LESSON 4

QUADRATIC EQUATIONS WITH FRACTIONAL DENOMINATORS

Learning Outcomes and Assessment Standards

Learning Outcomes 2: Functions and algebra
Assessment Standard
Solve quadratic equations by factorising.

Overview

In this lesson you will:

- Factorise the quadratic trinomial to solve the equation.
- Learn when equations are not defined.
- Use mathematical modelling – use algebra to prove conjectures.

Lesson

If $a \cdot b = 0$

Then $a = 0$ or $b = 0$

Note: For this to be true, the product must equal zero. Note that only one of the factors need to be zero, to make the product zero, since zero times anything is zero.

Rules for solving quadratic equations

- If there are denominators , factorise each denominator and find the LCD.
- Write down the LCD and make note of the restrictions. (No denominator can be zero because division by zero is undefined.)
- Multiply each term by the LCD.
- Make the quadratic equation equal to zero.
- Factorise and solve.

Example 1

Solve for x:

$x - 1 = \frac{6}{x}$

LCD $= x$ restriction: $x \neq 0$

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

Example 2

Solve for x:

$\frac{2x}{1-x} - \frac{x}{x-1} = \frac{2}{x+1}$ restrictions: $x \neq 1; x \neq -1$

$\therefore -2x(x + 1) - x(x + 1) = 2(x - 1)$

$\therefore -2x^2 - 2x - x^2 - x = 2x - 2$

$\therefore 3x^2 + 5x - 2 = 0$

$\therefore (3x - 1)(x + 2) = 0$

$x = \frac{1}{3}$ or $x = -2$

Example 3 (for you to practise)

$\frac{x}{x+1}-\frac{2x}{1-x}=\frac{x^2+3}{x^2-1}+\frac{9}{4}$

$\frac{x}{x+4}+\frac{2x}{x-1}=\frac{x^2+3}{(x-1)(x+1)}+\frac{9}{4}$ (change sign)

Restr: $x \neq \pm 1$ LCD $4(x-1)(x+1)$

$4x(x-1)+2x\cdot 4(x+1)=(x^2+3)4+9(x^2-1)$ (get rid of fraction)

$4x^2-4x+8x^2+8x=4x^2+12+9x^2-9$

$\therefore x^2-4x+3=0$ (standard form)

$\therefore (x-1)(x-3)=0$

$x = 1$ or $x = 3$

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since $x \neq 1$

The denominator may not be zero

Special results

If $5x+2=5x+3$

$0 = 5$ or $2 = 3$

This is not possible since 2 can never equal 3.

So there is no solution

But if $5x-2=5x-2$, we have many solutions. No matter what we make x, the equation holds true. This is referred to as an indetermined equation.

$0 = 0$. This is always true.

So $x \in \mathbb{R}$

In other words, *x* can be any real number

Example 4

$2-\frac{2x-3}{x-2}=\frac{1}{2-x}$

$\therefore 2-\frac{2x-3}{x-2}=\frac{-1}{x-2}$

Restr: $x \neq 2$ LCD $(x-2)$

$\therefore 2(x-2)-2x+3=-1$

$\therefore 2x-4-2x+4=0$

$0 = 0$

and $0 = 0$ is always true.

So an infinate number of values for x will satisfy this equation.

So $x \in \mathbb{R} \setminus \{2\}$

Activity 1

Solve for x:

1. $x(x + 4) = 21$
2. $x(x - 1) = 4(3x - 10)$
3. $(2x - 5)(3x + 2) = 2(3x - 1)$
4. $\frac{x+2}{x+1} - \frac{3}{x-2} = \frac{1}{x+1}$
5. $\frac{30}{x-2} - \frac{1}{2} = \frac{30}{x}$
6. $\frac{4x}{3x+12} - \frac{1}{2} = \frac{1}{2x-2}$
7. $\frac{21}{8(x-6)} - \frac{5}{8(x+2)} + 1 = 0$
8. $\frac{x+1}{x} - \frac{5x}{3x+3} = \frac{2}{3}$
9. $\frac{x+1}{x^2-4} + \frac{1-x}{x+2} = \frac{2}{5(x-2)}$
10. $\frac{2+\frac{1}{x}}{2} = \frac{3-\frac{1}{x}}{3+\frac{1}{x}}$

Solve for x if

a) x is an integer
b) is a rational number
c) x is an irrational number
d) x is a real number

Mathematical modelling

The use of algebra to prove conjectures

Example 1 (for you to practise)

Look at any five consecutive numbers

Let's take 1 2 3 4 5

Explore the difference between the product of the last two and the product of the first two (4)(5) – (1)(2) = 18

Investigate many more and what do you notice each time?

Now you will need to make a conjecture – this means you must state in words exactly what you have observed.

A conjecture is not a tested and proved fact, it is merely an inferred conclusion based on an observation.

"In a sequence of five consecutive numbers, the difference between the product of the largest two and the smallest two will always be six times the middle number."

Now we are going to use algebra to prove this conjecture.

Proof

Let the five consecutive numbers be $(x - 2)$; $(x - 1)$; x; $x + 1$; $x + 2$

$(x + 2)(x + 1) - (x - 2)(x - 1)$

$= x^2 + 3x + 2 - (x^2 - 3x + 2)$

$= x^2 + 3x + 2 - x^2 + 3x - 2$

$= 6x$

Some useful results you should know

$2n$ will always be an even number so $2n \pm 1$ will always be an odd number.

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Activity 2

1. Prove that the sum of two odd numbers is always even.
2. Prove that the difference between any two consecutive square numbers is an odd number.
3. a) Show that the difference between the 7^{th} square number and the 4^{th} square number is a multiple of 3.
   b) Show that the difference between the 10^{th} square number and the 6^{th} square number is a multiple of 4.
   c) Show that the difference between the 12^{th} square number and the 7^{th} square umber is a multiple of 5.
   d) Make a conjecture.
   e) Prove your conjecture.