proportion
- Work with ratios in indirect proportion
- Calculate percentages
- Multiply and divide exponents
- Work with zero exponents and negative exponents
- Simplify products and quotients raised to a power
- Raise an exponent to a power
- Work with number sequences
- Find the value of missing terms in a sequence
- Decide whether numbers are rational numbers or not
- Decide whether numbers are irrational numbers or not
- Simplify surds
- Decide whether numbers are real or not
- Use the real number diagram to answer questions

This chapter covers material from Topic 1: Numbers

SUBJECT OUTCOME 1.1:

Use Computational Tools and Strategies and Make Estimates and Approximations

Learning Outcome 1: Use a scientific calculator competently and efficiently

Learning Outcome 2: Execute algorithms appropriately in calculations

SUBJECT OUTCOME 1.2:

Demonstrate understanding of numbers and relationships among numbers and number systems and represent numbers in different ways

Learning Outcome 1: Identify rational numbers and convert between terminating or recurring decimals like $\frac{a}{b}$; $a; b \in Z; b \neq 0$

Learning Outcome 2: Know, understand and apply the laws of exponents

Learning Outcome 3: Convert surds into rational forms

Learning Outcome 4: Identify and work with arithmetic progressions, sequences and series

§ 3.1 RATIO

✧ A ratio compares two or more quantities. In the nine identical rectangles below, three are shaded and six are unshaded.
We say that:

- the ratio of the shaded to the unshaded rectangles is 3 to 6 which we write 3 : 6
- the ratio of the unshaded to the shaded rectangles is 6 to 3 which we write 6 : 3
- the ratio of the shaded rectangles to all the rectangles is 3 to 9 which we write 3 : 9
- the ratio of the unshaded rectangles to all the rectangles is 6 to 9 which we write 6 : 9

Note:

- The ratios remain the same no matter which three of the nine rectangles are shaded
- The **order** in which the ratio is written is **important**
- The ratio is written **without units, since we are comparing rectangles with rectangles**
- The ratio 3 : 6 can be written as the fraction $\frac{3}{6} = \frac{1}{2} = 1 : 2$
- 3 : 6 = 1 : 2 are **equivalent ratios**

EXAMPLE	**SOLUTION**
Fruit juice is made by mixing 1 cup of concentrate with 3 cups of water. Find the ratio of the number of cups of: 1) concentrate to water	1 : 3
2) concentrate to mixture	1 : 4
3) mixture to water	4 : 3
4) water to mixture	3 : 4
5) water to concentrate	3 : 1
6) mixture to concentrate	4 : 1

Note:

- the unit here is the cup
- A ratio is always written without units

Exercise 3.1

In a class of 30 learners, 12 are boys. Find the following ratios and write in simplest form:

1) number of boys : number of girls ………………………………………………

2) number of girls : number in class ……………………………………………….

3) number of boys : number in class ……………………………………………….

4) number of girls : number of boys ………………………………………………..

5) number in class : number of boys ………………………………………………..

6) number in class : number of girls ………………………………………………..

7) number of boys and girls : number in class ……………………………………

§ 3.2 EQUIVALENT RATIOS

EXAMPLE	SOLUTION
1) A recipe uses $1\frac{1}{2}$ cups of flour, 1 cup of sugar and $\frac{3}{4}$ of a cup of milk. Express the ratio of flour to sugar to milk in its simplest form.	Flour : sugar : milk $= 1\frac{1}{2} : 1 : \frac{3}{4}$ $= \frac{3}{2} : 1 : \frac{3}{4}$ *The LCM of 2 and 4 = 4, so multiply by 4.* $= 6 : 4 : 3$ **Note:** • The quantities to be compared have the **same unit** (in this case cups) • A ratio has **no units**
2) Palesa receives R 90 pocket money per month. The ratio of the pocket money received by Palesa and Thabo is 3 : 4. How much does Thabo receive per month?	Let Thabo receive Rx. P : T = 3 : 4 = 90 : x $\frac{3}{4} = \frac{90}{x}$ $\frac{3 \times 4x}{4} = \frac{90 \times 4x}{x}$ (The LCM of 4 and x is $4x$) $3x = 4 \times 90 = 360$ $x = \frac{360}{3}$ = R 120 Thabo receives R 120.

Exercise 3.2

1) Mortar (for bricklaying) is made by mixing 3 wheelbarrows of cement with 2 wheelbarrows of sand and 1 wheelbarrow of water.

a) Find the ratio of cement : sand : water ………………………………

b) How much sand will I need to order to mix with 9 wheelbarrows of cement?

………………………………………………………………………………………

c) How much cement and water will I need to order to mix with 8 wheelbarrows of sand?

………………………………………………………………………………………

d) How much cement and sand will I need to order to mix with 6 wheelbarrows of water?

………………………………………………………………………………………..

2) John, who is 16 years old and Thandi, who is 11 years old, receive pocket money in the ratio of their ages. John is given R 80 per month. How much is Thandi given?

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

§ 3.3 DIVIDING A QUANTITY IN A GIVEN RATIO

✧ To divide something in a given ratio, find
  a) The total number of parts from the ratio
  b) What one part is

EXAMPLE
Divide 66 sweets between Jim and Busi in the ratio of 6 : 5.
SOLUTION
Suppose that the sweets are divided into 6 + 5 = 11 **equal shares**. Jim gets $\frac{6}{11} \times 66 = 36$ sweets. Busi gets $\frac{5}{11} \times 66 = 30$ sweets. **Note:** • As a calculation check, 36 + 30 = 66 sweets

Exercise 3.3

1) Divide R650 between Susan and James in the ratio of 7 : 3.

2) The perimeter of a triangle is 612 cm. The sides of the triangle are in the ratio 3 : 4 : 5. What is the length of the longest side?

3) A green paint is mixed from blue and yellow paint in the ration 3 : 5. How much of each colour is needed to make 40 litres of this green paint?

§ 3.4 RATE

- A rate links two quantities. It tells us how one quantity compares or changes with another.
- The unit of a rate is usually given as "somethings" per "something" e.g. kilometres per hour, cents per litre, revolutions per minute.
- "Per" means "for each" or "for every". It is often shortened to ***p*** or **/**.

EXAMPLE	SOLUTION
1) A box of 15 naartjies costs R16,65. A bag of 8 naartjies of the same size costs R9,10. Which is the best deal?	BOX: 15 naartjies cost R16,65 so 1 naartjie costs R$\frac{16,65}{15}$ = R1,11 BAG: 8 naartjies cost R9,10 so 1 naartjie costs R$\frac{9,10}{8}$ = R1,14 The bag is the best deal. The larger quantity is more expensive. ***Note:*** *In each case you calculated the price per 1 naartjie or the unit price, which is a rate.*
2) A motorist travels 56 km in 29 minutes. What is his average speed in km/hour?	In 29 minutes, 56 km is travelled. So in 1 minute, 56 $\div$ 29 = 1,931….km are travelled *(do not round off)* In 60 minutes 60 $\times$ 2,115….= 115,862… km are travelled. The average speed $\approx$ 116 km per hour. **Note: Average speed =** $\frac{\textbf{distance}}{\textbf{time}}$

Exercise 3.4

1) A 500g tin of jam is marked R15,98. The 300g tin of the same jam costs R9,50. Which size is the better buy?

2) A car travels at a speed of 120 km per hour. How long will a trip of 2 520 km take if he travels at the same speed all the way?

3) My car has a petrol consumption of 8,5 litres per 100 km.

a) How many litres of petrol will I need to travel 1 km?

b) How many litres of petrol will I need to travel 420 km?

c) How far, to the nearest km, can I travel on 20 litres?

§ 3.5 DIRECT (or linear) PROPORTION

- When two ratios are equal to each other, we say that they are in proportion.
- When quantities are in **direct proportion**, one quantity increases in the same ratio as the other quantity increases, **or** one quantity decreases in the same ratio as the other quantity decreases.

EXAMPLE	SOLUTION
1) The price of a sweet is R3,45. What is the price of two sweets?	Two sweets will cost $2 \times$ R3,45 = R6,90. **Note:** The number of sweets and the money both doubled – they **increased** in the same ratio, so they are in **direct (linear) proportion**
2) Andile can run 5 km in 20 minutes. How far can he run at the same speed in 15 minutes?	In 20 minutes Andile runs 5 km In 1 minute (less time) he runs $\frac{5}{20}$ km (less distance) In 15 minutes (more time) he runs $\frac{5}{20} \times 15 = 3{,}75$ km ….. (more km) **Note:** The distance (km) **decreased** in the same ratio as the time (minutes) decreased, so the distance and the time are in **direct (linear) proportion**

Exercise 3.5

1) Ben's computer can print 1 800 words in 5 minutes. Working at the same rate, how many words can he print in

a) 1 minute? ………………………………………………………………………..

………………………………………………………………………..

b) 7 minutes? ………………………………………………………………………..

……………………………………………………………………….

2) My diesel motorcar has a petrol consumption of 6,5 litres per 100 km.

a) How many litres of petrol will I need to travel 1 km? …………………………….…

………………………………………………………………………………………..

b) How many litres of petrol will I need to travel 350 km? ……………………………….

………………………………………………………………………………………..

c) How far can I travel on 10 litres (correct to 1 decimal place)? …………………………

………………………………………………………………………………………..

3) A factory produces 600 washers every 3 minutes. How many washers will it produce in 8 minutes? ………………………………………………………………………………..

……………………………………………………………………………………………

…………………………………………………………………………………………...

§ 3.6 INDIRECT (inverse) PROPORTION

✧ When quantities are in **indirect proportion**, one quantity decreases in the same ratio as the other quantity increases.

EXAMPLE	SOLUTION
Betty has enough sandwiches to give 5 children, 4 sandwiches each. However, 10 children arrive. How many sandwiches will each child receive, if they are shared equally?	The total number of sandwiches available is $5 \times 4 = 20$. The number of sandwiches per child will be $20 \div 10 = 2$. **<u>Note:</u>** The number of children has doubled and the number of sandwiches has halved, so the number of children and the number of sandwiches are in **indirect (inverse) proportion.**

<u>Exercise 3.6</u>

1) 3 campers have enough food to last them for 15 days. 2 others join them. How many days will they take to finish the food if all eat the same quantity?

2) Seven builders complete a building project in 4 weeks.
   a) How long will 14 builders, working at the same rate, take to complete the project?

   b) How long will 2 builders, working at the same rate, take to complete the project?

3) It is estimated that a group of 6 plasterers can plaster a house in 9 days. Only 4 arrive for work. How long will it take these plasterers to finish the job?

§ 3.7 PERCENTAGES

✧ "Percent" means "per hundred", and is written %. We can convert a **percentage to a fraction** or a **percentage to a decimal number.**

EXAMPLE	SOLUTION
1) Convert 6% to a fraction	6% means 6 equal parts per 100 equal parts, so $6\% = \frac{6}{100} = \frac{3}{50}$.
2) Convert to a decimal number a) 9% b) 4,5%	 9% means $\frac{9}{100}$ or $9 \div 100 = 0{,}09$. $4{,}5\% = \frac{4{,}5}{100} = \frac{45}{1000} = 0{,}045$

✧ It is useful to know the following percentage to decimal conversions off by heart:

1% = 0,01	10% = 0,1	25% = 0,25	100% = 1
5% = 0,05	50% = 0,5	75% = 0,75	12,5% = 0,125

✧ We can also convert any **fraction to a percentage,** even those whose denominators do not divide exactly into 100.

<table>
<tr><th>EXAMPLE</th><th colspan="3">SOLUTION</th></tr>
<tr><td>1) Express $\frac{1}{4}$ as a percentage</td><td colspan="3">$\frac{1}{4} = \frac{1}{4} \times \frac{25}{25} = \frac{25}{100} = 25\%$
This was easy since 100 is exactly divisible by 4</td></tr>
<tr><td rowspan="2">2) Use a calculator to write the fraction $\frac{15}{43}$ as a percentage, rounded to 1 decimal place.</td><th>Key Sequence</th><th>Display</th><th>Answer</th></tr>
<tr><td>$\boxed{\frac{\blacksquare}{\square}}$ 15 $\downarrow$ 43 $\boxed{\%}$ $\boxed{=}$
or
15 $\boxed{\div}$ 43 $\boxed{\times}$ 100 $\boxed{=}$</td><td>38.888……</td><td>38,9 %</td></tr>
</table>

<u>Exercise 3.7</u>

1) Convert 24% to a fraction …………………………………………………………….

2) Convert 46,5% to a fraction …………………………………………………………..

3) Convert 45% to a decimal …………………………………………………………….

4) Convert 32,9% to a decimal …………………………………………………………..

5) Convert $\frac{3}{25}$ to a percentage …………………………………………………………..

6) Convert $\frac{34}{77}$ to a percentage correct to 2 decimal places …………………………………..

7) Convert $2\frac{5}{9}$ to a percentage correct to 1 decimal place ……………………………………

§ 3.8 EXPONENTS

- In 2^3, 2 is the **base** and 3 is the **exponent**
- $2^4 = 2\times2\times2\times2$. There are **4 factors**. We have **expanded** 2^4.
- If we **contract** $3\times3\times3 \ldots$ 7 factors, we have 3^7.
- **Multiplication:**
  $2^3\times2^4=(2\times2\times2)\times(2\times2\times2\times2)=2^7$. But 3 + 4 = 7. We can **add the exponents** rather than expand, provided the bases are the same.
  Note:
  - $4 = 4^1$

EXAMPLE	SOLUTION
Simplify 1) $2^3 \times 3^2 \times 2^4 \times 3$	$2^3 \times 3^2 \times 2^4 \times 3^1 = 2^{3+4} \times 3^{2+1} = 2^7 \times 3^3$
2) $a^3 \times a^4$	$a^3\times a^4 = a^{3+4} = a^7$
3) $2^b\times2^{3b}\times2^4\times2$	$2^b\times2^{3b}\times2^4\times2 = 2^{b+3b+4+1} = 2^{4b+5}$
4) $x^{-4}\times x^5\times x^0\times x$	$x^{-4}\times x^5\times x^0\times x = x^{-4+5+0+1} = x^2$

- **Division:**
  You know that $\dfrac{2^5}{2^3}=\dfrac{2\times2\times2\times2\times2}{2\times2\times2}=2^2$. But 5 – 3 = 2. We can **subtract the exponents** rather than expand, provided the bases are the same.

EXAMPLE	SOLUTION		EXAMPLE	SOLUTION
1) $\dfrac{5^8}{5^6}$	$\dfrac{5^8}{5^6} = 5^{8-6} = 5^2$		2) $\dfrac{a^6}{a^2}$	$\dfrac{a^6}{a^2} = a^{6-2} = a^4$
3) $\dfrac{2^3\times2^8}{2^4\times2}$	$\dfrac{2^3\times2^8}{2^4\times2} = 2^{3+8-4-1} = 2^6$		4) $\dfrac{2^3}{2^5}$	$\dfrac{2^3}{2^5} = \dfrac{1}{2^{5-3}} = \dfrac{1}{2^2}$
5) $9x^3 \div 3x^2 \times x$	$9x^3 \div 3x^2 \times x$ $= 3x^{3-2+1} = 3x^2$		6) $\dfrac{d^3c^4e}{cd^2e^2}$	$\dfrac{d^3c^4e}{cd^2e^2} = \dfrac{d^{3-2}c^{4-1}}{e^{2-1}} = \dfrac{dc^3}{e}$

Exercise 3.8
Simplify:

1) $5^2\times5^4\div5^3$ = ……………………………………………………………………………..
2) $2c^2 \times 4c^4 \div 8c^5$ = ……………………………………………………………………………..
3) $\dfrac{4^2\times4^5}{4^8}$ = ……………………………………………………………………………
4) $a^5\times\dfrac{a^2}{a^4}\div a$ = ……………………………………………………………………………..
5) $\dfrac{2^4}{3^2}\times\dfrac{2}{2^3}\times3$ = ……………………………………………………………………………
6) $3^2 \times 3^4 \div 3^3 \times 3^6 \times 3 \div 3^7$ = ……………………………………………………………

§ 3.9 ZERO EXPONENTS AND NEGATIVE EXPONENTS

- You know that $\frac{4^2}{4^2} = 4^{2-2} = 4^0$. But $\frac{4^2}{4^2} = 1$, since any non-zero number divided by itself is 1, so $4^0 = 1$.

  Note:
  - 0^0 is **undefined**. The base of a zero exponent may not be zero
  - Any number other than zero raised to the power of zero is 1
  - Zero raised to any other power is zero, e.g. $0^3 = 0$

- $\frac{2^4}{2^7} = \frac{2\times2\times2\times2}{2\times2\times2\times2\times2\times2\times2} = \frac{1}{2\times2\times2} = \frac{1}{2^3}$. But $\frac{2^4}{2^7} = 2^{4-7} = 2^{-3}$. So $2^{-3} = \frac{1}{2^3}$

  In this way we can **convert negative exponents to positive exponents**.

EXAMPLE	SOLUTION		EXAMPLE	SOLUTION
Write with positive exponents: 1) 3^{-2}	$3^{-2} = \frac{1}{3^2}$		2) x^{-3}	$x^{-3} = \frac{1}{x^3}$
3) p^{-a}	$p^{-a} = \frac{1}{p^a}$		4) $(-4)^{-2}$	$(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$
5) $\frac{x^2}{x^5}$	$\frac{x^2}{x^5} = x^{-3} = \frac{1}{x^3}$		6) $\frac{1}{c^{-1}}$	$\frac{1}{c^{-1}} = 1 \div \frac{1}{c} = 1 \times \frac{c}{1} = c$

Exercise 3.9

Simplify, giving your answers with positive exponents:

1) $0^3 =$ ……………………..
2) $(-7)^0 =$ ………………..
3) $\frac{3^2}{3^6} =$ …………………
4) $\frac{c^3\times c}{c^2\times c^6} =$ ……………
5) $\frac{1}{x^{-4}} =$ …………………
6) $-3^0 =$ …………………
7) $\left(\frac{3}{4}\right)^{-1} =$ ……………….
8) $(-3)^{-2} =$ ……………….
9) $-(3)^{-2} =$ …………….
10) $45^{a-a} =$ ……………….
11) $\frac{1}{3^{-2}} =$ …………………..
12) $\frac{1}{x^{-1}} + \frac{1}{y^{-1}} =$ …………
13) $(a+b)^{-1} =$ …………….
14) $\frac{2^3}{2^3} =$ …………………..
15) $\frac{a^2b^3c}{a^4b^2c^0} =$ ……………
16) $4^{-3} \times 4^3 =$ …………..
17) $5^{-4} \times 5^2 \times 5 =$ ………..
18) $\frac{2^{-3}\times2^2}{2^{-4}} =$ …………….

§ 3.10 PRUDUCTS AND QUOTIENTS RAISED TO A POWER

✧ You know that $(5\times3)^2=(5\times3)\times(5\times3)=5^2\times3^2$.
In the same way, $(a\times b)^3=(a\times b)\times(a\times b)\times(a\times b)=a^3\times b^3$

✧ Also $\left(\frac{2}{3}\right)^4=\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)=\frac{2^4}{3^4}$

EXAMPLE	SOLUTION
1) Write without brackets a) $(3.4)^2$	$(3.4)^2=3^2.4^2$
b) $(x\times y)^3$	$(x\times y)^3=x^3\times y^3$
c) $(x^2y^3z)^4$	$(x^2y^3z^1)^4=x^{2\times4}y^{3\times4}z^{1\times4}=x^8y^{12}z^4$
d) $\left(\frac{x}{y}\right)^3$	$\left(\frac{x}{y}\right)^3=\frac{x^3}{y^3}$
e) $\left(\frac{2x}{3y}\right)^2$	$\left(\frac{2x}{3y}\right)^2=\frac{2^2x^2}{3^2y^2}=\frac{4x^2}{9y^2}$
2) Write in brackets a) $a^2\times b^2$	$a^2\times b^2=(ab)^2$
b) $3^4\times2^4$	$3^4\times2^4=(3\times2)^4$
c) $(x+y)\times(x+y)$	$(x+y)\times(x+y)=(x+y)^2$
d) $a^6\times b^4\times c^8$	$a^6\times b^4\times c^8=(a^3\,b^2\,c^4)^2$
e) $\left(\frac{2}{3}\right)^3\times\left(\frac{3}{4}\right)^3$	$\left(\frac{2}{3}\right)^3\times\left(\frac{3}{4}\right)^3=\left(\frac{2\times3}{3\times4}\right)^3=\left(\frac{1}{2}\right)^3$
f) $\left(\frac{3a^2}{2b}\right)^2\times\left(\frac{a^3}{b}\right)^2$	$\left(\frac{3a^2}{2b}\right)^2\times\left(\frac{a^3}{b}\right)^2=\left(\frac{3a^2}{2b}\times\frac{a^3}{b}\right)^2=\left(\frac{3a^5}{2b^2}\right)^2$

Exercise 3.10

1) Remove the brackets and simplify where possible:

a) $(4+5)^2=$ ………………………………… **b)** $(2\times3)^2=$ ………………………………..

c) $(a^3\,b^2\,c)^3=$ ……………………………... **d)** $(p\times q)^3=$ ………………………………..

e) $(2p^4\times6q^2)^2=$ ………………………… **f)** $-(-4a^3b)^2=$ ……………………………

g) $\left(\frac{3y^2}{4x^3}\right)^3=$ ……………………………….. **h)** $\frac{(5a^2)^2}{5a^2}=$ ………………………………

i) $\left(\frac{2x^2}{4x^3}\right)^2=$ ……………………………….. **j)** $\left(\frac{10z^4}{5z^2}\right)^3=$ ………………………………

2) Contract to 1 bracket and simplify where possible:

a) $2^3\times5^3=$ ……………………………… **b)** $2^4\times5^4\times5^4\times2^4=$ …………………..

c) $\left(\frac{2}{3}\right)^4\times\left(\frac{3}{2}\right)^4=$ ………………………… **d)** $\left(\frac{25}{7}\right)^2\times\left(\frac{7^2}{25^2}\right)^2=$ ……………………

§ 3.11 POWER OF AN EXPONENT

- From the exponent definition, $\left(3^2\right)^4 = (3\times3)\times(3\times3)\times(3\times3)\times(3\times3) = 3^8$

  As a shortcut, we can **multiply the exponents**, keeping the existing base.

  Note:

  - $3^2\times3^4 = 3^6$, the **bases are both 3**, so we **add the exponents, but**

    $\left(3^2\right)^4 = 3^8$, the **base of the 2 is 3** and the **base of the 4 is 3 squared**, so the bases are different and we **cannot add** the exponents.

  - $\left(x\times y\right)^2 = x^2\times y^2$ So $(2 \times 3)^2 = 6^2 = 36 = 4 \times 9 = 2^2 \times 3^2$

    But $\left(x+y\right)^2 \neq x^2 + y^2$ So $(2 + 3)^2 = 5^2 = 25$, but $2^2 + 3^2 = 4 + 9 = 13$

- We can factorise the ***base*** of an exponent and write is as the ***product of prime factors***
  For example: $6^3 = (2 \times 3)^3 = 2^3 \times 3^3$ Check: $6^3 = 216$ and $2^3 \times 3^3 = 8 \times 27 = 216$

EXAMPLE	**SOLUTION**
Remove the brackets and simplify: 1) $\left(2^4\right)^3$	$\left(2^4\right)^3 = 2^{4\times3} = 2^{12}$
2) $\left(x^2\right)^3$	$\left(x^2\right)^3 = x^{2\times3} = x^6$
3) $x^2 \times x^3$	$x^2 \times x^3 = x^5$
4) $\left(a^b\right)^c$	$\left(a^b\right)^c = a^{b\times c} = a^{bc}$
5) $\left(\dfrac{2^4}{3^2}\right)^5$	$\left(\dfrac{2^4}{3^2}\right)^5 = \dfrac{2^{4\times5}}{3^{2\times5}} = \dfrac{2^{20}}{3^{10}}$
6) $\left(\dfrac{3a^2}{5b^3}\right)^3$	$\left(\dfrac{3a^2}{5b^3}\right)^3 = \dfrac{3^3a^{2\times3}}{5^3b^{3\times3}} = \dfrac{27a^6}{125b^9}$
7) $\dfrac{3^2\times2}{6^2}$	$\dfrac{3^2\times2}{6^2} = \dfrac{3^2\times2}{3^2\times2^2} = \dfrac{1}{2}$

Exercise 3.11

Simplify the following. Where necessary write the base as the product of prime factors.

1) $\left(3x^5\right)^2$ = …………….. **2)** $\left(s^2\times t^5\right)^3$ = ………… **3c)** $(2^2 + 3^2)^2$ = ……………

4) $\left(8^0\right)^5$ = ……………… **5)** $(-4^3)^2$ = ……………… **6)** $\left(\left(2^2\right)^2\right)^3$ = ………………

7) $\left(\dfrac{3a^2}{2b}\right)^4$ = …………….. **8)** $(4x - x)^3$ = …………… **9)** $\left(\dfrac{5^2b^4}{5b^5}\right)^3$ = ………………

10) $\dfrac{12^2}{8^2}$ = ………………………………………………………………………………..

11) $\dfrac{6^3}{27\times4^2}$ = ………………………………………………………………………………..

12) $\dfrac{18^2\times2}{4^2\times3^3}$ = ……………………………………………………………………………….

§ 3.12 NUMBER SEQUENCES

- Look at the number sequence: 1; 4; 7; 10; ….. ; ……. ; …
  Each number MINUS the preceding number gives three: $4 - 1 = 3$, $7 - 4 = 3$, $10 - 7 = 3$, OR, each number PLUS 3 gives us the next number in the sequence.
- Sequences like the one above, where adding the same quantity each time gives us the terms of the sequence, are called **arithmetic sequences**.

EXAMPLE	SOLUTION
1) Is the following an arithmetic sequence: 13; 8; 3; –2; –7; … ?	To find out whether this is an arithmetic sequence, calculate ***Term 2 – Term 1*** and then calculate ***Term 3 – Term 2***. If both answers are the same, the sequence is an arithmetic sequence. Term 2 – Term 1 = 8 – 13 = –5 Term 3 – Term 2 = 3 – 8 = –5 So this is an arithmetic sequence.
2) If so, write down the next three numbers in the sequence.	We subtract 5 in order to find the next three terms in the sequence. –7 – 5 = – 12; –12 – 5 = –17 and – 17 – 5 = –22 So the next three terms are –12; –17 and –22.

<u>Exercise 3.12</u>

Determine whether the following are arithmetic sequences. If they are, write down the next three numbers in the sequence.

1) 2; 7; 12; ………………………………………………………………………………

………………………………………………………………………………

2) 3; 6; 12; ………………………………………………………………………………

………………………………………………………………………………

3) 0; -4; -8; ………………………………………………………………………………

………………………………………………………………………………

4) 1,2; 2,3; 3,4; ………………………………………………………………………

………………………………………………………………………………

5) $\frac{1}{2}; \frac{1}{4}; 0;$ ………………………………………………………………………

………………………………………………………………………………

6) 1; 4; 9; 16; ………………………………………………………………………

………………………………………………………………………………

§ 3.13 TERMS OF A SEQUENCE

- Each number in a sequence is called a **term.** T_1 is the first term of the sequence, T_2 is the second term and so on.
  In the sequence 1; 4; 7; 10; …….. $\mathbf{T_1} = 1$, $\mathbf{T_2} = 4$, $\mathbf{T_3} = 7$ and $\mathbf{T_4} = 10$.
  The dots show that there are an infinite number of terms.
- Sometimes a general term (T_x) of a sequence is given. You have to use it to find a specific term or the number of a term.

EXAMPLE

1) Find the 1st, 2nd, 4th, 8th and 12th terms of a sequence with the general term $T_x = 3x - 1$

Term	T_1	T_2	T_4		T_8		T_{12}
$\mathbf{T_x = 3x - 1}$				17		29	

2) Which terms have the values 17 and 29?
3) Fill the missing values on the table

SOLUTION

1) $T_1 = 3(1) - 1 = 2$ $T_2 = 3(2) - 1 = 5$ $T_4 = 3(4) - 1 = 11$ $T_8 = 3(8) - 1 = 23$ $T_{12} = 3(12) - 1 = 35$

2)

$$\begin{aligned} T_x &= 3x - 1 \\ 17 &= 3x - 1 \\ 17 + 1 &= 3x - 1 + 1 \\ 18 &= 3x \\ \frac{18}{3} &= \frac{3x}{3} \\ \mathbf{6} &= x \end{aligned}$$

$$\begin{aligned} T_x &= 3x - 1 \\ 29 &= 3x - 1 \\ 29 + 1 &= 3x - 1 + 1 \\ 30 &= 3x \\ \frac{30}{3} &= \frac{3x}{3} \\ \mathbf{10} &= x \end{aligned}$$

3)

Term	T_1	T_2	T_4	$\mathbf{T_6}$	T_8	$\mathbf{T_{10}}$	T_{12}
$\mathbf{T_x = 3x - 1}$	**2**	**5**	**11**	17	**23**	29	**35**

Exercise 3.13

Complete the following two tables:

1)

n	1	2		7	10	
$\mathbf{T_n = 2n + 3}$			13			27

2)

x	1		4	5		10
$\mathbf{T_x = 3x - 4}$		5			20	

§ 3.14 THE SET OF RATIONAL NUMBERS

- The letter that stands for **the set of rational numbers** is ***Q***.
- We define rational numbers in the following way:

All rational number can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Things to remember:

- Division of a number by zero is undefined in any number system
- Every rational number can be written as **an integer divided by a non-zero integer.**
- If you can't write the number in this form, it is not rational.

EXAMPLE

If possible, write each of the following numbers in the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$. Hence say whether the number is rational or not.

Number	Write in the form $\frac{a}{b}$	$a, b \in Z, b \neq 0$ yes/no	Rational yes/no
1) $2\frac{3}{4}$	$2\frac{3}{4} = \frac{11}{4}$	Yes	Yes
2) 1,5	$1,5 = \frac{15}{10} = \frac{3}{2}$	Yes	Yes
3) $-\frac{12}{5}$	$-\frac{12}{5} = \frac{-12}{5}$ or $\frac{12}{-5}$	Yes	Yes
4) $\sqrt{25}$	$\sqrt{25} = 5 = \frac{5}{1}$	Yes	Yes
5) $\sqrt[3]{-27}$	$\sqrt[3]{-27} = -3 = \frac{-3}{1} = \frac{3}{-1}$	Yes	Yes
6) -7	$-7 = \frac{-7}{1}$	Yes	Yes

Exercise 3.14

Use the table to help you decide whether the following numbers are rational numbers.

Number	Write number in the form $\frac{a}{b}$	*Are $a, b \in Z, b \neq 0$?*	Rational?
1) $-2,5$			
2) $\frac{22}{7}$			
3) $-\sqrt{49}$			
4) 1			
5) 0,05			
6) $4\frac{2}{3}$			
7) 0			
8) -5^3			
9) $\sqrt{8}$			

§ 3.15 IRRATIONAL NUMBERS

- Real numbers that are not rational are called **irrational**.
- We use the rational number definition to decide whether numbers are rational or irrational.

EXAMPLE

Decide whether the following numbers are rational or irrational:

Number	Test for a rational number	Rational / Irrational
1) $\sqrt{49}$	$\sqrt{49} = 7 = \frac{7}{1}$	Rational
2) $\sqrt{7}$	$\sqrt{7} = \frac{\sqrt{7}}{1}$ **but** $\sqrt{7}$ is not an integer	Irrational
3) $\sqrt[3]{64}$	$\sqrt[3]{64} = 4 = \frac{4}{1}$	Rational

- Other examples of irrational numbers are $\sqrt{3}$, $\sqrt{11}$, $\sqrt[3]{24}$ etc. They are the square roots of non-perfect squares or the cube roots of non-perfect cubes. We call these irrational numbers are **surds**.

  Find $\sqrt{5}$ on your calculator. $\sqrt{5}$ = 2,236 067 978 ……..
  On your calculator, the digits after the decimal comma seem to come to an end.
  This is because your screen can only hold a certain number of digits.
  In actual fact, these digits never end and do not recur.
  These surds are irrational numbers.

Note:

- π = 3,141 592 654 ……… is irrational and is a non-terminating, non-recurring decimal.
- $\frac{22}{7}$ is an approximation for π, and is rational. π = 3,142 857 142 … and is a recurring decimal.

Exercise 3.15

Are the following numbers rational or irrational?

Number	Test for a rational number	Rational / irrational
1) $9,\dot{8}$		
2) $5\sqrt{2}$		
3) $\sqrt{4}$		
4) 7,148 148 148 ...		
5) $1,\overline{54}$		
6) 2,14		
7) π		
8) $\frac{22}{7}$		
9) $\sqrt[4]{16}$		

§ 3.16 SURDS

- Each irrational number lies between two rational numbers, and so can be shown approximately on a number line.
  You know that 16 < 18 < 25. By taking the square root of these numbers, $4 < \sqrt{18} < 5$.
  Check this by finding $\sqrt{18}$ on your calculator. $\sqrt{18} = 4{,}242\ 640$…….

EXAMPLE	SOLUTION
1) Simplify a) $\sqrt{20}$	$\sqrt{20} = \sqrt{5\times4} = 2\sqrt{5}$
b) $(\sqrt{7})^3$	$(\sqrt{7})^3 = \sqrt{7} \times \sqrt{7} \times \sqrt{7} = 7\sqrt{7}$
c) $\sqrt{1{,}21}$	$\sqrt{1{,}21} = \sqrt{1{,}1\times1{,}1} = 1{,}1$
2) Simplify a) $\sqrt{7}+\sqrt{7}$	$\sqrt{7}+\sqrt{7} = 2\sqrt{7}$
b) $3\sqrt{2} - 2\sqrt{2}$	$3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$
c) $\sqrt{7} \times \sqrt{7}$	$\sqrt{7} \times \sqrt{7} = \sqrt{49} = 7$
d) $\sqrt{2} \times \sqrt{3}$	$\sqrt{2} \times \sqrt{3} = \sqrt{6}$
e) $-2\sqrt{5} \times 3\sqrt{5}$	$-2\sqrt{5} \times 3\sqrt{5} = -6 \times 5 = -30$
f) $\sqrt{6} \times \sqrt{12}$	$\sqrt{6} \times \sqrt{12} = \sqrt{6} \times \sqrt{6\times2} = 6\sqrt{2}$
g) $\frac{\sqrt{8}}{\sqrt{32}}$	$\frac{\sqrt{8}}{\sqrt{32}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

<u>Rationalisation of the denominator:</u>

- If the denominator of a faction is a surd, we can make the denominator a rational number.
  <u>Example:</u>
  $\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ *Multiply numerator and denominator by the same surd*
  $= \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

<u>Exercise 3.16</u>

1) Simplify:

a) $\sqrt{8} \times \sqrt{8} =$ …………………………… **b)** $2\sqrt{3} \times 3\sqrt{2} =$ ………………………

c) $4\sqrt{3} - \sqrt{3} =$ …………………………… **d)** $3\sqrt{5} + 4\sqrt{5} =$ …………………………

e) $4\sqrt{3} \times 3\sqrt{3} =$ ………………………… **f)** $\sqrt{6} \times \sqrt{27} =$ …………………………

g) $\frac{\sqrt{18}}{6\sqrt{2}} =$ ………………………………….. **h)** $\sqrt{625} \div \sqrt[3]{125} =$ ……………………..

2) Rationalise the denominators:

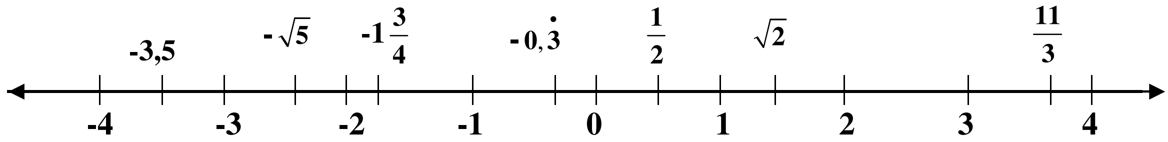
a) $\frac{6}{\sqrt{3}} =$ …………………………………… **b)** $\frac{14}{\sqrt{7}} =$ ……………………………………

3) Between which two integers does $\sqrt{7}$ lie? ………………………………………………..

4) Between which two integers does $\sqrt{3}$ lie? ………………………………………………..

§ 3.17 THE SET OF REAL NUMBERS

- All the rational numbers together with all the irrational numbers form the set of **Real numbers, *R***. Here are some of them on a number line.



Note:

- $\sqrt{-4}$ is not a real number, since $2 \times 2 = 4$ and $-2 \times -2 = 4$. It is a **complex number**.

EXAMPLE

On the table, mark with an X the number system/s to which each number belongs:

SOLUTION

Number	***Natural Number***	***Whole Number***	***Integer***	***Rational Number***	***Irrational Number***	***Real Number***
1) -56			×	×		×
2) 0		×	×	×		×
3) $13\frac{1}{8}$				×		×
4) $\sqrt{5}$					×	×
5) $\sqrt[3]{27}$	×	×	×	×		×
6) $-\frac{12}{5}$				×		×
7) π					×	×
8) $\frac{22}{7}$				×		×

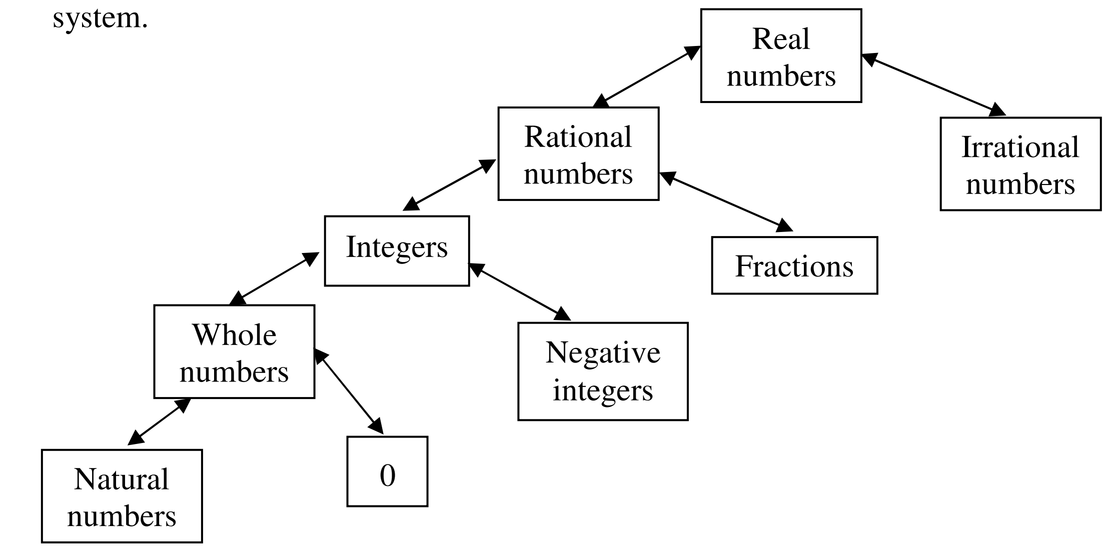
Exercise 3.17

On the table, mark with an X the number system/s to which each number belongs:

Number	***Natural Number***	***Whole Number***	***Integer***	***Rational Number***	***Irrational Number***	***Real Number***
1) $19,\dot{2}$						
2) π						
3) $-3\frac{2}{3}$						
4) $\sqrt{2}$						
5) $\frac{22}{7}$						
6) 1,148796…						
7) $\sqrt[4]{16}$						
8) 15,78						
9) 0						
10) $\sqrt{144}$						

§ 3.18 THE REAL NUMBER DIAGRAM

✧ Here is a diagram to help you remember what you have learned about the real number system.



Exercise 3.18

Use the number sets diagram to help you answer these questions:

Statement	True (T) / False (F)
1) $\sqrt{25}$ is rational.	
2) $\sqrt{13}$ is rational.	
3) Any recurring decimal number is rational.	
4) $\sqrt[3]{-27}$ is irrational	
5) $4,56 = 4\frac{56}{99}$	
6) $0,\overline{6} = 0,6$	
7) $\sqrt[3]{4}$ is rational because 4 is a perfect square	
8) All surds are rational	
9) $3,11111.... = 3,\dot{1}$	
10) $8,2 = 8\frac{2}{10}$	
11) All rational numbers are real numbers	
12) All real numbers are rational numbers	
13) All whole numbers are rational numbers	
14) All irrational numbers are real numbers.	
15) All real numbers are whole numbers.	
16) Zero is a real number	
17) Some real numbers are irrational	